ARITHMETIC

BY THE SAME AUTHOR.

ARITHMETIC

FOR THE

USE OF SCHOOLS AND COLLEGES.

BY

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PREFACE TO REVISED EDITION.

Since the book was first published over forty years ago, there have naturally been some changes in the ideas of teaching Arithmetic. In revising this book I have steadily kept these in view and have tried to embody them here as far as possible.

The careful reader will observe many additions and alterations. Among others the following may be mentioned:

1. Stress has been laid on oral work.

2. Graphical illustrations have been given where necessary, specially in the section on Fractions.

3. A new section on Decimalization of Money has been inserted.

4. The sections on Approximation and Contracted Methods have been entirely re-written and considerably enlarged and the latest methods have been given.

5. A large number of new and interesting examples mainly taken from University and Public Examination Papers has been inserted.

These improvements will, it is hoped, increase the usefulness of the book, and render it still more worthy of the approbation which it has hitherto received.

I shall be grateful for information of any errors and misprints.

CALCUTTA,
January, 1933.

P. C. C.

PREFACE TO REVISED THIRD EDITION.

A new chapter on Graphs of Statistics has been added to increase the usefulness of the book.

CALCUTTA,
October, 1934.

A. C. C.

PREFACE TO THE FIRST EDITION.

THIS work has been written with the view of providing a book for class use in our Schools and Colleges, which shall suit the capacities of the young beginner and at the same time meet the requirements of the advanced student.

So far as has been possible within the necessary limits of the book, I have carefully avoided laying down arbitrary *rules* and have endeavoured to establish the leading propositions of the science of Arithmetic by a process of simple reasoning, being fully convinced that a mere mechanical facility in manipulating figures, sufficient though it may be for the calculations necessary in every-day life, is in no way conducive to a healthy development of the reasoning faculty. I have accordingly explained the processes of Arithmetic by means of specimen examples fully worked out, and in every division of the subject I have begun with simple principles and have tried to proceed by gradual and natural steps to those of a more complex nature.

Compound quantities have been assigned a somewhat earlier place than is usually given them; in other parts of the subject however there is but little departure from the common order. Decimals have been treated as a natural extension of the common system of notation; but the principles of vulgar fractions have been made use of here and there for purposes of explanation. The method adopted for the addition and subtraction of Recurring Decimals requires no conversion and re-conversion to vulgar fractions. A little more space than usual has been devoted to the subject of Problems, and I venture to hope that I have been able to make it simpler and more attractive by means of careful arrangement and classification. Although I have adopted the Unitary Method (a method so simple in its application and so suitable for young learners) in the section on

Problems, I have not abandoned the Rule of Three as some writers have done, because I do not consider it to be a misleading process, if properly understood. The sections on Stocks and other branches of Commercial Arithmetic I have tried to make in some degree complete. And I may add that although the book contains nothing that might strictly be called original, yet it will be found to differ in many ways from any existing text-book on the subject.

The book contains a large number of examples for exercise ; these have been worked out several times from the printed sheets, yet it would be presumptuous to hope that no error has escaped notice. I shall be grateful to Teachers and Students for any correction that they may send me.

I have to thank some friends for valuable criticism and advice and also for correcting and revising many of the proof-sheets. I have the pleasure of expressing my thanks also to some of the students of the M. A.-O. College, Aligarh, for great assistance in verifying the answers to many of the examples.

ALIGARH, N. W. P.,
January, 1890.

J. C. C.



CONTENTS.

NOTE. The easier part of the Section on Problems may be taken at a much earlier stage than is indicated by its position in the book ; and Examples 37 and 38 may be omitted on the first reading.

TABLES OF MEASURES.

For further information turn to the pages referred to.

English Money Table. [*Page* 51.]

4 Farthings (*q.* or *f.*)	make	1 Penny (1*d.*).
12 Pence	...	1 Shilling (1*s.* or 1/-).
20 Shillings	...	1 Pound or Sovereign (£1).

2 Shillings = 1 Florin. 5 Shillings = 1 Crown.
21 Shillings = 1 Guinea. 27 Shillings = 1 Moidore.

Indian Money Table. [*Page* 51.]

3 Pies (*p.*).	make	1 Pice.
4 Pice or 12 Pies	...	1 Anna (1*a*).
16 Annas	...	1 Rupee (R1).

1 Rupee = 1*s.* 6*d.*

English Jewellers' or Troy Weight. [*Page* 66.]

(Chiefly used for weighing gold, silver and jewels.)

24 Grains (gr.)	make	1 Pennyweight (1 dwt.).
20 Pennyweights	...	1 Ounce (1 oz.).
12 Ounces	...	1 Pound (1 lb.).

So that a Pound Troy = 5760 Grains.

English Standard or Avoirdupois Weight. [*Page* 67.]

16 Drams (dr.)	make	1 Ounce (1 oz.).
16 Ounces	...	1 Pound (1 lb.).
28 Pounds	...	1 Quarter (1 qr.).
4 Quarters	...	1 Hundredweight (1 cwt.)
20 Hundredweights	...	1 Ton (1 ton).
A stone (st.)	=	14 lb.
A Pound Avoir.	=	7000 Grains Troy.

Indian Bazar Weight. [*Page* 68.]

4 Sikis	make	1 Tola.
Sikis	...	1 Kancha (Powa-chatak).
Kanchas or 5 Tolas	...	1 Chatak (1 ch.).
16 Chataks	...	1 Seer.
40 Seers	...	1 Maund (1 md.).

4 Chataks = 1 Powa. 4 Powas = 1 Seer.
5 Seers = 1 Punsbury. 8 Punsburies = 1 Maund.

Madras Local Weight. [*Page* 69.]

3	Tolas	make	1 Pollum.
8	Pollums	...	1 Seer.
5	Seers or 40 Pollums	...	1 Viss.
8	Viss	...	1 Maund.
20	Maunds	...	1 Candy or Barum.
	A Madras maund	=	25 lb. Avoir.

Bombay Local Weight. [*Page* 69.]

4	Dhans	make	1 Raktika.
8	Raktikas	...	1 Masha.
4	Mashas	...	1 Tank.
72	Tanks	...	1 Seer.
40	Seers	...	1 Maund.
20	Maunds	...	1 Candy.
	A Bombay maund	=	28 lb. Avoir.

English Linear Measure. [*Page* 70.]

12	Inches (in.)	make	1 Foot (1 ft.).
3	Feet	...	1 Yard (1 yd.).
5½	Yards	...	1 Pole, Rod or Perch (1 po.)
40	Poles or 220 yards	...	1 Furlong (1 fur.).
8	Furlongs or 1760 yards	...	1 Mile (1 mi.).
3	Miles	...	1 League (1 lea.).

1 Pole	=	5 yd. 1 ft. 6 in.	
9 Inches	=	1 Span.	
2 Spans or 18 Inches	=	1 Cubit (*Hath*).	
∴ 2 Cubits	=	1 Yard.	
6 Feet	=	1 Fathom.	
4 Poles or 22 Yards	=	1 Chain	Used in land surveying.
100 Links	=	1 Chain	

English Square Measure. [*Page* 72.]

144	Square Inches (sq. in.)	make	1 Square Foot (1 sq. ft.).
9	Square Feet	...	1 Square Yard (1 sq. yd.).
30¼	Square Yards	...	1 Square Pole, Rod or Perch. (1 sq. po.).
40	Square Poles	...	1 Rood (1 ro.).
4 or 4840	Roods or sq. yards	...	1 Acre (1 ac.).
640	Acres	...	1 Square Mile (1 sq. mi.).

A square chain = 22 × 22 sq. yards or 484 sq. yards.
∴ 10 sq. chains = 1 acre.
1 sq. pole = 30 sq. yd. 2 ft. 36 in.

*** For LAND MEASURES OF BENGAL see pages 74 and 75.

Measures of Solidity. (*English*) [*Page* 75.]

1728 Cubic Inches	make	1 Cubic Foot (1 cu. ft.).
27 Cubic Feet	...	1 Cubic Yard (1 cu. yd.).

Measures of Capacity. (*English*) [*Page* 75.]

4 Gills	make	1 Pint (1 pt.).	
2 Pints	...	1 Quart (1 qt.).	
4 Quarts	...	1 Gallon (1 gall.).	
2 Gallons	...	1 Peck (1 pk.).	For *dry* goods only.
4 Pecks	...	1 Bushel (1 bus.).	For *dry* goods only.
8 Bushels	...	1 Quarter (1 qr.).	For *dry* goods only.
5 Quarters	...	1 Load (1 ld.).	For *dry* goods only.
2 Loads	...	1 Last (1 last).	For *dry* goods only.

A *Barrel* contains 36 gallons.

Note. A gallon of distilled water weighs exactly 10 lb. Avoir. **A pint of water weighs a pound and a quarter.** [A gallon contains 277·274 cubic inches]. A cubic foot of water weighs *about* 1000 oz. Avoir.

Measures of Time. (*English*) [*Page* 76.]

60 Seconds (sec.)	make	1 Minute (1 min.).
60 Minutes	...	1 Hour (1 hr.).
24 Hours	...	1 Day (1 da.).
7 Days	...	1 Week (1 wk.).
365 Days	...	1 Year (1 yr.).
366 Days	...	1 Leap-year.
100 Years	...	1 Century.

Measures of Angles. [*Page* 77.]

60 Seconds (60″)	make	1 Minute (1′).
60 Minutes	...	1 Degree (1°).
90 Degrees	...	1 Right Angle (1 rt. gle.).

Measures of Number. [*Page* 78.]

	12 Units	make	1 Dozen.
	12 Dozen	...	1 Gross.
	12 Gross	...	1 Great Gross.
	20 Units	...	1 Score (*Kurri*).
Also	24 Sheets of paper	...	1 Quire.
	20 Quires	...	1 Ream.
	10 Reams	...	1 Bale.

Apothecaries' Weight. [*Page* 78.]

(i) *Measures of Weight.*

Druggists use the *grain* to weigh small quantities and the *pound* and *ounce* Avoir. to weigh large quantities. Some physicians in prescribing use the following table :

20 Grains	make	1 Scruples (1 scr.).
3 Scruples	...	1 Drachm (1 dr.).
8 Drachms	...	1 Ounce Troy.

(ii) *Measures of Capacity.*

60 Minims (m.) or drops	make	1 Fluid drachm (fl. dr.).
8 Fluid drachms	...	1 Fluid ounce (fl. oz.).
20 Fluid ounces	...	1 Pint (O.).
8 Pints	...	1 Gallon (C.).

A teaspoonful	1 Fluid drachm.
A dessertspoonful	2½ Fluid drachms.
A tablespoonful	4 Fluid drachms.

Note. Since a pint of water weighs a pound and a quarter, the weight of a fluid ounce of distilled water is an *ounce* Avoir.

Metric Measures. [*Pages* 384—389.]

Fraction or Multiple of Standard Measure.	Length.	Volume.	Weight
0·001	Millimetre	Millilitre	Milligram
0·01	Centimetre	Centilitre	Centigram
0·1	Decimetre	Decilitre	Decigram
1	**Metre**	**Litre (1000 c. c.)**	**Gram**
10	Decametre	Decalitre	Decagram
100	Hectometre	Hectolitre	Hectogram
1000	Kilometre	Kilolitre	Kilogram

ARITHMETIC.

I. INTRODUCTION.

1. A **quantity** is anything which may be regarded as being made up of parts like the whole. [*Hamblin Smith*.

Thus, a sum of money, the length of a rod, the weight of a sack of rice, a number of men, are quantities.

2. A quantity is called a **unit** quantity [or simply a *unit*] when it is used for the purpose of comparing the magnitudes of other quantities of the same kind. [*J. B. Lock*.

Thus, a rupee is used as the unit of money when we speak of a certain sum as three *rupees*. A boy is the unit when we speak of a certain class in a school as containing fifteen *boys*.

3. That which indicates the magnitude of a quantity relatively to its unit is called a **number**.

Thus, the number *three* indicates the relative magnitude of the quantity *three rupees* as compared with its unit *a rupee*.

4. The **Measure** or **numerical value** of a quantity is the *number* which expresses how many times the unit is contained in the quantity.

Thus, if we use a yard as the unit of length, and speak of a certain length as five yards, the number five is the *measure* or *numerical value* of that length.

Note. The numerical value of a quantity indicates its *relative* magnitude. The *absolute* magnitude of a quantity is indicated by its numerical value and unit together.

5. A number is called an **abstract** number, when it is not attached to any particular unit; as, *four*, *five*, *seven*.

6. A number is called a **concrete** number, when it is attached to some particular unit; as, *four* horses, *five* men, *seven* yards.

7. **Arithmetic** is a part of the Science which teaches the use of numbers.

II. THE METHOD OF REPRESENTING NUMBERS BY FIGURES.

8. In Arithmetic we represent all numbers by means of the ten symbols or *figures* 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, called **digits**. The first nine of these figures are called the **significant** digits; the last is called **zero, cipher** or **nought**.

9. Numbers from *one* to *nine* are represented by the nine significant digits taken in order. Thus

one	two	three	four	five	six	seven	eight	nine
1	2	3	4	5	6	7	8	9

10. All higher numbers are represented by two or more of the figures, the following *convention* being adopted :

It is agreed that in a line of figures, the figure in the first place *towards the right* shall have its *simple value*,* and shall represent so many **units** ; the figure in the second place from the right shall have *ten times its simple value*, and shall represent so many *tens of units*, or **tens** ; the figure in the third place shall have *ten times the value it shou'd have in the second place* or *one hundred times its simple value*, and shall represent so many *tens of tens*, or *hundreds, of units*, or **hundreds** ; thus 435 shall express one hundred times four units, together with ten times three units and also five units more ; or in other words, it shall express four hundreds, three tens and five units : and so on, the value of a figure increasing tenfold at each step of removal towards the left.

11. The following table, called the **Numeration Table**, gives the respective names of places of figures representing a number.

Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1	9	8	7	6	5	4	3	2	1

* The value of a figure which it has when it stands by itself is called its *simple* or *intrinsic* value. The value of a figure which it has in consequence of its position in a line of figures is called its *local* or *accidental* value.

The periods which follow those in the above table are trillions, quadrillions, quintillions, sextillions, septillions, octillions, etc.

12. The symbol o has no value in itself and represents no number. In a line of figures, o in the first place (towards the right) indicates the absence of units ; in the second place, absence of tens ; in the third place, absence of hundreds ; and so on.

Thus

30 represents three tens and *no units* ;

400 represents four hundreds, *no tens*, also *no units* ;

309 represents three hundreds, *no tens*, and nine units.

13. It appears then, that numbers from *one* to *nine* are represented by one figure ; numbers from *ten* to *ninety-nine* are represented by two figures ; numbers from *one hundred* to *nine hundred and ninety-nine* are represented by three figures : numbers from *one thousand* to *nine thousand, nine hundred and ninety-nine* are represented by four figures ; and so on.

14. The method above explained of representing numbers by means of ten figures and their combinations was invented by the Hindus. But Europeans call it the Arabic Notation because it was introduced into Europe by the Arabs who had learnt it from the Hindus.

NUMERATION.

15. Numeration is the art of reading a number expressed in figures.

Art. 9 enables the learner to read the numbers expressed by one figure ; and the following table will enable him to read the numbers expressed by two figures.

10 ten	23 twenty-three	36 thirty-six
11 eleven	24 twenty-four	37 thirty-seven
12 twelve	25 twenty-five	38 thirty-eight
13 thirteen	26 twenty-six	39 thirty-nine
14 fourteen	27 twenty-seven	40 forty
15 fifteen	28 twenty-eight	41 forty-one
16 sixteen	29 twenty-nine	42 forty-two
17 seventeen	30 thirty	43 forty-three
18 eighteen	31 thirty-one	44 forty-four
19 nineteen	32 thirty-two	45 forty-five
20 twenty	33 thirty-three	46 forty-six
21 twenty-one	34 thirty-four	47 forty-seven
22 twenty-two	35 thirty-five	48 forty-eight

49 forty-nine
50 fifty
51 fifty-one
52 fifty-two
53 fifty-three
54 fifty-four
55 fifty-five
56 fifty-six
57 fifty-seven
58 fifty-eight
59 fifty-nine
60 sixty
61 sixty-one
62 sixty-two
63 sixty-three
64 sixty-four
65 sixty-five
66 sixty-six
67 sixty-seven
68 sixty-eight
69 sixty-nine
70 seventy
71 seventy-one
72 seventy-two
73 seventy-three
74 seventy-four
75 seventy-five
76 seventy-six
77 seventy-seven
78 seventy-eight
79 seventy-nine
80 eighty
81 eighty-one
82 eighty-two
83 eighty-three
84 eighty-four
85 eighty-five
86 eighty-six
87 eighty-seven
88 eighty-eight
89 eighty-nine
90 ninety
91 ninety-one
92 ninety-two
93 ninety-three
94 ninety-four
95 ninety-five
96 ninety-six
97 ninety-seven
98 ninety-eight
99 ninety-nine

16. When a number is expressed by *three* figures, the third figure from the right is read as so many *hundred*, the two remaining figures being read together as in the above table. Thus

the number expressed by 100 is read *one hundred* ;
the number expressed by 340 is read *three hundred and forty* ;
the number expressed by 452 is read *four hundred and fifty-two* ;
the number expressed by 607 is read *six hundred and seven.*

17. If a number is expressed by more than three figures, divide the line of figures by commas into periods of three figures each, commencing from the right ; and read the first period (towards the right) as in Art. 16, read the second period as so many *thousand*, the third period as *million*, the fourth as *thousand*, the fifth as *billion*, the sixth as *thousand*, and so on. *The periods must be read off from left to right in order.*

Thus

2,435 is read 'two *thousand*, four hundred and thirty-five' ;
23,204 is read 'twenty-three *thousand*, two hundred and four' ;
234,021 is read 'two hundred and thirty-four *thousand* and twenty-one' ;
324,103,200 is read 'three hundred and twenty-four *million*, one hundred and three *thousand*, two hundred' ;
36,204,340,432,004 is read 'thirty-six *billion*, two hundred and four *thousand*, three hundred and forty *million*, four hundred and thirty-two *thousand* and four'.
1,000 represents a thousand ;
1,000,000 represents a million ;
1,000,000,000,000 represents a billion.

EXAMPLES. I.

To be done first orally, then in writing.

Express each of the following numbers in words :

1. 10 ; 16 ; 48 ; 99 ; 76 ; 43 ; 50 ; 31 ; 62.
2. 100 ; 111 ; 902 ; 620 ; 300 ; 103 ; 234 ; 130.
3. 9216 ; 5409 ; 5004 ; 1011 ; 1210 ; 9000 ; 9999.
4. 12345 ; 20103 ; 40040 ; 50001 ; 90600 ; 89346.
5. 500000 ; 708900 ; 102030 ; 309809 ; 379586.
6. 7234651 ; 7090709 ; 9000000 ; 7800040 ; 3567891.
7. 32567892 ; 34083092 ; 90009000 ; 55500055.
8. 789345621 ; 390085000 ; 222000000.
9. 7009056700 ; 3259287891 ; 8070088200.
10. 325000340001 ; 3085060008230 ; 13579864 28123.
11. What is the local value of each of the significant digits in the numbers, 72, 359, 4203, 70809, 1300450789 and 3079004078023 ?
12. What does each of the zeroes in the numbers 20103, 307005050 and 300508230509 indicate ?
13. Express in words the least number of five figures and the greatest number of four figures.
14. What is the greatest number that can be formed with the figures 1, 3, 5, 8 ? What is the least ?
15. What is the greatest number that can be formed with the figures 2, 8, 0, 4, 7 ? What is the least ?
16. What is the greatest number that begins with 8 and ends with 1 and has four figures altogether ? What is the least ?

NOTATION.

18. Notation is the art of representing by figures a number expressed in words.

The method is as follows :

Begin at the left hand, and put down the required figures in the places necessary to express the number, according to the Numeration Table ; and fill up the vacant places, if any, with ciphers.

Thus, to represent by figures the number, ***five million, twenty-eight thousand, three hundred and four***, we put down 5 in the place of ***millions*** or in the seventh place from the right, 2 in the place of ***tens of thousands*** or in the fifth place, 8 in the place of ***thousands*** or in the fourth place, 3 in the place of ***hundreds*** or in the third place, and 4 in the place of ***units*** or in the first place ; and then we fill up the sixth and second places with ciphers ; and the number expressed in figures is 5028304.

EXAMPLES. 2.

State in figures :

1. Thirteen ; seventeen ; nineteen ; twelve ; eleven.

2. Twenty-three ; thirty-four ; forty ; twenty-seven.

3. Seventy-seven ; ninety ; eighty-four ; sixty-three.

4. Three hundred and forty-two ; four hundred and eighty-six ; five hundred and four ; nine hundred.

5. Two hundred and three ; four hundred and thirty ; five hundred and fifty-five ; four hundred.

6. Eight hundred and ninety-two ; seven hundred and four : six hundred and forty ; five hundred and twelve.

7. Seven thousand, eight hundred and thirty-five ; nine thousand and twenty-eight ; six thousand and nine ; four thousand : six thousand and eighty-five.

8. Five thousand, nine hundred and ninety-two ; eight thousand and seventy-four ; two thousand and three ; four thousand and forty ; three thousand, four hundred and three.

9. Twelve hundred ; eighty thousand and eight ; eighteen thousand, four hundred and fifty-four ; thirty-six thousand and twelve ; ninety-thousand.

10. Twenty thousand and seventy ; thirty thousand and eight ; fifty-four thousand, four hundred ; sixteen thousand and four.

11. Four hundred and five thousand ; eight hundred thousand and forty ; seven hundred and two thousand and seventy-four.

12. Three million, nine hundred and four ; nine million, four hundred ; fifteen million and fifty ; one hundred and eight million, three thousand and four ; four million and five thousand.

13. Five thousand million, seven hundred thousand and twenty-eight ; three hundred and fifteen thousand seven hundred and sixty-four million, nine thousand and three.

14. Three billion and fifty ; four hundred and five billion, ten million, twenty thousand and seven ; one billion, one million, one thousand ; six billion and six.

15. Five hundred and twelve billion, two hundred and fifty-five thousand seven hundred and sixty-two million, seven hundred and thirteen thousand, four hundred and seventy-three.

16. Twelve billion and twelve ; seven hundred billion, seven hundred thousand and seven hundred ; three billion, three million, three thousand, three hundred and three.

17. Seven thousand three hundred and five billion, five hundred and two million, six thousand and twenty-four ; forty-seven billion, forty-seven million, forty-seven thousand and forty-seven.

18. State in figures the least number of seven figures and the greatest number of five figures.

19. One boy wrote 70007007 and another wrote 777 when told to write 'seven thousand, seven hundred and seven' in figures; what mistakes did they commit?

THE INDIAN METHOD OF NUMERATION.

19. The following is the Indian Numeration Table in common use:

Hundreds of crores.	Tens of crores.	Crores.	Tens of lacs.	Lacs.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1	9	8,	7	6,	5	4,	3	2	1

The above number is read thus:

One hundred and ninety-eight crores, seventy-six lacs, fifty-four thousand, three hundred and twenty-one.

Note. The Hindu names of places of figures are as follow:— *aka*, *dasha*, *shata*, *sahasra*, *oyut*, *laksha* (lac), *niyut*, *coti* (crore), *arbud*, *padma*, *kharba*, *nikharba*, *mohapadma*, *sanku*, *jaladhi*, *antya*, *madhya*, *parardhya*.

EXAMPLES. 3.

Express in words according to the Indian Numeration:

1. 345543; 3020050; 7990570; 7050304.

2. 12345678; 305750080; 45000000.

3. 2300780001; 7080904080; 3794857612.

4. 8274057009; 3500001230; 3103705040.

5. 1234567890; 6000789000; 5010702009.

Express in figures:

6. One lac, fourteen thousand; seventy-eight lacs; fifteen lacs, four thousand and thirty; seven lacs and seven.

7. One crore, five hundred; twenty-eight crores, three lacs and four; twenty crores; one crore, one lac, one thousand and one.

8. Three hundred crores, five lacs, four thousand; one hundred and one crores, one lac, one hundred and one.

9. Three hundred and twenty-eight crores, seventeen lacs, forty-five thousand, seven hundred and fifteen.

10. Seven hundred and five crores, seventeen lacs, twenty-four thousand, seven hundred and thirty-eight.

11. How many thousands are in a lac? How many lacs in a million? How many millions in a crore?

12. Read according to the Indian numeration the number—one hundred and three million, twenty-eight thousand, four hundred and one.

13. Read according to the English numeration the number—one hundred and three crores, seven lacs, seven hundred and four.

THE ROMAN SYSTEM OF NOTATION.

20. In this system the symbols chiefly employed are I, V, X, L, C, D and M which represent 1, 5, 10, 50, 100, 500 and 1000 respectively. Again a bar placed over a letter increases its value a thousand-fold; thus $\overline{X}$ represents 10,000.

The following table will explain the method of representing any number by means of the above symbols.

I	1	XI	11	XXX	30	CD	400
II	2	XII	12	XL	40	D	500
III	3	XIII	13	L	50	DC	600
IV	4	XIV	14	LX	60	DCC	700
V	5	XV	15	LXX	70	DCCC	800
VI	6	XVI	16	LXXX	80	CM	900
VII	7	XVII	17	XC	90	M	1000
VIII	8	XVIII	18	C	100	MCD	1400
IX	9	XIX	19	CC	200	MCM	1900
X	10	XX	20	CCC	300	MM	2000
MDCCCLXXXIX	1889					$\overline{DLX}$DCCXLII	560742

EXAMPLES. 4.

Express in Arabic notation:

1. VI. **2.** IX. **3.** XLIX. **4.** XCIX.
5. LXXV. **6.** CCLXIV. **7.** DCIX. **8.** DCLXIV.
9. MCMXC. **10.** $\overline{L}$XX. **11.** MMDCCLXIV.

Express in Roman notation:

12. 44. **13.** 66. **14.** 79. **15.** 83.
16. 149. **17.** 436. **18.** 990. **19.** 1351.
20. 5670. **21.** 3149. **22.** 45978. **23.** 1000000.

III. ADDITION.

21. Addition is the method of finding a single number which is equal to two or more given numbers taken together.

The given numbers are called **summands**, and the single number obtained by adding them is called their **sum** or **amount**.

22. The sign + signifies that the two numbers between which it is placed are to be *added*. Thus, $7+2$ signifies that 2 is to be added to 7. The sign + is called the **plus sign**, and $7+2$ is read "seven *plus* two".

The sign = stands for the words "is equal to" or "equals". Thus, $2+3=5$ states that the sum of 2 and 3 *is equal to* 5. The sign = is called the **sign of equality**, and $2+3=5$ is read "two plus three **is equal to** five" or "two plus three **equals** five".

23. The numbers *one*, *two*, *three*, *four*, *five*, etc. being taken in order, if we add the number *one* to any one of them, we get the number next following: thus $1+1=2$; $2+1=3$; $3+1=4$; and so on.

We obtain the sum of 5 and 3 thus:

$$\begin{aligned} 5+3 &= 5+2+1 \\ &= 5+1+1+1 \\ &= 6+1+1 \\ &= 7+1 \end{aligned}$$

Results thus obtained are registered in the following table, called the Addition Table, which the learner should commit to memory.

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 ... 3	2 ... 4	2 ... 5	2 ... 6	2 ... 7	2 ... 8	2 ... 9	2 ...10	2 ...11
3 ... 4	3 ... 5	3 ... 6	3 ... 7	3 ... 8	3 ... 9	3 ...10	3 ...11	3 ...12
4 ... 5	4 ... 6	4 ... 7	4 ... 8	4 ... 9	4 ...10	4 ...11	4 ...12	4 ...13
5 ... 6	5 ... 7	5 ... 8	5 ... 9	5 ...10	5 ...11	5 ...12	5 ...13	5 ...14
6 ... 7	6 ... 8	6 ... 9	6 ...10	6 ...11	6 ...12	6 ...13	6 ...14	6 ...15
7 ... 8	7 ... 9	7 ...10	7 ...11	7 ...12	7 ...13	7 ...14	7 ...15	7 ...16
8 ... 9	8 ...10	8 ...11	8 ...12	8 ...13	8 ...14	8 ...15	8 ...16	8 ...17
9 ...10	9 ...11	9 ...12	9 ...13	9 ...14	9 ...15	9 ...16	9 ...17	9 ...18

Example. Add 7+8+9+8.

Process: 7+8=15; 15+9=24; 24+8=32. *Ans.*

Note. As facility in mental addition is the basis of all accurate facility in the subsequent processes of Arithmetic, the pupil should have a sufficient number of exercises in mental addition before he proceeds further. The use of fingers should be strictly prohibited.

EXERCISES IN MENTAL ADDITION.

N. B. The following exercises are not considered sufficient; they are intended only to show the nature of the questions that might be asked.

1. What is the sum of

(*a*) 2 and 9; 3 and 4; 8 and 7; 7 and 5; 9 and 9; 9 and 7; 3 and 7; 8 and 5; 9 and 6; 6 and 8; 8 and 9; 7 and 3?

(*b*) 10 and 7; 20 and 8; 30 and 6; 50 and 9; 70 and 5?

(*c*) 11 and 6; 12 and 7; 26 and 4; 36 and 3; 72 and 7?

*(*d*) 15 and 7; 16 and 8; 22 and 9; 37 and 6; 85 and 9; 43 and 8; 49 and 9; 28 and 7; 68 and 7; 98 and 7; 99 and 9?

2. Add

(*a*) 5 to 7, to 17, to 27, to 37, etc.

(*b*) 7 to 9, to 19, to 29, to 39, etc.

(*c*) 8 to 8, to 18, to 28, to 38, etc.

3. (*a*) How much do 1 and 2 make? 3 and 2? 5 and 2? etc.

(*b*) How much do 2 and 3 make? 5 and 3? 8 and 3? etc.

(*c*) How much do 3 and 5 make? 8 and 5? 13 and 5? etc.

N. B. When the pupil has acquired a little facility the above question may, with advantage, be put in the following form:

4. Count by increments of 6 starting at 4.

Answer. 4, 10, 16, 22, 28, 34, etc.

5. I have 10 marbles in one hand and 7 in the other; how many marbles have I in all?

*The following process in mental addition may be recommended for beginners:—

15+7=15+5+2=20+2=22.

But the process should be abandoned as soon as facility in addition has been acquired.

6. Twelve articles make a dozen ; how many in two dozen ?

7. Ram had 19 marbles and he has won 8 ; how many marbles has he now ?

8. I have purchased a table for 16 rupees and a chair for 7 rupees ; how many rupees have I spent in all ?

9. If mangoes are selling at the rate of 13 for the rupee, how many shall you get for two rupees ?

10. John bought 25 mangoes and 9 oranges ; how many fruits did he buy in all ?

11. You are 13 years old ; your brother is 7 years older than you ; what is the age of your brother ?

12. If I give you 20 rupees I shall have 15 rupees left in my purse ; how many rupees have I ?

13. A boy has lost 8 marbles and has 27 left ; how many had he at first ?

14. You have 23 marbles in your pocket ; I give you 9 ; how many have you now in all ?

15. A man bought 35 maunds of rice on a certain day, and 9 maunds on the next day ; how many maunds did he buy in all ?

16. A man's age is 47 years ; how old will he be 7 years hence ?

17. If you buy 56 mangoes and your brother 8 more than you, how many does your brother buy ?

18. What is the number from which if I take 15 there will remain 60 ?

19. A man bought a table for 75 rupees and gained 5 rupees by selling it ; for how many rupees did he sell it ?

20. A man gave 19 rupees to his wife, 7 rupees to his son and 4 rupees to his daughter ; how many rupees did he give away in all ?

21. What is the united length of five roads which are 1, 2, 3, 4 and 5 miles long respectively ?

22. I bought a book for 6 annas and a bottle of ink for 4 annas more than the book ; how much did I spend in all ?

23. A man sold 9 oranges to *A*, to *B* 7 more than to *A* : how many did he sell in all ?

24. Ram bought 2 mangoes at 4 annas each and 8 oranges at one anna each ; how much did he pay to the fruit-seller ?

25. From a rope are cut off first 27 yards, then 8 yards, and there are 7 yards left ; what was the length of the rope ?

24. In the case of large numbers the process of addition is as follows :

Example 1. Add together 378, 409 and 56.

We write down the numbers, one under another, thus

Hundreds.	Tens.	Units.
3	7	8
4	0	9
	5	6
8	**4**	**3**

placing units under units, tens under tens, hundreds under hundreds, and so on ; and then draw a line under the lowest line of figures. Under this line we place the sum which is found in the following way :

We first add the units, thus (8+9+6) units=23 units=2 tens+**3** units ; we place the **3** under the column of units and *carry on* the 2 tens for adding to the column of tens. Next we add the tens, thus (2+7+0+5) tens=14 tens=1 hundred+**4** tens ; we place the **4** under the column of tens and *carry on* the 1 hundred for adding to the column of hundreds. We then add the hundreds, thus (1+3+4) hundreds=**8** hundreds ; and we place the **8** under the column of hundreds.

Mental Process :		8,	17,	**23** ;	set down **3**,
	carry	2,	9,	**14** ;	set down **4**,
	carry	1,	4,	**8**.	

In actual practice the work is written down thus :

$$\begin{array}{r} 378 \\ 409 \\ 56 \\ \hline 843 \end{array}$$

Note 1. Accuracy and rapidity are the fundamental things in addition. For rapid work the student should acquire the habit of looking ahead and adding two or three numbers according to his choice and convenience. When the eye is trained for that purpose the process of addition may be much shortened as the following example will show.

Example 2. Add together 89763, 25954, 73896, 589263, 32157, 98756.

3	9	7	6	3
2	5	9	6	4
7	3	8	9	6
5	8	9	2	6
3	2	1	5	7
9	8	7	5	6
37	**9**	**4**	**6**	**2**

Method of Convenient Grouping.

Mental Process :

1st col. : 3, 13, 26, 32 ; set down **2**, carry 3.
2nd col. : 3, 15, 26, 36 ; set down **6**, carry 3.
3rd col. : 3, 10, 27, 37, 44 ; set down **4**, carry 4.
4th col. : 4, 18, 29, 39 ; set down **9**, carry 3.
5th col. : 3, 13, 25, **37**

Note 2. It is often convenient to add number horizontally from right to left or from left to right without writing them one under another in columns, *i.e.*, without placing the units under units, the tens under tens and so on. But care should be taken to add units to units, tens to tens, hundreds to hundreds and so on.

Example 3. Find the value of 3125 + 6309 + 7844 + 8623.

The sum = **25901.**

Mental Process :

Units digits (from right to left)—3, 7, 16, **21**, set down **1**, carry 2.
Tens digits " " " " 2, 4, 8, **10**, set down **0**, carry 1.
Hundreds digits " " " " 1, 7, 15, 18, **19**, set down **9**, carry 1.
Thousands digits " " " " 1, 9, 16, 22, **25**, set down **25**.

25. Checking of addition results.—Add the numbers in the *reverse* order, that is, if the addition has been done from top to bottom, add again from bottom to top and *vice versa*. If the addition has been done horizontally from right to left, add again from left to right and *vice versa*. In each case see if the two results agree.

EXAMPLES. 5.

N. B. Sums should be dictated and the pupils required to read out the answers in words. The same sum may be given several times by altering the order of the summands.

Add together

1.		2.		3.		4.		5.	
	3		6		8		7		8
	5		9		7		5		9
	9		8		9		8		8
	4		7		7		9		9

6.	7.	8.	9.	10.
56	73	40	90	79
42	26	37	50	84

11.	12.	13.	14.	15.
375	879	79	936	984
208	82	40	742	76
740	190	673	999	940

16.	17.	18.	19.
7643	429	3098	4807
248	7	207	309
5004	84	40	4
1234	9476	329	500

20.	21.	22.	23.
28	58073	839	38756
4007	9705	2058	50952
350	368	476	78095
9	78000	8205	34560
302	29	47460	32308

24.	25.	26.	27.
89763	38760	467895	79
25964	5807	58009	3025
73896	304	5555	329
58926	19	795073	876502
32157	7	567982	39879
98756	374	368000	300

28.	29.	30.	31.
9038	7	3578924	9357350
30054	7000007	5893679	2984721
5028	34003	8279563	8305902
76	404040	9528789	7650729
9	36000	3474923	8472038
938050	38	8923463	5679824

Find the sumof

32. 804, 97056, 48, 397834 and 909.

33. 73568, 9340, 8654, 76, 703 and 98.

34. 74, 79048, 309, 8000386, 43 and 3002.

35. 300, 785, 897634, 12345, 207 and 20708.

Without writing down the numbers one under another, find the value of

36. 432398 + 7867 + 83989 + 7030.

37. 70 + 8200 + 7396 + 5678920 + 97 + 2.

38. 3 + 309 + 29 + 307895 + 3253 + 500.

39. 87 + 9800000 + 80234 + 10201 + 34567 + 9.

40. 3456 + 456 + 56 + 6 + 76000 + 984530789.

41. Add together the following numbers: seventy-nine; three thousand, four hundred and fifty; sixty-six thousand, six hundred and ninety-four; four thousand and four; eighty.

42. Find the total of—six hundred and ninety-two; four lacs, forty-five thousand and seven; ninety-eight lacs, seven hundred; forty-five; seven.

43. Find the amount of—seven hundred and forty-six million, seventy-four thousand, nine hundred and sixty-two; eighty-six thousand, five hundred and four; twelve million, seven thousand and three; ninety-one; seven million and seven.

44. How much are nineteen + seven lacs, seven thousand and seven + three hundred and four crores, seventy-four lacs and twent-nine + eight crores, eight lacs, eight thousand and eight + seven thousand, seven hundred and forty-two + six + three lacs, four hundred and seven?

45. Find the amount of 76, 378046, 30567, 8, 9345, 300009, 3708, 309, 37805892, 28, 7923000 and 342.

46. What is the number from which if 3457 be taken 479 is left?

47. A man was born in 1856; in what year was he 34 years of age?

48. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30 and December 31; how many days are there in the whole year?

49. State how many boys are in a school in which there are 125 in the first class, 87 in the second, 95 in the third, 107 in the fourth, 70 in the fifth and 256 in the other classes.

50. A garden contains 327 mango trees, 704 cocoanut trees, 456 date trees, 528 orange trees and only 25 tamarind trees: how many trees are there in all?

51. A certain town contains 87,903 Hindus, 48,093 Mahomedans, 723 Europeans, 1,309 Anglo-Indians and 159 other races: what is the total population of the town?

52. A gentleman bought three pieces of land in a town for 9,700 rupees; he built a house on one piece at a cost of 7,825 rupees, another on the second piece at a cost of 21,750 rupees, and a third on the remaining piece at a cost of 2,729 rupees: what sum did he spend in all?

53. We imported 53,89,082 maunds of salt in January 1885; 7,09,280 maunds in February and 10,94,803 maunds in March: what was the centre weight imported in the first 3 months of 1885?

54. I bought four baskets of mangoes ; the first contained 246 mangoes ; the second 619 ; the third 19 more than the second : and the fourth as many as the first and second together : how many mangoes did I buy ?

55. What is the number from which if I first take 70835 and then 85679, there will remain 7040 ?

56. In the following square, shew that the sum of numbers in each row, in each column and along either diagonal is the same and give the sum.

15	5	9	3
2	10	8	12
7	13	1	11
8	4	14	6

A square like the one given above is called a **magic square**.

57. Shew that each of the following is a magic square.

4	2	6
6	4	2
2	6	4

5	3	7
7	5	3
3	7	5

58. Shew that the following is a magic square.

3	4	5	1	2
2	3	4	5	1
1	2	3	4	5
5	1	2	3	4
4	5	1	2	3

IV. SUBTRACTION.

26. **Subtraction** is the method of finding the number which is left when the *smaller* of two given numbers is taken from the *greater*.

The greater of the two given numbers is called the **minuend**, the less is called the **subtrahend**, and the number found by subtraction is called the **remainder** or **difference**.

The sign $-$, placed between two numbers, signifies that the second number is to be *subtracted* from the first. Thus $7-4$ signifies that 4 is to be subtracted from 7. The sign $-$ is called the **minus sign**, and $7-4$ is read "seven *minus* four".

27. It follows from the definition of subtraction that it is the process of finding the number which must be *added* to a given number to make a larger given number. Hence subtraction is sometimes called *complementary addition*.

We are able to subtract a small number from another, from the known results of the Addition Table.

Example. $7-4=3$, because $4+3=7$.

EXERCISES IN MENTAL SUBTRACTION.

1. Take 3 from 8 ; 4 from 9 ; 5 from 7 ; 6 from 9 ; 5 from 8.

2. What is the difference between 10 and 6 ; 12 and 8 ; 16 and 9 ; 13 and 7 ; 11 and 6 ; 16 and 8 ; 18 and 9 ; 15 and 7 ; 17 and 8 ?

3. How many does 7 leave from 28 ; 5 from 27 ; 6 from 56 ; 7 from 99 ; 3 from 57 ; 8 from 88 ; 6 from 49 ; 4 from 26 ?

4. Subtract 9 from 22 ; 8 from 35 ; 7 from 42 ; 6 from 51 ; 5 from 60 ; 4 from 73 ; 8 from 86 ; 6 from 92 , 5 from 81.

5 (*a*) What remains when we take 6 from 30, 6 from 24, 6 from 18, 6 from 12, 6 from 6 ?

(*b*) What remains when we take 7 from 100, 7 from 93, 7 from 86, etc ?

(*c*) Count by decrements of 6 commencing at 100.

Ans. 100, 94, 88, etc.

6. Take 7 from the sum of 5 and 6 ; 9 from the sum of 6 and 8 ; 6 from the sum of 5 and 4 ; 8 from the sum of 6 and 7.

7. A boy who had 15 marbles has lost 8 : how many has he left ?

8. I have 17 rupees in my purse ; if I give you 9 rupees, how many rupees shall I have left ?

9. Your brother's age is 14 years; you are 5 years younger than he: how old are you?

10. In a class there are 19 boys on the roll; on a certain day 6 boys were absent: how many were present?

11. A man had 16 rupees; he gave 7 rupees to his wife and the rest to his son: how much did the son get?

12. A man bought a table for 19 rupees and sold it for 25 rupees: how much did he gain?

13. There are 37 mangoes on a tree: if 8 be plucked, how many will be left?

14. Ram has 48 marbles; if Gopal had 9 more than what he now has, he would have as many as Ram: how many has Gopal?

15. I have 16 marbles; John has 28: how many more should I get to have as many as John?

28. In the case of large numbers the process of subtraction is as follows:

Example 1. Subtract 34 from 86.

We place the smaller number under the greater, as in Addition, placing units under units and tens under tens. We now take 4 units from 6 units, and set down the result, which is 2 units, under the column of units; next, we take 3 tens from 8 tens, and set down the result, 5 tens, under the column of tens. Thus the remainder obtained is 52.

Tens.	Units.
8	6
3	4
5	2

In actual practice the work is written down thus

```
86
34
--
52
```

as will be seen from the next two examples.

Example 2. Subtract 368 from 952.

```
952
368
---
584
```

Here, proceeding as in the previous example, we meet with the difficulty of taking a greater digit from a less, and to get over this difficulty we avail ourselves of the following principle, usually termed *borrowing: The minuend and subtrahend may be increased by the same number without altering their difference*; and we reason thus:

We cannot take 8 units from 2 units; we therefore add 10 units to the 2 units, making 12 units, and we take 8 units from the 12 units and set down the result, 4 units, under the column of units.

Having increased the upper number by 10 units, we add, by way of compensation, 1 ten to the lower number, changing 6 tens into 7 tens. We have now to take 7 tens from 5 tens, and as we cannot do so, we add 10 tens to the 5 tens, making 15 tens, and we take 7 tens from the 15 tens, and set down the result, 8 tens, under the column of tens. Having increased the upper number by 10 tens, we add, by way of compensation, 1 hundred to the lower number, changing 3 hundreds into 4 hundreds. We now take 4 hundreds from 9 hundreds, and set down the result, 5 hundreds, under the column of hundreds.

Note. Instead of the above process it will be practically convenient to determine how much must be added to the subtrahend to make up the minuend.

Example. Subtract 576 from 829.

We are to find the number which being added to 576 makes up 829.

We place the smaller number under the greater, as in Addition. We now see that 6 units + **3** units = 9 units ; we therefore set down the **3** under the column of units : next, 7 tens + **5** tens = 12 tens ; we set down the **5** under the column of tens, and **carry** 1 hundred : then, (1 + 5) hundreds + **2** hundreds = 8 hundreds ; we set down the **2** under the column of hundreds.

```
829
576
253
```

Mental Process : 6 and **3** are 9 ;
7 and **5** are 12 ;
carry 1, 6 and **2** are 8 ;

29. Checking of subtraction results.—Add the result to the subtrahend. If the sum is the same as the minuend the work is correct.

EXAMPLES. 6.

Perform the following subtractions :

1.	78 35	**2.**	95 43	**3.**	356 134	**4.**	789 246	**5.**	7825 3504
6.	64 39	**7.**	97 48	**8.**	86 78	**9.**	94 85	**10.**	93 60
11.	795 606	**12.**	480 390	**13.**	977 799	**14.**	843 384	**15.**	904 589
16.	5380 739	**17.**	54090 7073	**18.**	84321 53789	**19.**	85858 58585	**20.**	54321 12345

21.	**22.**	**23.**	**24.**	**25.**
20004	789356	708093	805400	7000203
17325	99999	20503	70053	500950

26. 82439 – 76893.

27. 93405 – 7990.

28. 790256 – 82789.

29. 80000 – 76438.

30. 1000000 – 999999.

31. 777770 – 88889.

32. 780004 – 389210.

33. 100956 – 39897.

Find the missing figures in the following sums on subtraction :

34.	**35.**	**36.**	**37.**
789356	7*8*9*	8*54**	70*02*3
******	20503	70053	50095*
689357	*875*9	*3**47	**9*247

Fill up the missing lines in the following addition sum :

38.	**39.**	**40.**	**41.**
12345	617532	999999	3187019
....	....	....	....
67890	700835	1000000	5007009

42. What number must be added to each of the following numbers to make the sum equal to a million ?—19, 305, 9475, 99446 and 43500.

43 What number must be taken from 93867 to leave 903 ?

44. By how much does a lac exceed twenty-nine ?

45. By how much is a crore greater than one thousand and one ?

46. By how much is seventy-nine less than ten thousand ?

47. The Duke of Wellington was born in 1769 and died in 1852 ; how old was he at his death ?

48. Sir Isaac Newton died in 1727 aged 85 years : when was he born ?

49. Mount Everest is 29,100 feet high ; Kinchinjunga is 28,177 feet high : by how many feet is the former higher than the latter ?

50. If the receipts of a railway company are 3,98,450 rupees and the expenses 2,80,769 rupees, what are the profits ?

51. A merchant bought goods for 3,000 rupees and sold them for 3,325 rupees : how much did he gain ?

52. If I had 540 rupees more than I have, I should be able to clear a debt of 10,000 rupees : how much have I ?

53. The sum of two numbers is 93875, and the greater number is 77359 : what is the smaller number ?

54. The smaller of two nnmbers is 3799, and their sum is 780900 : what is the greater number ?

55. What number must be subtracted from 7389 that the remainder may be 999 ?

56. Find the difference between the sum and difference of a million and a thousand.

57. *A* has 39,876 rupees ; *B* has 3,758 rupees less than *A* ; and *C* has 876 rupees less than *B* ; how much has *C* ?

58. A boy when told to write 'three thousand, four hundred and five' in figures wrote 30004005, how much more did he write ?

59. A boy wrote 500403 when he was told to write 'fifty lacs, four thousand and three' in figures ; how much less did he write ?

30. The number to which the sign + is prefixed is called a **positive** number ; and the number to which the sign − is prefixed is called a **negative** number. If no sign is prefixed to a number it is to be considered as *positive*. Numbers connected by the sign + or − are called **terms**.

The most convenient method of finding the value of an *expression* (in which several numbers are connected by the sign + or −) is to find the sums of the *positive* and *negative* numbers separately and then to take their difference.

Example. Find the value of $473-369+621-403$.

Now, $473+621=1094$; and $369+403=772$;

$\therefore$ the result required $=1094-772=322$.

31. Combined Addition and Subtraction.

Example. Subtract the sum of 2435, 4748, 2246 and 1027 from 20302.

20302	Process :	
2435	7, 13, 21, 26 and 6, 32.	Set down 6 and carry 3.
4748	3, 5, 9, 13, 16 and 4, 20.	Set down 4 and carry 2.
2246	2, 4, 11, 15 and 8, 23.	Set down 8 and carry 2.
1027	2, 3, 5, 9, 11 and 9, 20.	Set down 9 and carry 2.
9846	2 and 0, 2.	

Note. The sum of the figures in the 1st column is 26. Adding 6 to this we obtain 32 which has got 2 in the units' place and which is the *least* of the numbers (with in the units' place) greater than 26.

EXAMPLES. 7.

Find the value of each of the following expressions :

1. 973 − 724 + 209.
2. 78965 − 8795 − 7386.
3. 8703 − 7935 + 3002 − 1030
4. 1600 − 924 − 300 − 88.
5. 94567 + 3285 − 77777 − 304 + 64.
6. To 753 − 98 + 7 I first add 329, and then take the difference of 720 and 699 from the sum ; what is the result ?
7. By how much is the difference of 7203 and 4980 less than their sum ?
8. By how much does the sum of 7985 − 899 and 7003 exceed their difference ?
9. The greater of two numbers is 94047, and their difference is 909 + 350 ; what is the other ?
10. What number must be added to 329 + 408 − 540 that the sum may be one lac ?
11. Subtract by the method of Art. 31 the sum of 2546, 4758, 3254 and 1017 from 13681.
12. Write down the missing digits in each of the following examples in addition :

(1)		(2)		(3)	
	2861		46807		45*8*
	4942		37752		32*1
	50892		*70845		*2831
	*****		*****		*346
	73151		271021		73125

13. A boy found that the sum of 673, 32186, 7985 and another number was 61321 ; find the missing number.

V. MULTIPLICATION.

32. Multiplication is a short method of finding the sum of a certain number of repetitions of a given number.

The number to be repeated is said to be *multiplied* by the number which indicates *how often* it is to be repeated. Thus, when 4 is multiplied by 3, the result is 4 + 4 + 4 or 12.

The number which is multiplied is called the **multiplicand** ; the number by which it is multiplied is called the **multiplier** ; and the resulting number is called the **product**.

The sign of multiplication is ×. Thus 7×4 signifies that 7 is to be multiplied by 4, and is read "seven *into* four" or "four times seven". Sometimes a dot (.) is used instead of ×.

33. The multiplier and the multiplicand may be interchanged without altering the value of the product. Thus $3\times4=4\times3$; for, $3\times4=3+3+3+3=12$, and $4\times3=4+4+4=12$.

This proposition can also be proved otherwise.

For example, to prove that $5\times4=4\times5$.

Place 5 dots in a line, and repeat this line 4 times. The number of dots in a row is 5, and there are 4 rows; therefore the number of dots altogether is 5 multiplied by 4. Again the number of dots in a column is 4, and there are 5 columns; therefore the number of dots altogether in 4 multiplied by 5. But the total number of dots is the group is just the same however they may be counted. Hence $5\times4=4\times5$.

• • • • •
• • • • •
• • • • •
• • • • •

The multiplier and multiplicand are called **factors** of the product.

34. The following Multiplication Tables must be committed to memory by the pupil.

First Table.

	1	2	3	4	5	6	7	8	9	10
Once	1	2	3	4	5	6	7	8	9	10
Twice	2	4	6	8	10	12	14	16	18	20
Thrice	3	6	9	12	15	18	21	24	27	30
4 times	4	8	12	16	20	24	28	32	36	40
5 times	5	10	15	20	25	30	35	40	45	50
6 times	6	12	18	24	30	36	42	48	54	60
7 times	7	14	21	28	35	42	49	56	63	70
8 times	8	16	24	32	40	48	56	64	72	80
9 times	9	18	27	36	45	54	63	72	81	90
10 times	10	20	30	40	50	60	70	80	90	100

Second Table.

	1	2	3	4	5	6	7	8	9	10
11 times	11	22	33	44	55	66	77	88	99	110
12 times	12	24	36	48	60	72	84	96	108	120
13 times	13	26	39	52	65	78	91	104	117	130
14 times	14	28	42	56	70	84	98	112	126	140
15 times	15	30	45	60	75	90	105	120	135	150
16 times	16	32	48	64	80	96	112	128	144	160
17 times	17	34	51	68	85	102	119	136	153	170
18 times	18	36	54	72	90	108	126	144	162	180
19 times	19	38	57	76	95	114	133	152	171	190
20 times	20	40	60	80	100	120	140	160	180	200

Third Table.

	11	12	13	14	15	16	17	18	19	20
11 times	**121**	132	143	154	165	176	187	198	209	220
12 times		**144**	156	168	180	192	204	216	228	240
13 times			**169**	182	195	208	221	234	247	260
14 times				**196**	210	224	238	252	266	280
15 times					**225**	240	255	270	285	300
16 times						**256**	272	288	304	320
17 times							**289**	306	323	340
18 times								**324**	342	360
19 times									**361**	380
20 times										**400**

EXERCISES ON THE MULTIPLICATION TABLE.

(*Oral.*)

1. How much is 7 times 6 ? 8 times 9 ? 12 times 12 ? etc.

2. Multiply 12 by 8 ; 9 by 7 ; 16 by 9 ; etc.

3. What is the product of 9 and 9 ? of 16 and 6 ? etc.

4. What is the sum of 6 repeated 9 times ? 15 repeated 8 times ? etc.

5. What number is as great as 10 times 11 ? 7 times 9 ? etc.

6. If 9 boys have 6 marbles each, how many have they al together ?

7. How many rupees are there in 12 boxes, each containing 11 rupees ?

8. Sixteen annas make a rupee ; how many annas are there in 5 rupees ?

9. Fifteen boys sit on each form in a school, and there are fifteen forms ; how many boys are there ?

10. The multiplicand is 11 and the multiplier is 13 ; what is the product ?

11. The factors of a product are 9 and 19, what is the product ?

12. When mangoes are 20 for a rupee, how many can you buy for 5 rupees ?

13. There are 7 days in a week ; how many days are there in 8 weeks ?

14. In a house of 4 stories there are 15 rooms on each story ; how many rooms are there in the house ?

15. If a cow be worth 15 rupees, how much will you have to pay for 9 cows ?

16 On a page of a book there are 17 lines, and each line contains 19 letters ; how many letters are there in the page ?

17. By how much is 7 times 11 less than 90 ?

18. By how much is 3 times 16 greater than 35 ?

19. What number exceeds 9 times 9 by 19 ?

20. How many legs have 7 horses and 3 cows got altogether ?

35. We now proceed to show how large numbers are multiplied.

Example 1. Multiply 2095 by 3.

We arrange the numbers thus :

$$\begin{array}{r} 2095 \\ \underline{3} \\ 6285 \end{array} \text{ product.}$$

The product is found in the following way :

3 times 5 units is **15** units ; we set down **5** in the place of units, and *carry on* 1 for adding to tens : next, 3 times 9 tens is 27 tens, and adding 1 *carried*, the result is **28** tens ; we set down **8** in the place of tens, and *carry on* 2 for adding to hundreds : next, 3 times 0 is 0,* and adding 2 *carried*, the result is **2** hundreds ; we set down **2** in the place of hundreds : then, 3 times 2 thousands is **6** thousands ; and we set down **6** in the place of thousands. Thus the product is 6285.

Mental Process : 3 times 5, **15** ;
carry 1, 3 times 9, **28** ;
carry 2, **2** ;
3 times 2, **6** ;

N.B. The student will see that the above short process is substantially the same as the following extended process of addition.

```
2095
2095
2095
----
6285
```

Example 2. Multiply 12149 by 5.

```
12 14 9        Process :
      5           9×5=45 ; set down 5, carry 4,
60 74 5           14×5=70 ; 70+4=74 ; set down 74,
                  12×5=60 ; set down 60.
```

EXAMPLES. 8.

Multiply

1. 23 by 2.	**2.** 32 by 3.	**3.** 21 by 4.
4. 39 by 5.	**5.** 47 by 6.	**6.** 58 by 9.
7. 98 by 8.	**8.** 76 by 9.	**9.** 85 by 9.
10. 329 by 3.	**11.** 405 by 7.	**12.** 879 by 9.
13. 3245 by 6.	**14.** 7089 by 5.	**15.** 9206 by 8.
16. 78956 by 4.	**17.** 89035 by 7.	**18.** 85503 by 9.

19. 34079 by 2, 3, 4, 5, 6, 7, 8, 9.

20. Find the value of 725+725+725+725+725.

Supply the missing figures in the following multiplication sums

```
21.   4 * 2 7 *          22.   * 2 * 6 *
              9                        7
    -----------              -----------
    * * 1 * * 7              2 * * 2 * 3
```

*0×3=0 ; for 0+0+0=0.

36. If we write a cipher to the right of a number its value is increased tenfold ; hence, when we multiply a number by 10, the product is obtained by annexing 0 to the number. Thus $23 \times 10 = 230$. Similarly, when we multiply a number by 100, 1000,... the product is obtained by annexing 00, 000,... to the number.

Also, if we have to multiply a number by 30, we may first multiply it by 3, and then annex 0 to the result ; the final result will be the product required. So also, if we have to multiply by 300, we may first multiply by 3 and then annex 00 to the result.

Example. Multiply 329 by 600.

Process :
```
 329
 600
------
197400  Ans.
```

EXAMPLES. 9.

Find the product of

1. 359 by 30.
2. 7035 by 40.
3. 3905 by 50.
4. 703 by 600.
5. 39 by 900.
6. 8229 by 700.
7. 3005 by 8000.
8. 9004 by 9000.
9. 30303 by 6000.
10. 7295 by 90, 800, 7000, 60000, 500000.

37. It is clear from the definition of multiplication that, if we have to multiply a number by 5, we may multiply it separately by 2 and 3, and then add the two results : the final result will be the product required : if we have to multiply a number by 23 we may multiply it separately by 3 and 20, and then add the two results.

Example 1. Multiply 728 by 329.

```
(A)    728                              (B)     728
       329                                      329
      ------                                   ------
      6552 = product by   9.                   6552
     14560 =    „      „  20.                 1456
    218400 =    „      „  300.               2184
    ------                                   ------
    239512 = product by 329.                 239512
```

Here, to obtain the product of 728 by 329, we multiply 728 by 9, 20 and 300 separately, and add the three results. The partial products are found by the methods explained in the two preceding articles.

In practice we do not annex the zeroes in multiplying by 20 and 300 (because they have no effect in the addition which we perform afterwards) and our work stands as at (B).

OBSERVE that the multiplier must be placed under the multiplicand as in Addition ; also that, *in all cases*, the first figure on the right of each partial product must be placed in the same vertical column with the figure by which the product is obtained.

Note 1. We may multiply by the figures of the multiplier in any order we like, bearing in mind the foregoing observation.

(1)		(2)	
728		728	
329		329	
1456	by 2.	2184	by 3.
2184	by 3.	1456	by 2.
6552	by 9.	6552	by 9.
239512		239512	

Note 2. When the multiplier or multiplicand or both end with ciphers, it is convenient first to omit them in working and then to annex as many ciphers to the product as have been omitted.

Example 2. Multiply 37008 by 4203 4309 by 12300 ; 290 by 243 ; and 40300 by 4370.

(1)	(2)	(3)	(4)
37008	4309	290	40300
4203	12300	243	4370
111024	12927	87	2821
74016	8618	116	1209
148032	4309	58	1612
155544624	53000700	70470	176111000

38. Checking of multiplication results.—(i) Interchange multiplier and multiplicand and repeat multiplication. (ii) For the method of "**casting out the nines**" see Art. 39.

EXAMPLES. 10.

Perform the following multiplications :

1. 375 × 54. **2.** 904 × 98. **3.** 740 × 69.
4. 4972 × 345. **5.** 8762 × 904. **6.** 8072 × 972.
7. 708 × 708. **8.** 8463 × 340. **9.** 8239 × 5009.
10. 89025 × 8007. **11.** 90407 × 6050. **12.** 123456 × 70809.
13. 863400 × 70600. **14.** 820078 × 90072. **15.** 480390 × 8907.
16. 8573056 × 900082. **17.** 7390250 × 3009000.
18. 9876507 × 39421. **19.** 3700 × 809025000.
20. 8976543 × 978653. **21.** 370304 × 6070370.
22. 307650 × 90060. **23.** 784692 × 80075.
24. 830038 × 700208. **25.** 3257650 × 3257650.
26. 35756 × 6570002. **27.** 209030 × 400800600.

Obtain the following products by using one line of multiplication only :

28. 4329 × 11. **29.** 3809 × 12. **30.** 7204 × 13.

31. 7082 × 14. **32.** 4890 × 15. **33.** 8789 × 16

34. 13570 × 17. **35.** 28070 × 18. **36.** 4356 × 19.

37. There are 192 pies in a rupee ; how many pies are there in 3705 rupees ?

38. A book contains 570 pages, and each page contains 3749 letters ; how many letters are there in the whole book ?

39. If the price of one cottah of land in Calcutta be 975 rupees, what is the price of 325 cottahs ?

40. If 29390 persons cross the Hughly Bridge daily, how many cross in a year of 365 days ?

41. What is the weight of 739 bags of rice, each weighing 28 maunds ?

42. How many rupees must be paid for 6 elephants at 3479 rupees each, and 16 horses at 765 rupees each ?

43. A cistern has a leak by which 78 tolas of water come out per hour ; if the full cistern is emptied in 48 hours, how many tolas of water does the cistern hold ?

44. Supply the missing figures in the following multiplication sums :

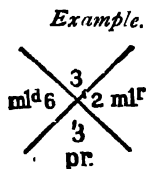
```
(i)   8 2 3           (ii)  * 5 2 *
        * *                   3 *
    ---------             ---------
    * * * 4               7 * * 2
      * * * 5               * * * 8
    ---------             ---------
    * * * * *             8 * * * *
```

CASTING OUT THE NINES.

39. The following method called **"casting out the nines"** is frequently employed in testing the correctness of the result of multiplication.

From the sum of the digits in the multiplicand cast out the 9's (i.e., subtract 9 as many times as possible) and set down the remainder ; do the same thing with the multiplier ; multiply the two remainders together ; cast out the 9's from the result, and set down the remainder ; then if the multiplication has been performed correctly, the last remainder will be the same as he remainder obtained by casting out the 9's from the sum of e digits in the product.

Example. $186 \times 47 = 8742$.



The sum of digits in $186 = 15$; $15 - 9$ gives rem. 6 ; the sum of digits in $47 = 11$; $11 - 9$ gives rem. 2 ; $6 \times 2 = 12$; $12 - 9$ gives rem. 3.
Sum of digits in $8742 = 21$; $21 - 9 = 12$; $12 - 9$ gives rem. 3.

N. B. The test will fail if such a mistake has been committed, as does not affect the sum of the digits of the product, or, increases or decreases it by 9 or a multiple of 9.

EXAMPLES. 11.

Multiply, and test the result of multiplying :

1. 3756 by 738. 2. 8943 by 826. 3. 3789 by 989.
4. 30804 by 3080. 5. 78093 by 8034. 6. 73980 by 3001.
7. 39400 by 3900. 8. 803075 by 390. 9. 823794 by 8234.

40. *Example.* Find the **continued product** of 28, 8 and 3.

We multiply 28 by 8, and the product by 3, the final result being 672.

```
 28
  8
---
224
  3
672 Ans.
```

EXAMPLES. 12.

Find the following continued products :

1. $27 \times 8 \times 2$. 2. $703 \times 85 \times 79$ 3. $8050 \times 70 \times 30$.
4. $59 \times 85 \times 76 \times 5$. 5. $3205 \times 9 \times 8 \times 5$. 6. $99 \times 88 \times 77 \times 66$.
7. How much is twice nine times seventy-three ?

8. A day contains 24 hours, an hour contains 60 minutes and a minute contains 60 seconds ; how many seconds are there in a day ?

9. 5 tolas make a chatak ; 16 chataks make a seer ; 40 seers make a maund ; how many tolas are there in a maund ?

10. A book contains 329 pages, each page contains 27 lines, and each line contains 45 letters ; how many letters are there in the whole book ?

11. How many mangoes are there on a tree which has 29 branches, each branch containing 325 mangoes ?

12. In a railway train there are 46 carriages ; each carriage has 6 compartments ; and each compartment contains 8 persons : how many persons are there in the train ?

41. The *second*, *third*, *fourth*... **power** of a number is the product of *two*, *three*, *four*,... factors each equal to that number. There the second power of $2=2\times2=4$; the third power of $2=2\times2\times2=8$. The second power of a number is called its **square**, the third power its **cube**. The number itself is often called its *first* power.

The symbol 4^2 is used to express 4×4; also, 4^3 is used to express $4\times4\times4$; and so on. The small figures 2, 3, are called **indices** or **exponents** of the power.

The process of finding any power of a number is called **involution**.

EXAMPLES. 13.

Find the square of

1. 1, 2, 3, 4, 5,...19, 20. **2.** 24. **3.** 50. **4.** 68.
5. 100. **6.** 112. **7.** 248. **8.** 729. **9.** 874.

Find the cube of

10. 1, 2, 3, 4,...19, 20. **11.** 93. **12.** 100. **13.** 879.
14. 555. **15.** 309. **16.** Find the value of $25^2+40^3-12^3+2^4$.

42. Combined Multiplication and Subtraction.—It is important that the student should acquire facility in combining multiplication and subtraction in *one* operation in a question like the following :

Example. Subtract 7 times 347 from 3283.

Mental Process : 7 times 7, 49, and **4**, 53. *Carry* 5.

7 times 4, 28, 33, and **5**, 38. *Carry* 3.

7 times 3, 21, 24, and **8**, 32.

$$\begin{array}{r} 3283 \\ 347 \\ 7 \\ \hline 854 \end{array}$$

N. B. In the first step we have taken the least number greater than 49 with 3 in the units' place, *i.e.*, 53. In the second step, we have taken the least number greater than 33 with 8 in the units' place, *i.e.*, 38 and so on.

EXAMPLES. 14.

Subtract

1. 329×8 from 4827. **2.** 732×9 from 82170.
3. 3798×6 from 894670. **4.** 9378×7 from 369812.
5. 7384×11 from 100000. **6.** 369×12 from 89468.

Add

7. 389×4 to 39. **8.** 894×9 to 786.
9. 7345×12 to 3940. **10.** 39874 to 329×16.

IV. DIVISION.

43. **Division** is the operation by which we find **how often** one given number, called the **Divisor,** must be subtracted from another given number, called the **Dividend,** so that the **Remainder,** if any, may be less than the first given number.

The *number* of times the subtraction is performed is called the **Quotient**.

It will be found that 7 units can be subtracted from 30 units, 4 times, and that then 2 units out of 30 remain over. Hence, when 30 is divided by 7, 30 is the *dividend*, 7 is the *divisor*, the *quotient* is 4 and the *remainder* is 2.

The sign of divison is $\div$. Thus $30 \div 7$ signifies that 30 is to be divided by 7, and is read "30 *divided by* 7" or simply "30 *by* 7". The symbol $\frac{30}{7}$ is also used to denote the same operation of division.

44. It follows from the definition of division that

Divisor × Quotient + Remainder = Dividend.

When there is no remainder the division is said to be **exact**. In this case division may be explained as the *inverse* of multiplication, the quotient being the number whose product by the divisor is the dividend.

45. By division we break up a number (dividend) into equal parts : if the divisor represents the magnitude of a part, the quotient gives the number of the parts ; if the divisor represents the number of the parts, the quotient gives the magnitude of one of the parts.

Example 1. 30 oranges are divided among boys so that each boy gets 7 oranges ; how many boys get a share ? (*Ans.* 4 boys, 2 oranges remainder.)

Example 2. 30 oranges are divided equally among 7 boys ; how many does each boy get ? (*Ans.* 4 oranges each, 2 oranges remainder.)

N. B. **The teacher should explain how in both of these cases the result may be obtained by repeated subtractions.**

46. The division of numbers not greater than 400 by numbers not greater than 20 is effected by means of the Multiplication Table.

Example. Divide 59 by 7.

Here, we have to find how often 7 may be subtracted from 59, or in other words, *how many times 7 is contained* in 59.

We may find the quotient and the remainder by successive subtractions of 7 from 59. But we are saved the trouble of repreated subtractions by using a known result of the Multiplication Table. Thus, since 8 times 7 is 56, 59÷7 gives 8 as quotient and 3 as remainder.

EXERCISES IN MENTAL DIVISION.

1. How many times is 5 contained in 20? 8 in 72? 9 in 54? 14 in 14? 16 in 128? etc.

2. How many times can you subtract 7 from 56? 6 from 48? 9 from 81? 18 from 306? etc.

3. Divide 84 into 7 equal parts; 104 into 17 equal parts; etc.

4. What is the fourth part of 36? sixth part of 54? twelfth part of 108? etc.

5. In 54 how many times 4, and how many over? how many times 5, and how many over? etc.

6. What is the remainder when 7 is subtracted as often as possible from 64? 6 from 42? 8 from 84? etc.

7. Find the quotient and remainder when 43 is divided by 6; 70 by 8; 85 by 9; 190 by 16; etc.

8. How many times does the fourth part of 72 contain 3? fifth part of 70 contain 7? etc.

9. 135 mangoes were divided equally among 15 boys; how many did each get?

10. 54 oranges are distributed equally among the children of a family, and each one gets 9; how many children are there in the family?

11. There are 16 annas in a rupee; how many rupees are there in 144 annas?

12. I bought a dozen chairs for 72 rupees; what is the price of a single chair?

13. How many yards of cloth at 12 annas each can I buy for 180 annas?

14. How many dogs have 80 legs?

47. When the dividend and divisor are any numbers, the process of division is as follows:

Example. Divide 88909 by 24.

The form of the operation is

24) 88909 (3704 Quotient.
72
169
168
109
96
13 Remainder.

The explanation is this :

We first take 8, and we find that 24 is not contained in it : we therefore take 88 and find how often 24 is contained in 88, and as it is contained three times, we set down 3 as the first figure in the quotient ; then multiply 24 by 3 and subtract the result 72 from 88 : to the remainder 16 we bring down the next figure in the dividend ; then, as 24 is contained in 169 seven times, we set down **7** as the second figure in the quotient ; then multiply 24 by 7 and subtract the result 168 from 169 : to the remainder 1 we bring down the next figure in the dividend ; then, as 24 is not contained in 10 we set down **0** as the third figure in the quotient and bring down 9, the next figure in the dividend ; then, as 24 is contained in 109 four times, we set down 4 as the fourth figure in the quotient ; then multiply 24 by 4 and subtract the result 96 from 109. We thus obtain 3704 as quotient and 13 as remainder.

N. B. The student will see that in the above process what we really do is this : from the dividend we first subtract 3000 times 24, next from the remainder we subtract 700 times 24, and then from the second remainder we subtract 4 times 24 ; we therefore altogether subtract (3000 + 700 + 4) or 3704 times 24 from 88909. The form of this extended operation is shown at the side.

24) 88909 (3000
72000
16909 (700
16800
109 (4
96
Remr. 13 3704 Qt.

48. The Position of the Quotient.—In working out the above division sum we have placed the quotient to the right of the dividend. The following operation in which the qnotient has been placed *above* the dividend will be found more useful.

37*4
24) 88909
72 (thousands)
169 (hundreds)
168 „
109 (units)
96 „
13 „

3704 Quotient.
24) 88909
72
169
168
109
96
13 Remainder.

The process of division is the same as before. But it should be observed that

(i) The first digit (3) of the quotient is placed just above the last digit (8) of the partial dividend, (88 thousands), used in the first operation.

(ii) There must be a digit above each of the remaining digits of the dividend. A check is, thus, obtained on the number of digits in the quotient. The mistake of omitting a zero in the quotient is, therefore, avoided. As for example, here the tens figure of the dividend has not been used ; so there is a gap in the tens place of the quotient which is filled up with a zero.

(iii) This method gives the *local value* of each digit of the quotient as soon as it is written down. Thus, the first digit (3) of the quotient is placed above the thousands digit (8) of the dividend and, therefore, its local value is 3000.

THE ITALIAN METHOD OF DIVISION.

49. This is an abbreviated method of performing ordinary Long Division. In it at each stage of the work corresponding to a digit in the quotient, the multiplication and subtraction are combined in one operation and all the working is done mentally ; only the successive remainders are written down. (See Art. 42).

The following example will explain and illustrate this method of division.

For convenience of comparison it has also been worked out in full according to the ordinary method of Long Division.

Example. Divide 740619 by 592.

Ordinary Method.	*Italian Method.*
1251	1251
592) 740619	592) 740619
592	1486
1486	3021
1184	619
3021	27
2960	
619	
592	
27	

The mental work in the Italian Method of Division is as follows :—

(i) 1 two, 2, and 8, 10 ; set down 8, carry 1. 1 nine, 9, 10, and 4, 14 ; set down 4, carry 1. 1 five, 5, 6 and 1, 7. Set down 1. Bring down 6 from the dividend.

(ii) 2 twos, 4, and **2**, 6 ; carry 0. 2 nines, 18, and **0**, 18 ; carry 1. 2 fives, 10, 11, and **3**, 14. Bring down 1 from the dividend.

(iii) 5 twos, 10, and **1**, 11 ; carry 1. 5 nines 45, 46, and **6**, 52 ; carry 5. 5 fives, 25, 30. Bring down 9 from the dividend.

(iv) 1 two, 2, and **7**, 9 ; carry 0. 1 nine, 9, and **2**, 11, carry 1. 1 five, 5, 6.

Thus the quotient is 1251 and the remainder is 27.

Note. The student is advised to try this method for a few days. The practice will convince him of its superiority to the ordinary method. But it must be pointed out that the Italian Method is much more liable to error, and an error is difficult to detect. So beginners should take great care when using this method.

50. Checking of Division results.—(1) Multiply quotient by divisor, add remainder to the product ; result should be the dividend. (2) Divide dividend by quotient ; result should be the original divisor, with the same remainder as in question set. (3) Subtract remainder from dividend ; result should be the product of divisor and quotient. Use the method of '**casting out the nines**'.

EXAMPLES. 15.

Divide

1. 376 by 2.	**2.** 9234 by 2.	**3.** 7085 by 2.
4. 7000 by 3.	**5.** 8025 by 3.	**6.** 90126 by 3.
7. 82045 by 4.	**8.** 32813 by 4.	**9.** 45678 by 4.
10. 12345 by 4.	**11.** 100200 by 5.	**12.** 77777 by 5.
13. 90403 by 6.	**14.** 87345 by 6.	**15.** 78934 by 6.
16. 3789 by 7.	**17.** 45986 by 7.	**18.** 32480 by 7.
19. 38474 by 8.	**20.** 34509 by 8.	**21.** 16042 by 8.
22 72124 by 9.	**23.** 90001 by 9.	**24.** 78000 by 9.
25. 38972 by 10.	**26.** (1) 24560 by 10	(2) 32000 by 10.

27. Supply the missing figures so that

(1) 149* may be exactly divisible by 6 ;

(2) 33*9 " " " " " 9 ;

(3) 21*5 " " " " " 9 ;

(4) 64*6 " " " " " 8 ;

(5) 57*3 " " " " " 7.

Divide

28. 77777 by 11. **29.** 39042 by 16. **30.** 57084 by 19.

31. 72043 by 37. **32.** 10000 by 59. **33.** 70707[illegible] by 62.

34. 35896 by 88. **35.** 28923 by 329. **36.** 13013 by 269.

37. 89089 by 555. **38.** 398406 by 879. **39.** 700000 by 991.

40. 999999 by 8888. **41.** 3270457 by 1002.

42. 7766334 by 7634. **43.** 9997770 by 3906.

44. 47946387 by 7207. **45.** 987654321 by 8642.

46. 123456789 by 98765. **47.** 187654321 by 12345.

48. 1080924890 by 72034. **49.** 1200730092 by 897324.

50. 38407890901 by 90735. **51.** 208900563000 by 870056.

52. 297506823 by 708076. **53.** 567892314670 by 8976867.

54. 7801849202713 by 926 **55.** 9876540456789 by 999.

Perform the following divisions by the Italian method :

56. 38956 by 26. **57.** 95100 by 48 **58.** 10020 by 74.

59. 47500 by 91. **60.** 97856 by 141. **61.** 26534[illegible] by 584.

62. 36780 by 628. **63.** 30321 by 681. **64.** 809345 by 3456.

65. 2080400 by 5456.

66. The product of two numbers is 357435 ; one of them is 705 ; what is the other ?

67. How many men will receive 113 rupees each out of 4068 rupees ?

68. How often must 817 be taken to make up 431376 ?

69. What number multiplied by 493 will produce 6409 ?

70. I subtract 3405 from 780953, then subtract 3405 from the remainder, and so on : how often can I do this ?

71. The quotient is 307, the divisor 98 and the remainder 29 ; what is the dividen d ?

72. The population of a certain town is 345330, and one out of 45 dies annually ; how many die in a year ?

73. A gentleman's yearly income is 19500 rupees ; how much must he spend per week so that he may neither save nor borrow ? (There are 52 weeks in a year.)

74. A ship sails 125 miles a day ; how long will it take to sail a distance of 32000 miles ?

75. 2750 bottles are to be packed in boxes, each holding 125 bottles : how many boxes will be required.

SHORT DIVISION.

51. The process of division may be greatly shortened when the divisor does not exceed 20.

Example. Divide 8259 by 6.

6) 8259

Quot. 1376, rem. 3.

We draw a line under the dividend, and under this we set down the successive figures of the quotient, the multiplication, subtraction, etc being performed mentally.

EXAMPLES. 16.

Divide, employing Short Division,

1. 34561 by 2.	**2.** 78930 by 3.	**3.** 80358 by 4.
4. 12792 by 5.	**5.** 23057 by 6.	**6.** 98400 by 7.
7. 34567 by 8.	**8** 19870 by 9.	**9.** 34567 by 10.
10. 580046 by 11.	**11.** 807040 by 12.	**12.** 135689 by 13.
13. 450782 by 14.	**14.** 743080 by 15.	**15.** 935862 by 16.
16. 3890457 by 17.	**17.** 8207305 by 18.	**18.** 1234567 by 19.

19. Each of 3456789, 80704030 and 987654321 by 2, 3, 4, 5, 6,...19, 20 separately.

20. Work examples 1 to 26 of *Examples* 15 by Short Division.

VII. PROPOSITIONS IN THE FUNDAMENTAL OPERATIONS.

52. To find the sum of any number of the *natural numbers* beginning with 1.

RULE. *Multiply the last number by the next higher number, and divide the result by 2.*

Example 1. Add together 1+2+3+4+...+15.

Here the last number is 15, and the next higher number is 16; their product is 240: therefore the sum required = 240 ÷ 2 = 120.

Example 2. Add together 21+22+23+...+35.

Here, add together the numbers from 1 to 35, and also the numbers from 1 to 20; and subtract the latter sum from the former.

53. Given the sum and difference of two numbers, to find the numbers.

RULE. *To get the greater number, add the sum and difference, and divide the result by 2. To get the smaller number, subtract the difference from the sum, and divide the result by 2.*

Example 1. The sum of two numbers is 40 and their difference is 16 ; what is the greater number ?

Process : $40+16=56$; $56\div2=28$. *Ans.*

Example 2. The sum of two numbers is 59 and their difference is 11 ; what is the smaller number ?

Process : $59-11=48$; $48\div2=24$. *Ans.*

EXAMPLES. 17.

Find the value of

1. $1+2+3+\ldots+20$.

2. $1+2+3+\ldots+30$.

3. $1+2+3+\ldots+45$.

4. $1+2+3+\ldots+75$.

5. $1+2+3+\ldots+100$

6. $7+8+9+\ldots+50$.

7. $40+41+42+\ldots+90$.

8. $100+101+102+\ldots+200$.

9. The sum of two numbers is 376, and their difference is 114 ; what is the greater number ?

10. Find the greater of two numbers of which the sum is 89251 and the difference is 385.

11. The sum of two numbers is 83957, and their difference is 74821 ; what is the smaller number ?

12. Find the smaller of two numbers, of which the sum is 79358 and the difference is 3456.

13. The sum of two numbers is 8527 and their difference is 729 ; find the numbers.

14. Find the two numbers, of which the sum is 10000 and the difference is 888.

54 Multiplication by factors.

Example 1. Multiply 329 by 35. Here $35=7\times5$.

Process :

$$\begin{array}{r} 329 \\ 7 \\ \hline 2303 \\ 5 \\ \hline 11515 \end{array}$$ *Ans.*

Example 2. Multiply 1725 by 217, and by 721, making in each case only two partial multiplications.

(1)		(2)	
	1725		1725
	217		721
	12075		12075
	36225		36225
	374325 *Ans.*		1243725 *Ans.*

Here, we multiply by 7, and by 21; but the product by 21 is obtained by multiplying the first product by 3.

55. Abbreviated methods of multiplication.

(*a*) To multiply a number by 5, annex 0 to the number, and divide the result by 2. Thus, $172 \times 5 = 1720 \div 2 = 860$.

Example. Multiply 172 by 15.

	2) 1720	=	product by 10.	(1)
	860	=	product by 5.	(2)
Adding (1) and (2),	2580	=	product by 15.	

(*b*) To multiply a number by 25, annex 00 to the number, and divide the result by 4. Thus, $38 \times 25 = 3800 \div 4 = 950$.

Example 1. Multiply 38 by 35.

	4) 3800			
	950	=	product by 25.	(1)
	380	=	product by 10.	(2)
Adding (1) and (2),	1330	=	product by 35.	

Example 2. Multiply 38 by 75.

	4) 3800	=	product by 100.	(1)
	950	=	product by 25.	(2)
Subtracting (2) from (1),	2850	=	product by 75.	

(*c*) To multiply a number by 125, annex 000 to the number, and divide the result by 8. Thus, $89 \times 125 = 89000 \div 8 = 11125$.

(*d*) To multiply a number by 9, 99, 999, 9999,..., annex as many 0's as there are 9's in the multiplier, and from the result subtract the number itself. Thus, $345 \times 99 = 34500 - 345 = 34155$.

(*e*) To multiply by a number which differs but little from 10, 100, 1000, 10000,..., we employ a method similar to the above.

Example. Multiply 345 by 998.

345×1000	=	345000
345×2	=	690
By subtraction,		344310 *Ans.*

56. Abbreviated method of squaring a given number.

If the given number contains two figures :—To and from the given number add and subtract the unit figure ; multiply the two results together, and to the product add the square of the unit figure. If the given number contains three (or more) figures, take from the end two (or more) figures instead of the unit figure.

Example 1. Find the square of 47.

$$47+7=54\ ;\ 47-7=40\ ;$$
$$54\times 40=2160\ ;\ 7^2=49\ ;$$
$$\therefore\quad 47^2=2160+49=2209.$$

Example 2. Find the square of 346.

$$346+46=392\ ;\ 346-46=300\ ;\ 392\times 300=117600\ ;$$
$$\therefore\quad 346^2=117600+46^2.$$

Now, $46+6=52\ ;\ 46-6=40\ ;\ 52\times 40=2080\ ;\ 6^2=36\ ;$

$$\therefore\quad 46^2=2080+36=2116.$$

Hence $346^2=117600+2116=119716.$

EXAMPLES. 18.

Multiply, using factors not greater than 20,

1. 728 by 24. **2.** 8025 by 42. **3.** 9345 by 72.

4. 921 by 144. **5.** 872 by 280. **6.** 742 by 125.

Obtain the following products by two lines of multiplication only.

7. 7925×328. **8.** 825×729. **9.** 3842×321.

10. 392×365. **11.** 526×848. **12.** 734×4812

13. Obtain the product of 2356 by 125255 by three lines of multiplication.

14. Multiply 8273 by 147497 making only three partial multiplications.

Obtain the following products by the method of Art. 55.

15. 725×5. **16.** 329×5. **17.** 812×5. **18.** 84×25.

19. 729×25. **20.** 92×25. **21.** 93×125. **22.** 125×125.

23. 207×125. **24.** 112×99. **25.** 282×999. **26.** 204×9999.

27. 421×998. **28.** 4268×980. **29.** 827×9997.

30. 739×50. **31.** 371×15. **32.** 892×35.

33. 709×75. **34.** 304×15. **35.** 789×75.

Find, by the method of Art. 56, the square of

36. 35. **37.** 55. **38.** 86. **39.** 97.

40. 325. **41.** 465. **42.** 779. **43.** 895.

57. Division by factors.

When the divisor is a number which can be broken up into two or more small factors, the division may be effected conveniently, both for rapidity and accuracy, by dividing successively by these factors employing the method of Short Division. The smallest factor should be taken as the first divisor. But the great difficulty with beginners, in connection with division by factors, is the determination of the true **remainder**. The following examples explain and illustrate the processes to be used.

Example 1. Divide 15792 by 48. Here $48=6\times 8$.

Process :

```
6 ) 15792
8 )  2632
      329 quotient.
```

Example 2. Divide 934 by 24. Here $24=4\times 6$.

Process :

```
4 ) 934
6 ) 233 groups of 4, and 2 units remainder.
     38 groups of 24, and 5 groups of 4 remainder.
```

The quotient is 38.

The remainder $=2$ units $+5$ groups of 4

$=2+(5\times 4)$

$=2+20$

$=22.$

Example 3. Divide 78254 by 105. Here $105=3\times 5\times 7$.

Process :

```
3 ) 78254
5 ) 26084 groups of 3, and 2 units remainder.
7 )  5216 groups of 15, and 4 groups of 3 remainder.
      745 groups of 105, and 1 group of 15 remainder.
```

The quotient is 745.

The remainder $=2$ units $+4$ groups of $3+1$ group of 15

$=2+(4\times 3)+(1\times 15)$

$=2+12+15$

$=29.$

The work in *Examples* 2 and 3 may be set down conveniently as under :

Example 2.	*Example* 3.
4) 934	3) 78254
6) 233...2... 2	5) 26084...2... 2
Qt. 38...5...20	7) 5216...4...12
22 remainder.	Qt. 745...1...15
	29 remainder.

The following rule is easily deduced from the above examples:

In all cases,

The true remainder = 1st R + (2nd R × 1st dvr.)
+ (3rd R × 1st dvr. × 2nd dvr.) + etc.

38. Abbreviated methods of division.

(1) To divide a number by 10, 100, 1000,........., cut off one, two, three,.........,figures from the right of the number ; the figures cut off will give the Remainder and the remaining figures the Quotient. Thus, when we divide 53274 by 100, the quotient is 532, and the remainder is 74.

(2) To divide by any number ending with ciphers, cut off the ciphers from the divisor and as many figures from the right of the dividend ; then divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder annex the figures cut off from the dividend to get the total remainder. Thus, if we have to divide 3754 by 700, we divide 37 by 7, which gives 5 as quotient and 2 as remainder ; the total remainder is 254.

(3) To divide a number by 5, 15, 35 or 45 multiply the number by 2 and divide the result by 10, 30, 70 or 90 (by the above method) : divide the remainder by 2 to get the true remainder. Thus to divide 78 by 5, we multiply 78 by 2, getting 156 as the result ; this divided by 10 gives 15 as quotient and 6 as remainder ; the true remainder is 6 ÷ 2 or 3. Hence 78 divided by 5 gives 15 as quotient and 3 as remainder.

(4) To divide a number by 25 or 75, multiply the number by 4 and divide the result by 100 or 300 ; divide the remainder by 4 to get the true remainder.

(5) To divide a number by 125, multiply the number by 8 and divide the result by 1000 ; divide the remainder by 8 to get the true remainder.

(6) To divide by a number consisting of 9's only.

(i) Draw a barrier line as many digits from the right of the dividend as there are 9's in the divisor.

(ii) Take the digits to the left of the barrier line and set down as many digits as there are 9's in the divisor to the right, and the remaining digits to the left, of the barrier line.

(iii) Repeat the process so long as the number of digits in the dividend admits.

(iv) Add the numbers so obtained. The sum of the digits to the left of the barrier line is the quotient ; the sum of the digits to the right is the remainder.

(v) If in the process of addition any number is carried from the right to the left of the barrier line, add that number to the remainder.

Example 1. Divide 82361 by 99.

```
          823|61
            8|23
             | 8
Quotient  831|92  Remainder.
```

Example 2. Divide 84361 by 99.

```
     (A)               (B)

   843|61            843| 61
     8|43              8| 43
      | 8                |  8
   852|12            851|112  Subtract 99 from the remainder
      | 1              1| 99    and add 1 to the quotient.
Qt. 852|13 Rem.   Q. 852| 13  Remainder.
```

Example 3. Divide 186453 by 999

```
          186|453
             |186
Quotient  186|639  Remainder.
```

EXAMPLES. 19.

In the following examples employ Short Division.

1. 936 ÷ 24.	2. 736 ÷ 32.	3. 1890 ÷ 45.
4. 2856 ÷ 42.	5. 3312 ÷ 144.	6. 8274 ÷ 25.
7. 38920 ÷ 72.	8. 23456 ÷ 63.	9. 74829 ÷ 99.
10. 32034 ÷ 121.	11. 704568 ÷ 240.	12. 824506 ÷ 8^2.
13. 123456 ÷ 7^3.	14. 987654 ÷ 480.	15. 888888 ÷ 5^4.

Divide by the method of Art. 58 :

16. 3894 ÷ 10.	17. 3456 ÷ 100.	18. 89345 ÷ 1000.
19. 82746 ÷ 100.	20. 89346 ÷ 1000.	21. 123456 ÷ 10000.
22. 3892 ÷ 30.	23. 7892 ÷ 50.	24. 98467 ÷ 800.
25. 73568 ÷ 1900.	26. 736894 ÷ 16000.	27. 9876543 ÷ 12600.
28. 354693 ÷ 2900.	29. 7689246 ÷ 790.	30. 9234587 ÷ 3400.
31. 378 ÷ 5.	32. 4689 ÷ 5.	33. 1276 ÷ 5.
34. 7845 ÷ 25.	35. 82769 ÷ 25.	36. 137892 ÷ 25.

37. $83764 \div 125$.	**38.** $137891 \div 125$.	**39.** $3792 \div 125$.
40. $374 \div 15$.	**41.** $789 \div 35$.	**42.** $921 \div 45$.
43. $1234 \div 75$.	**44.** $1394 \div 65$.	**45.** $9246 \div 85$.
46. $5675463 \div 99$.	**47.** $6484536 \div 99$.	**48.** $45637290 \div 99$.
49. $17280639 \div 999$	**50.** $8932512 \div 999$.	**51.** $30822915 \div 9999$.

59. In a chain of operations of addition and subtraction the *order* of the operations is *from left to right.* Thus $8-5+4-2$ means that 5 is to be subtracted from 8, then 4 is to be added to the result, and then 2 is to be subtracted from the last result. But we shall get the same result if we subtract the sum of the negative terms from the sum of the positive terms; and this method is often more convenient.

In a chain of operations of multiplication and division the *order* of the operations is *from left to right.* Thus $24\times 4\div 2$ means that 24 is to be multiplied by 4, and then the result is to be divided by 2; $24\div 4\times 2$ means that 24 is to be divided by 4, and then the result is to be multiplied by 2; and $24\div 4\div 2$ means that 24 is to be divided by 4, and then the result is to be divided by 2.

When an expression contains all (or some of) the signs $+$, $-$, $\times$, $\div$, *the multiplication and division are to be worked before addition and subtraction.* Thus, in $7-6\div 2+5\times 3$, 6 must be divided by 2 before subtraction, and 5 must be multiplied by 3 before addition.

Example 1.
$$\begin{aligned} 8\div 2\times 6\div 2\div 3 &= 4\times 6\div 2\div 3 \\ &= 24\div 2\div 3 \\ &= 12\div 3 \\ &= 4. \end{aligned}$$

Example 2.
$$\begin{aligned} 7+2\times 6\div 4-12\div 6 &= 7+12\div 4-2 \\ &= 7+3-2 \\ &= 10-2 \\ &= 8. \end{aligned}$$

Example 3. Find the value of $3\times 10^3+4\times 10^2+5\times 10+7$.
$$\begin{aligned} 3\times 10^3+4\times 10^2+5\times 10+7 &= 3\times 1000+4\times 100+5\times 10+7 \\ &= 3000+400+50+7=3457. \end{aligned}$$

Example 4. Express 4567 in terms of 10.
$$\begin{aligned} 4567 &= 4000+500+60+7 \\ &= 4\times 1000+5\times 100+6\times 10+7 \\ &= 4\times 10^3+5\times 10^2+6\times 10+7. \end{aligned}$$

EXAMPLES. 20.

Find the value of each of the following expressions :

1. $6 \times 7 \div 3$ 2. $16 \div 8 \times 3$. 3. $20 \div 5 \div 2$.
4. $10 \div 5 \times 3 \div 2$. 5. $6 \times 5 \div 3 \times 2$. 6. $8 \times 6 \div 4 \div 3$.
7. $7 \times 3 + 5 \times 2$. 8. $16 \div 2 - 3 \times 2$. 9. $8 \div 2 - 6 \div 3$.
10. $6 \times 5 - 8 \div 4$. 11. $9 + 6 \div 2 - 8$. 12. $9 - 6 \div 2 + 8$.
13. $12 \div 4 \div 3 + 7 - 2 \times 4$. 14. $7 \times 6 - 3 \times 4 - 4 \times 5$.
15. $7 \times 8 \times 9 - 12 \times 3 - 18$. 16. $18 \div 2 - 6 \div 3 + 14 \div 2$.
17. $10^2 - 7 \times 3 + 6^2 \div 3^2$. 18. $828 \div 18 - 100 \div 5^2 + 23$.
19. $639 \div 9 \times 3 - 720 \div 8 \div 15 - 53 \times 2 + 22 \div 2 \times 9$.
20. $204 \times 3 \div 4 + 630 \div 7 \times 2 \div 3 - 4 \times 4 \times 9 \div 2 - 47 \times 3$.
21. Find the value of
  (i) $4 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 7$.
  (ii) $5 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 4 \times 10 + 8$.
22. Express in terms of 10 the following numbers :
  (1) 456 ; (2) 5078 ; (3) 6891 ; (4) 70891.

THE USE OF BRACKETS.

60. When an expression is enclosed in a **bracket** ( ), { }, or [ ], or placed under a **vinculum** ‾‾‾‾, the whole expression is affected by the sign that precedes or follows the bracket or vinculum.

Thus,

$2 \div (3+4)$ means that 2 is to be divided by the sum of 3 and 4.

$(2+3) \times 4$ means that the sum of 2 and 3 is to be multiplied by 4.

$13 - (3+5)$ means that the sum of 3 and 5 is to be subtracted from 13.

$7 - (3 + 4 - 2)$ means that the difference between 4 and 2 is to be added to 3, and the result to be subtracted from 7.

Hence to simplify an expression like the above, we are to perform the operations indicated inside the brackets before performing operations indicated outside the brackets.

Note. In a product the sign of multiplication is often omitted when one or more of the factors are enclosed in brackets.

Thus, $3(5-4)$ means $3 \times (5-4)$;

$(3+2)(4-2)$ means $(3+2) \times (4-2)$.

61. A bracket may be removed if it is preceded by the sign + : thus $8+(7-5+2)=8+7-5+2$.

A bracket preceded by the sign − may also be removed if the sign of every term within the bracket is changed, namely, + to − and − to + : thus $8-(7-5+2)=8-7+5-2$.

When several brackets occur one within the other, we must remove the innermost bracket first, then the next one and so on.

Example. Simplify $9-[3+\{7-(5-2)\}]$.

The expression

(i) $=9-[3+\{7-3\}]$	or (ii) $=9-[3+\{7-5+2\}]$
$=9-[3+4]$	$=9-[3+7-5+2]$
$=9-7$	$=9-3-7+5-2$
$=2.$	$=14-12$
	$=2.$

EXAMPLES. 21.

Simplify

1. $7-(2+3)$. **2.** $6-(5-2)$. **3.** $(6-2)\times 3$.

4. $(16-4)(5-3)$. **5.** $16-4(5-3)$. **6.** $(16\div 4)(5-3)$.

7. $(16+4)\div 5-3$. **8.** $16+4\div(5-3)$. **9.** $(16+4)\div(5-3)$.

10. $3+12\div(2\times 3)$. **11.** $20+\{8+(5-2)\}$. **12.** $20-\{8+(5-2)\}$.

13. $20-\{8-(5-2)\}$. **14.** $20-\{8-(5+2)\}$.

15. $17-\{9+2(3-1)\}$. **16.** $17-\{9-2(3+1)\}$.

17. $9-[7+\{4-(5-2)\}]$. **18.** $9+[7-\{4+(5-2)\}]$.

19. $4\div[3+4\div\{2+4\div(4-2)\}]$. **20.** $(10-2\times 3)\div(6\div 3)$.

21. $6-[10\div\{20-3(7-\overline{5-3})\}]$. **22.** $6-[6-5\{3-(2\div\overline{3-2})\}]$.

MISCELLANEOUS EXAMPLES. 22.

1. What number must be added to 3452 to make 6000 ?

2. What number must be taken from 3021 to leave 999 ?

3. The sum of two numbers is 8920, and the smaller number is 309 ; what is the greater number ?

4. The difference between two numbers is 379, and the greater number is 1000 ; what is the smaller ?

5. The difference between two numbers is 79 and the smaller number is 709 ; what is the greater number ?

6. What is the difference between the least number of five figures and the greater number of three figures ?

7. The dividend is 3792, the quotient 12 and the remainder 0; find the divisor.

8. What number multiplied by 304 will produce 3344?

9. The divisor is 321, the quotient 11 and the remainder 260; find the dividend.

10. What is the divisor when the dividend is 345, the remainder 5 and the quotient 20?

11. Find the sum of all the numbers of 3 digits, which you can form with the figures 3, 0, 4.

12. Find the difference between the greatest and the least number of 4 digits, that you can form with the figures 3, 2, 7, 8.

13. There are two numbers, of which the product is 7243491, and the greater number is 34007; find the difference between the two numbers.

14. Find the sum of the products, two and two, of 369, 217 and 648.

15. How many times can 23 be subtracted from 920550, and what will be the final remainder?

16. The product of two numbers is 173432, and half of one of them is 163; what is the other?

17. The product of two numbers is 123904, and double of one of them is 1408; what is the other?

18. How many times in succession must 201 be added to 3166 to make the final sum 10000?

19. How much must be added to the product of 75 and 83 to give the product of 75 and 85? How much must be subtracted to give the product of 74 and 83?

20. How often does the sum of 3692 and 2769 contain their difference?

21. What number multiplied by 37 will give the same product as 185 multiplied by 309?

22. In a division sum the divisor is 5 times and the quotient is 6 times the remainder which is 73; what is the dividend?

23. If, in dividing a number by 105, the operation be performed by short division by employing factors 3, 5, 7 in succession and the several remainders be 2, 4, 5, what is the complete remainder?

24. If when a number is divided continuously 7, 8 and 9 the remainders are 5, 3 and 6 respectively, what would be the remainder if the same number were divided by the continued product of 7, 8 and 9?

25. The quotient is 702, the remainder is 24, and the divisor 7 more than the sum of both ; what is the dividend ?

26. The sum of two numbers is 205, and one of them exceeds the other by 7 ; what are the numbers ?

27. Your age is 12 years ; your brother's age is 19 years ; what will be your brother's age when you are 16 years old ?

28. Find the sum of three numbers, the first of which is made up of 3908 and 78904, the second of which exceeds the first by 1740, and the third exceeds the difference of the other two by 7809.

29. There are two numbers ; the less is 94567, and the other exceeds it by 327 ; what is their sum ?

30. I have 3290 rupees in cash, 75000 rupees in Government promissory notes ; I owe 3525 rupees to *A* and 25 rupees less to *B* : how much am I worth ?

31. The sum of two numbers is 729, the less is 57 ; what is their difference ?

32. What number must be subtracted from the product of 329 and 412 to make it equal to their sum ?

33. A man sold 260 mangoes at 2 pice each, and 50 oranges at the rate of two for a pice ; how many pice did he get in all ?

34. Obtain the product of 3749 by 216636 by three lines of multiplication.

35. Multiply 7384 by 42428 in three lines.

36. If I had 300 rupees more, I could have paid a debt of 7[illegible]0 rupees and have 25 rupees over ; how much have I ?

37. In a game of cricket *A*, *B* and *C* together score 134 runs ; *B* and *C* together score 76 runs ; and *A* and *C* together score 100 runs ; find the number of runs scored by each.

38. *A* and *B* together have 79 rupees, *C* has 49 rupees less than what *A* and *B* together have, and *B* has 9 rupees more than *C* ; find what each has.

39. I bought a dog for 25 rupees, a cat for 15 rupees less, and a horse for 30 rupees more than twice the price of the cat and dog ; how much did I spend in all ?

40. A man, after selling oranges to three purchasers, found that he had a rupee worth left ; if he had sold 5 more oranges to each purchaser he would have only 3 left : at what rate per rupee did he sell the oranges ?

41. A cistern has two pipes attached to it ; by one of the pipes 24 seers of water enter into the cistern per minute, and by the other 14 seers go out in the same time : how much water will there be in the cistern if both the pipes are left open for 6 minutes ?

Also find how much the cistern holds if the empty cistern be filled in 10 minutes when both the pipes are open.

42. A gentleman's monthly income amounts to 250 rupees, and his monthly expenses amount to 175 rupees ; how much will he be able to save at the end of 2 years ? [A year = 12 months.]

43. A man's age is 59 years ; his brother is 7 years older than he, and his sister 12 years younger than his brother : what was the man's age when his sister was born ?

44. A man was 30 years old when his eldest son was born ; how old will his son be when he is 40 years old, and what will be the man's age when the son is 40 years old ?

45. Find a number such that if it be added 12 times to 60 the sum will be 780.

46. The distance from Calcutta to Goalundo is 152 miles ; a train starts from Calcutta at 7 A. M., and runs towards Goalundo at the rate of 19 miles an hour ; at what o'clock will it arrive there ?

47. Take any number, subtract from it the sum of its digits ; the result will be divisible by 9 without remainder.

48. If any number and the sum of its digits be each divided by 9, the remainders will be equal.

49. Take any number, multiply it by 2, add 16 to the product, divide the sum by 2, subtract the original number from the quotient ; the remainder will be 8.

50. The product of any three consecutive numbers is divisible by 6 without remainder.

51. Complete the division below by writing in the missing numbers in the first and second lines.

```
3|
 |______
7|______ remainder 1.
  3923  remainder 4.
```

52. Supply the missing figures in the following division sums :

(i) 7 | 3 * 1 *
    * 3 * —2.

(ii) 6 | 2 * 5 * 7
    * 2 5 * —1.

53. A boy had to divide 76428 by 123 He copied a figure wrong in the divisor, and obtained as his quotient 611 with remainder 53. What mistake did he make ?

54. In a long division sum the dividend is 4967 and the successive remainders from first to last are 3, 13, 22 ; find the divisor and quotient.

VIII. MEASURES OF MONEY AND REDUCTION.

62. In practice it is found convenient to use large units for measuring large quantities, and small units for measuring small quantities. Thus, we say that the price of a table is 20 rupees; the price of a book is 10 annas; the price of a toy is 3 pice.

A list of the relative magnitudes of the various units used for the measurement of quantities of the same kind is called a Table.

63. English Money Table.

4 Farthings (*q.* or *f.*)	make	1	Penny (1*d.*).
12 Pence	...	1	Shilling (1*s.* or 1/-).
20 Shillings	...	1	Pound or Sovereign (£1).

2 Shillings = 1 Florin. 5 Shillings = 1 Crown.
21 Shillings = 1 Guinea. 27 Shillings = 1 Moidore.

Note 1, 2 and 3 farthings are usually indicated by ¼*d.*, ½*d.* and ¾*d.* respectively.

The following coins are now in circulation in England:

Copper coins :—a farthing, a half-crown, a penny.

Silver coins :—a threepenny piece, a fourpenny piece (or *groat*) a sixpence (or *tester*), a shilling, a florin, a half-crown. a crown.

Gold coins :—a half-sovereign, a sovereign.

The *standard* of gold coin in England is 22 parts of pure gold and 2 parts of copper, melted together. Each of these 24 parts is called a *carat*. Pure gold is said to be 24 carats *fine* and standard gold 22 carats *fine*. The *standard* of silver coin is 37 parts of pure silver and 3 parts of copper. From a pound Troy of standard silver there are coined 66 *shillings*. In copper coinage 24 pennies are coined from one pound Avoirdupois of copper.

Gold coinage is the standard in England. Silver coinage is not a *legal tender* for more than 40*s.*, nor is copper coinage for more than 12*d.*

64. Indian Money Table.

3 Pies (*p.*)	make	1	Pice.
4 Pice or 12 Pies	...	1	Anna (1*a.*).
16 Annas	...	1	Rupee (R1).

Copper coins :—a pie, a half-pice, a pice, a double-pice.

Nickel coins :—an anna piece, a two-anna piece, a four-anna piece.

Silver coins :—a two-anna piece, a four-anna piece or quarter-rupee, an eight-anna piece or half-rupee, a rupee.

Gold coins :—a sovereign, a half-sovereign.

The *standard* of gold or silver coin in India is 11 parts of pure gold or silver and 1 part of alloy. The weight of a rupee = 180 grains Troy. A double-pice weighs 200 grains Troy.

The rupee and half-rupee are *legal tender* for any amount, other silver coins and the nickel and copper coins being a *legal tender* for the fractions of a rupee only.

The British Sovereign is now in circulation in India. But it is not a legal tender now. R1 = 1*s.* 6*d.*

REDUCTION.

65. A quantity expressed by means of a single unit is called **a simple quantity.** A quantity expressed by means of more than one unit is called a **compound quantity.** Thus, R7 is a simple quantity ; R3. 4*a.* 3*p.* is a compound quantity.

Reduction is the process by which we express (1) a simple or a compound quantity in terms of a lower unit, or (2) a simple quantity in terms of higher units.

66. I. DESCENDING REDUCTION.

***Example* 1.** Reduce R34. 7*a.* 6*p.* to pies.

Since R1 = 16*a.*, R34 = (16 × 34)*a.* = (34 × 16)*a.* = 544*a.*

∴ R34. 7*a.* = 544*a.* + 7*a.* = 551*a.*

Again, since 1*a.* = 12*p.*, 551*a.* = (12 × 551)*p.* = (551 × 12)*p.* = 6612*p.*

∴ R34. 7*a.* 6*p.* = 6612*p.* + 6*p.* = 6618*p.* *Ans.*

In practice the operations of multiplication and addition are combined, and the process stands thus :

```
 R     a.   p.
 34  .  7  .  6
 16
 ----
 551a.
  12
 -----
6618p.  Ans.
```

Note. Memorize : R1 = 16*a.* = 64 pice = 192*p.*

Example 2. Reduce £3. 7*s*. 4½*d*. to farthings.

Process :

£.	s.	d.
3 .	7 .	4½
20		
67*s*.		
12		
808*d*.		
4		
3234*f*. *Ans*.		

Note. Memorize : £1 = 20*s*. = 240*d*. = 960*f*.

EXAMPLES. 23.

Reduce to annas :

1. R39. 2. R104. 3. R7208. 4. R3698.
5. R7. 9*a*. 6. R23. 4*a*. 7. R37. 12*a*. 8. R51. 14*a*.

Reduce to pies :

9. R309. 10. R740. 11. R3402.
12. R201. 9*a*. 13. R112. 10*a*. 14. R704. 13*a*.
15. R27. 0*a*. 3*p*. 16. R39. 12*a*. 9*p*. 17. R67. 15*a*. 11*p*.

Reduce (i) to pice and (ii) to pies :

18. R3. 0*a*. 2 *pice*. 19. R7. 13*a*. 1 *pice*. 20. R9. 14*a*. 3 *pice*.

Reduce

21. R3705 to half-rupees. 22. R408 to quarter-rupees.
23. R78. 14*a*. to two anna pieces. 24. R3. 2*a*. to double-pice.
25. R30. 7*a*. to half-pice. 26. R7. 8*a*. 6*p*. to pice.

Reduce to shillings :

27. £720. 28. £240. 29. £709. 30. £305.
31. £20. 5*s*. 32. £26. 12*s*. 33. £30. 17*s*. 34. £35. 19*s*.

Reduce to pence :

35. £35. 36. £670. 37. £7020.
38. £45. 11*s*. 39. £50. 13*s*. 40. £76. 15*s*.
41. £3. 12*s*. 6*d*. 42. £9. 0*s*. 10*d*. 43. £7. 16*s*. 11*d*.

Reduce to farthings :

44. £1000. 45. £305. 17*s*. 46. £7. 12*s*. 9*d*.
47. £3. 7*s*. 3½*d*. 48. £7. 0*s*. 9¼*d*. 49. £2. 16*s*. 0¾*d*.

Reduce (i) to crowns, (ii) to sixpences and (iii) to fourpences :

50. £9. 5*s*. 51. £10. 10*s*. 52. £15. 15*s*.

Reduce

53. £2. 7*s.* 6*d* to half-crowns. **54.** £3. 3*s.* 9*d.* to threepences.

55. 300 half-crowns to farthings. **56.** 56 guineas to half-pence.

57. If the price of an orange be one pice, how many can you buy for R1. 9*a.* ?

58. A debt of £2. 7*s.* 7½*d.* is to be paid in farthings; how many will be required ?

59. How many one-anna books can be bought with R7. 13*a.* ?

60. For how many children can a treat be provided with R13. 12*a.* at 4*a.* a head ?

61. I gave away £1. 13*s.* to a number of beggars giving a penny to each; how many beggars were there ?

67. II. ASCENDING REDUCTION.

***Example* 1.** Reduce 1995 pies to R *a. p.*

Process :

12) 1995 *p.*
16) 166*a.* + 3*p.* rem.
R10. + 6*a.* rem.

Answer. R10. 6*a.* 3*p.*

***Example* 2.** Reduce 15723 farthings to £. *s. d.*

Process

4) 15723*f.*
12) 3930*d.* + 3*f.* rem.
20) 327*s.* + 6*d.* rem.
£16 + 7*s.* rem.

Answer. £16. 7*s.* 6¾*d.*

Note. Memorize :

1000 pies	R5 3*a.* 4*p.*
192 pies	R1.
100 pice	R1. 9*a.*
100 annas	R6. 4*a.*
240 pence	£1.
960 farthings	£1.

EXAMPLES. 24.

Reduce to R. *a. p.* :

1. 10000 *pies.* **2.** 30793 *pies.* **3.** 77777 *pies.*

4. 3948 *pies.* **5.** 7823 *pies.* **6.** 11111 *pies.*

7. 30303 *pies.* **8.** 47474 *pies.* **9.** 10001 *pies.*

10. 1090 *pice.* **11.** 3785 *pice.* **12.** 3082 *pice.*

13. 7082 *half-pice.* **14.** 8936 *half-pice.* **15.** 3840 *double-pice*

Reduce to £. *s. d.* :

16. 376 *pence.* **17.** 7023 *pence.* **18.** 8920 *pence.*

19. 1000 *farthings.* **20.** 10008 *farthings.* **21.** 3333 *farthings.*

22. 8040 *farthings.* **23.** 7929 *farthings.* **24.** 4408 *farthings.*

25. 379 *half-pence.* **26.** 3940 *threepences.* **27.** 27 *guineas.*

28. 390 *half-crowns.* **29.** 396 *sixpences.* **30.** 30 *moidores.*

31. I paid one pice to each of 960 beggars ; how many rupees did I spend ?

32. How much money will be required to buy 300 half-anna postage stamps ?

33. If you buy 720 oranges at one farthing each, how many shillings shall you have to pay to the fruit-seller ?

IX. COMPOUND ADDITION.

68. The following example will illustrate the method of adding together compound quantities.

Example. Add together R15. 11*a.* 6*p.*, R10. 9*a.* 9*p.*, R8. 13*a.* 10*p.*, and R26. 6*a.* 3*p.*

We first add the pies, and we find that there are 28 pies ; and this is equivalent to 2*a.*+4*a.* We place 4 under the column of pies and carry 2*a.* Next we add the annas, and we find that there are (with 2*a.* carried) 41 annas. Since 41*a.*= R2.+9*a.*, we place 9 under the column of annas and carry R2. Then we add the rupees and we find that there are (with R2 carried) 61 rupees. Thus the sum required is R61. 9*a.* 4*p.*

R.	*a.*	*p.*
15	11	6
10	9	9
8	13	10
26	6	3
61	9	4

EXAMPLES. 25.

Add together

	a.	*pice.*		*a.*	*pice.*		*a.*	*pice.*		*a.*	*pice.*
1.	3 .	2	**2.**	8 .	3	**3.**	12 .	3	**4.**	13 .	2
	7 .	3		12 .	1		7 .	1		10 .	3
	9 .	2		14 .	2		13 .	2		9 .	0
	6 .	3		10 .	3		15 .	3		8 .	1

	a.	*p.*		*a.*	*p.*		*a.*	*p.*		*a.*	*p.*
5.	9 .	9	**6.**	12 .	10	**7.**	7 .	6	**8.**	8 .	3
	10 .	4		7 .	7		12 .	7		9 .	11
	7 .	0		11 .	11		14 .	10		15 .	7
	13 .	11		14 .	8		13 .	4		12 .	9

9.

R.	a.	p.
9	12	3
15	7	1
9	0	2
10	2	3
8	7	0

10.

R.	a.	p.
12	13	3
7	12	9
20	8	7
31	14	3
12	12	0

R.	a.	p
22	12	3
33	13	8
14	14	0
3	9	2
7	7	11

12.

R.	a.	pice.
13	7	3
107	13	2
39	12	1
7	0	3
19	14	0
12	8	1
317	9	2

13.

R.	a.	p.
8	7	9
11	11	11
309	14	8
39	0	10
604	8	4
89	13	4
824	7	2

14.

R.	a.	p.
100	13	4
29	7	8
7	12	3
309	0	11
76	7	9
770	7	7
86	9	10

15.

R.	a.	p.
8	8	8
17	4	7
309	12	11
1234	13	10
239	8	9
26	4	3
7	3	6
29	14	5
100	7	8

16.

R.	a.	p.
349	15	4
1207	13	8
740	9	6
39	4	9
123	12	11
8	7	10
1286	13	7
836	9	2
63	10	8

17.

R.	a.	p.
896	9	8
64	11	2
42	9	11
4276	13	4
7624	3	7
72	8	3
726	12	10
3725	7	8
346	10	5

18.

£.	s.	d.
7	12	3
19	19	7
100	13	9
76	7	8
304	8	2

19.

£.	s.	d.
39	18	10
76	2	9
300	17	3
49	16	8
4	3	6

20.

£.	s.	d.
100	13	9
376	3	3
489	14	7
39	4	6
4	9	8

21.

£.	s.	d.
392	8	3
76	9	9
1396	7	8
300	13	2
39	19	1
4	12	3
7892	10	4

22.

£.	s.	d.
9	12	0
72	4	8
384	17	7
4782	6	2
400	19	3
92	13	4
4	6	6

23.

£.	s.	d.
346	19	3
46	12	4
39	13	0
4	8	7
9	12	0
13	14	4
5	12	0

24.

£.	s.	d.
3	4	$5\frac{1}{2}$
13	14	$10\frac{1}{4}$
527	19	$7\frac{1}{4}$
12	13	$3\frac{1}{2}$
5	7	$8\frac{3}{4}$
8	9	$6\frac{1}{2}$
5	12	$0\frac{3}{4}$
300	15	$10\frac{1}{2}$

25.

£.	s.	d.
300	1	$0\frac{1}{2}$
29	5	3
31	7	$2\frac{1}{2}$
4	13	$5\frac{1}{2}$
5	15	$7\frac{3}{4}$
6	19	$9\frac{1}{4}$
81	12	$11\frac{3}{4}$
390	11	$0\frac{1}{4}$

26.

£.	s.	d.
432	9	9
73	12	$2\frac{3}{4}$
820	13	$0\frac{3}{4}$
70	14	$9\frac{1}{4}$
8	15	2
9	16	$3\frac{1}{2}$
12	17	4
329	18	$7\frac{1}{4}$

X. COMPOUND SUBTRACTION.

69. The following examples will explain and illustrate the methods of subtracting one compound quantity from another.

Example 1. Subtract R7. 9*a*. 6*p*. from R12. 3*a*. 9*p*.

R.	*a*.	*p*.	
12	3	9	
7	9	6	
R4	10	3	*Ans.*

Here we have to find the quantity which being added to R7. 9*a*. 6*p*. makes up R12. 3*a*. 9*p*. We see that 6*p*. + 3*p*. = 9*p*.; we therefore put down **3** under the column of *pies*. Next, 9*a*. + 10*a*. = 19*a*. = R1 + 3*a*.; we put down **10** under the column of *annas* and carry R1 for adding to the *rupees of the subtrahend*; now, R1 (*carried*) + R7 + R4 = R12; and we place **4** under the column of *rupees*.

Example 2. Subtract R5. 9*a*. 5*p* from R13. 3*a*. 4*p*.

Second Method.

	R.	*a*.	*p*.
	13	3	4
	5	9	5
Ans.	7	9	11

When the figure in any place in the minuend is less than the figure in the corresponding place in the subtrahend, in the case of *pies* always subtract the figure in the subtrahend from **12** and add the difference to the figure in the minuend. Similarly, in the case of *annas*, always subtract the figure in the subtrahend from **16** and add the difference to the figure in the minuend.

5*p*. and 7*p*., 12*p*.; 7*p*. and 4*p*., 11*p*. Set down **11** under the column of *pies* and carry 1*a*.

9*a* and 1*a*. (*carried*), 10*a*.; 10*a*. and 6*a*., 16*a*.; 6*a*. and 3*a*., 9*a*. Set down **9** under the column of *annas* and carry R1.

R5 and R1 (*carried*), R6; R6 and R7, R13 Set down **7** under the column of *rupees*.

70. Combined Addition and Subtraction.

Example. Subtract from R20 3*a.* 2*p.* the sum of R3. 4*a.* 8*p.*, R6. 5*a.* 10*p.* and R7. 8*a.* 9*p.*

16 + 3 = 19.
2 + 12 = 14.

R.	a.	p.
20	3	2
3	4	8
6	5	10'
7	8'	9'
2	15'	11'

Process :

In the column of pies, cast out '12' and in the column of annas, cast out '16'. Thus beginning from the top of the subtrahends :

Pies—8, 18 ; cast out 12 and carry 6 ; continuing 6, 15 ; cast out 12 and carry 3 ; „ , 3 (*carried*), and **11**, 14. Set down **11** and put against it a mark to show that 1 is to be carried to the next column. Carry 3 annas corresponding to the 3 marks (').

Annas—3 (*carried*), 7, 12, 20 ; cast out 16 and carry 4 ; 4 and **15**, 19 ; set down 15 and put a mark against it to show that 1 is to be carried to the next column. Carry R2 corresponding to the 2 marks (').

Rupees—2 (*carried*), 5, 11, 18 : 18 and **2**, 20 ; set down **2**.

Hence the difference = R2. 15*a.* 11*p.*

N. B. When the last partial subtrahend is greater than the partial minuend put a mark (') against the figure set down. Here is the column of pies the last partial subtrahend 3 is greater than the partial minuend 2. So a mark is put against 11.

EXAMPLES. 26.

Subtract

1. R7. 9*a.* 2 *pice* from R13. 12*a.* 3 *pice.*
2. R28. 12*a.* 3 *pice* from R30. 9*a.* 2 *pice.*
3. R3. 9*a.* 1 *pice* from R13. 4*a.*
4. R9. 7*a.* 6*p.* from R13. 3*a.* 3*p.*
5. R39. 13*a.* 9*p.* from R79. 12*a.* 6*p.*
6. R3. 7*a.* 8*p.* from R13.
7. R13. 12*a.* 7*p.* from R29.
8. R14. 14*a.* 3*p.* from R15. 12*a.*
9. R69. 15*a.* 2*p.* from R80. 8*a.*
10. R91. 12*a.* 11*p.* from R150. 0*a.* 7*p.*
11. R726. 15*a.* 5*p.* from R1000. 13*a.* 4*p.*
12. R109. 10*a.* 3*p.* from R110. 0*a.* 9*p.*
13. £7. 17*s.* 9*d.* from £13. 7*s.* 4*d.*
14. £13. 16*s.* 7*d.* from £27. 12*s.* 4*d.*

15. £45. 19*s.* 11*d.* from £66. 18*s.* 8*d.* **16.** £7. 7*s.* 7*d.* from £10.

17. £13. 13*s.* 8¾*d.* from £15. 17*s.* 0½*d.*

18. £37. 7*s.* 6¼*d.* from £49. 0*s.* 3*d.*

19. Subtract the sum of ₨9. 12*a.* 3*p.*, ₨15. 7*a.* 9*p.* and ₨8. 7*a.* 8*p.* from ₨37. 2*a.* 4*p.*

20. Subtract the sum of ₨8. 13*a.* 3*p.*, ₨12. 6*a.* 8*p.* and ₨8. 7*a.* 9*p.* from ₨38. 2*a* 3*p.*

21. A man went to a bazar with ₨46. 3*a.* 6*p.* in his bag. He spent ₨21. 4*a.* 6*p.* for cloth, ₨7. 9*a.* 3*p.* for coal and ₨13. 9*a.* 9*p.* for rice. How much had he left?

XI. COMPOUND MULTIPLICATION.

71. *Compound multiplication* is a short method of finding the sum of a certain number of repetitions of a given compound quantity.

The process is as follows :

Example. Multiply ₨5. 12*a.* 4*p.* by 7, and by 35.

First Method :

7 times 4*p.* = 28*p.* = 2*a.* + 4*p.* ; set down 4 and carry 2. 7 times 12*a.* = 84*a.*, which with 2*a.* (*carried*) = 86*a.* = ₨5 + 6*a.* ; set down 6 and carry 5. 7 times ₨5 = ₨35 ; this with ₨5 (*carried*) gives ₨40 ; and setting down this, the required product is ₨40. 6*a.* 4*p.*

₨.	*a.*	*p.*	
5	12	4	
		7	
₨40	6	4	*Ans.*

Second Method :

	₨	*a.*	*p.*
₨5 × 7 =	35	0	0
12*a.* × 7 =	5	4	0
4*p.* × 7 =		2	4
₨5. 12*a.* 4*p.* × 7 =	₨40	6	4

Note. To multiply by 35 we multiply first by 7 and the product by 5.

72. Application of Money Tables.

(i) To multiply pies by 12.

RULE. *Consider pies as annas.*

For, 1*p.* × 12 = 12*p.* = 1*a.*

(ii) To multiply annas by 16.

RULE *Consider annas as rupees.*

For, 1*a.* × 16 = 16*a.* = ₨1.

(iii) To multiply pence by 12.

RULE. *Consider pence as shillings.*

For, 1*d.* × 12 = 12*d.* = 1*s.*

(iv) To multiply shillings by 20.

RULE. *Consider shillings as sovereigns.*

For, 1*s.* × 20 = 20*s.* = £1.

(v) To multiply annas by 8.

RULE. *Consider annas as rupees and divide by 2.*

For, 1*a.* × 8 = 8*a.* = a half-rupee.

(vi) Find the product of R7. 14*a.* × 8.

R7. 14*a.* × 8 = R8 × 8 − 2*a.* × 8 = R64 − R1 = R63.

EXAMPLES. 27.

(*Oral.*)

Find the value of

1. 2*a.* 9*p.* × 12 ; R3. 2*a.* × 16 ; R2. 5*p.* × 12 ; R6. 3*a.* × 16 ; R9. 11*p.* × 12 ; R7. 6*a.* × 16.

2. 2*d.* × 12 ; 3*s.* 9*d.* × 12 ; £6. 8*d.* × 12 ; 19*s.* × 20 ; £5. 18*s.* × 20 ; £7. 3*s.* × 20.

3. 8*s.* × 10 ; 18*s.* × 10 ; 16*a.* × 8 ; 15*a.* × 8.

4. R6. 12*a.* × 3 ; R9. 8*a.* × 12 ; R6. 8*a.* × 16.

EXAMPLES. 28.

Multiply

1. R3. 8*a.* 3 *pice* by 3, 5, 7.

2. R9. 12*a.* 6*p.* by 5, 7, 9.

3. R39. 14*a.* 11*p.* by 11, 13, 16.

4. £29. 18*s.* 9*d.* by 3, 7, 9.

5. £37. 15*s.* 4*d.* by 6, 8, 20.

6. £40. 7*s.* 10½*d.* by 5, 9, 12.

[In the following examples use the method of multiplication by factors.]

7. R2. 4*a* 2 *pice* by 21, 32, 25.

8. R39. 12*a.* 9*p.* by 56, 99, 100

9. R48. 13*a.* 8*p.* by 125, 121, 144.

10. £34. 16*s.* 3*d.* by 81, 64, 800.

11. £48. 12*s.* 0¾*d.* by 99, 72, 420.

Find the value of

12. 9 things at 3*a.* 4*p.* each.

13. 56 things at R2. 4*a.* each.

14. 81 things at 2*s.* 6*d.* each.

15. 100 things at 7*s.* 6½*d.* each.

16. 1000 yards of broadcloth at R5. 7*a.* 6*p.* per yard.

17. 700 copies of a book at 7*s.* 7½*d.* each.

18. 3000 maunds of wheat at R3. 5*a.* 6*p.* per maund.

73. When the multiplier is a large number and cannot be split up into factors, the following method should be used.

Example. Multiply ₹12. 8*a*. 7*p*. by 473.

First Method :

	₹	*a*.	*p*.	
	12 .	8 .	7	
			10	
	125 .	5 .	10	
			10	
	1253 .	10 .	4	
			4	
	5014 .	9 .	4	product by 400,
Multiplying 3rd line by 7,	877 .	8 .	10	... ... 70.
Multiplying 1st line by 3,	37 .	9 .	9	... ... 3.
Adding last 3 results,	₹5929 .	11 .	11	product by 473.

Second Method :

7*p*.
473
12) 3311*p*.
275*a* —11*p*.
8*a*. × 473 = 3784*a*.
16) 4059*a*.
₹253—11*a*.
₹12 × 473 = ₹5676
₹5929

Hence the product is ₹5929. 11*a*. 11*p*.

EXAMPLES. 29.

Multiply

1. ₹3. 4*a*. 2 *pice* by 23, 37. **2.** ₹7. 12*a*. 9*p*. by 37, 47.

3. ₹3. 13*a*. 6*p*. by 421, 704. **4.** ₹2. 12*a*. 3*p*. by 2175, 3070.

5. £4. 7*s*. 6*d*. by 511, 112. **6.** £3. 9*s*. 3*d*. by 3684, 1237.

7. £6. 11*s*. 0¾*d*. by 753, 829. **8.** £7. 0*s*. 1½*d*. by 1111, 1231.

9. A gentleman spends ₹7. 8*a*. 9*p*. every day ; how much does he spend in a year of 365 days ?

10. Find the cost of 503 maunds of rice at ₹3 9*a*. 3*p*. per maund.

XII. COMPOUND DIVISION.

74. The process of dividing a compound quantity by an *abstract number*, that is, of dividing it into a given number of equal parts, is as follows :

Example 1. Divide R138. 3*a*. 3*p*. by 29.

R138 ÷ 29 gives R4 as quotient and R22 as remainder; this remainder, together with 3*a*. = 355*a*.:

355*a*. ÷ 29 gives 12*a*. as quotient and 7*a*. as remainder; this remainder, together with 3*p*. = 87*p*.:

87*p*. ÷ 29 gives 3*p*. as quotient and no remainder.

Hence the quotient is R4. 12*a*. 3*p*.

```
       R.    a.  p.
29 ) 138 . 3 . 3 ( R4
     116
      22
      16
29 ) 355 ( 12a.
     29
      65
      58
       7
      12
29 ) 87 ( 3p.
     87
```

∴ The quotient is R4. 12*a*. 3*p*.

EXAMPLES. 30.

Divide

1. R72. 3*a*. 3 *pice* by 23.
2. R286. 11*a*. 1 *pice* by 59.
3. R455. 14*a*. 7*p*. by 61.
4. R850. 14*a*. 4*p*. by 79.
5. R1025. 6*a*. 8*p*. by 80.
6. R583. 6*a*. 6*p*. by 98.
7. R4981. 10*a*. 3*p*. by 325.
8. R5049. 12*a*. 5*p*. by 499.
9. £97. 10*s*. 3*d*. by 29.
10. £29. 5*s*. by 52.
11. £1279. 13*s*. 3*d*. by 23.
12. £4476. 2*s*. 5*d*. by 83.
13. £946. 5*s*. 6*d*. by 279.
14. £859. 5*s*. $\frac{1}{2}$*d*. by 365.

In the 10 following examples use the method of Short Division.

15. R13. 15*a*. 8*p*. ÷ 2.
16. R225. 13*a*. 8*p*. ÷ 4.
17. R728. 14*a*. 6*p*. ÷ 5.
18. R1007. 10*a*. 2*p*. ÷ 7.
19. R329. 11*a*. 4*p*. ÷ 8.
20. R1243. 8*a*. ÷ 9.
21. £29. 7*s*. 6$\frac{3}{4}$*d*. ÷ 3.
22. £333. 19*s*. 3*d*. ÷ 6.
23. £378. 16*s*. 10*d*. ÷ 8.
24. £3781. 0*s*. 9$\frac{3}{4}$*d*. ÷ 9.

Employ the method of Division by Factors in the 6 following examples.

25. R27. 10*a*. ÷ 24.
26. R160. 0*a*. 3*p*. ÷ 49.
27. R323. 2*a*. 8*p*. ÷ 56.
28. R683. 2*a*. 6*p*. ÷ 54.
29. £3522. 1*s*. 7*d*. ÷ 28.
30. £543. 11*s*. ÷ 42.

31. The price of 140 quires of paper is R32. 13*a*.; find the price of one quire.

32. If 55 copies of a book are sold for ₨34. 6*a.*, what is the price of a single copy?

33. If the cost of 2880 articles be ₨480, what is the cost of one article?

34. A man's wages for 30 days are £5. 5*s.*; what does he earn per day?

Note. When the divisor is 10, 100, 1000,......, the following method should be used.

Example 2. Divide ₨1345. 13*a.* 4*p.* by 100.

The division in each step is effected by cutting off the two figures from the right; the figures cut off give the remainder and the remaining figures give the quotient. [See Art. 58, (1).]

```
       ₨    a.  p.
100)13,45 . 13 . 4(₨13 7a. 4p. Ans.
      16
   a. 7 33
       12
   p. 4 00
```

EXAMPLES. 31.

Divide

1. ₨135. 12*a.* 6*p.* by 10 **2.** ₨376. 2*a.* 4*p.* by 10.

3. ₨279. 11*a.* by 100. **4.** ₨1245. 13*a.* 4*p.* by 100.

5. ₨4067. 11*a.* 4*p.* by 100. **6.** ₨6100. 8*a.* 4*p.* by 100.

7. ₨203. 2*a.* by 1000. **8.** ₨2135. 6*a.* 8*p.* by 1000.

9. £438. 6*s.* 8*d.* by 10. **10.** £227. 16*s.* 8*d.* by 10.

11. £511. 2*s.* 11*d.* by 100. **12.** £3007. 5*s.* 10*d.* by 1000.

Example 3. Divide ₨97. 2*a.* 9*p.* into 31 equal parts.

```
     ₨.  a.  p.
31 ) 97 . 2 . 9 ( ₨3
     93
      4
     16
31 ) 66 ( 2a.
     62
      4
     12
31 ) 57 ( 1p.
     31
     26
```

Here we have a remainder (26*p.*) after division, and we observe that if the quotient, ₨3. 2*a.* 1*p.*, be multiplied by the divisor the

product will be less than the dividend by 26*p.*; again, if ₨3. 2*a.* 2*p.* be multiplied by the divisor the product will be greater than the dividend by (31 − 26)*p.* or 5*p.* The last therefore is nearest to the correct result. Hence **to the nearest pie** the result is ₨3. 2*a.* 2*p.*

RULE. When there is a remainder after division, the quotient or the quotient increased by 1*p.* is the result *correct to the nearest pie*, according as the divisor is greater or less than *twice* the number of pies in the remainder. If the divisor is equal to twice that number, both the results are equally correct.

EXAMPLES. 32.

Find, to the nearest pie, the result of dividing

1.	₨35. 7*a.* 8*p.*	by 7.	**2.**	₨49. 12*a.* 3*p.*	by 10.
3.	₨67. 13*a.* 11*p.*	by 41.	**4.**	₨327. 8*a.* 6*p.*	by 100.
5.	₨427. 10*a.* 7*p.*	by 56.	**6.**	₨394. 11*a.* 2*p.*	by 100.
7.	₨727. 15*a.* 10*p.*	by 67.	**8.**	₨923. 14*a.*	by 100.

Find, to the nearest farthing, the result of dividing

9.	£27. 17*s.* 9*d.*	by 5.	**10.**	£42. 18*s.* 3*d.*	by 10.
11.	£333. 19*s.* 4*d.*	by 29.	**12.**	£498. 15*s.*	by 100.
13.	£557. 16*s.* 11*d.*	by 210.	**14.**	£876. 12*s.*	by 300.

Divide

15.	₨4912. 8*a.* 8*p.*	by 24.	**16.**	₨7895. 4*a.* 5*p.*	by 55.
17.	₨47892	by 731.	**18.**	₨98765. 9*a.* 1*p.*	by 1000.
19.	£7829	by 539.	**20.**	£85632. 10*s.* 10*d.*	by 670.

75. To divide a compound quantity by another of the same kind, that is, to find how many times the latter is contained in the former, we proceed as in the following example :

Example 1. How many times is ₨8. 6*a.* 7*p.* contained in ₨25. 3*a.* 9*p.* ?

We reduce the compound quantities to the same *expressed* lowest denomination, and then proceed as in Simple Division.

First Method :

₨8. 6*a.* 7*p.* = 1615*p.*; ₨25. 3*a.* 9*p.* = 4845*p.*

Now $4845 \div 1615 = 3$.

∴ ₨8. 6*a.* 7*p.* is contained in ₨25. 3*a.* 9*p.*, 3 times.

Second Method :

```
                R.   a.   p.
R8. 6a. 7p. ) 25 . 3 . 9 ( 3
              25 . 3 . 9
```

Here 8 is contained in 25 *three* times and we find by trial that 3 is the correct quotient.

Compare with simple division :

```
H.  T.  U.   H.  T.  U.
8   6   7 )  25  3   9 ( 2
             17  3   4
              8  0   5
```

Here we find that 3 is too big, so we take 2 for the quotient.

N. B. In both cases we find the quotient by *trial.*

Example 2. How many times is R2. 3*a.* 6*p.* contained in R28. 5*a.* 3*p.* ? Find the remainder.

```
                R.   a.   p.
R2. 3a. 6p. ) 28 .  5 . 3 ( 10+2
              22 .  3 . 0
              R6 .  2 . 3
              R4 .  7 . 0
              R1 . 11 . 3
```

Ans. 12 ; Remainder, R1. 11*a.* 3*p.*

Note 1. First take 10 times, 20 times, 30 times and so on. In the case of large sums the first method of *Example* 1 should be employed.

Note 2. The method of Art. 74 is called **partition** and the above method is called **quotition.**

EXAMPLES. 33.

How many times is

1. R15. 7*a.* 3*p.* contained in R139. 1*a.* 3*p.* ?
2. R20. 12*a.* 6*p.* ... ... R311. 11*a.* 6*p.* ?
3. R53. 10*a.* 9*p.* ... ... R1288. 2*a.* ?
4. £30. 7*s.* 3*d.* ... ... £637. 12*s.* 3*d.* ?
5. £17. 12*s.* 4*d.* ... ... £986. 10*s.* 8*d.* ?

Find the quotient and remainder in the division of

6. R211. 15*a.* 10*p.* by R7. 7*a.* 7*p.*
7. R376. 8*a.* 7*p.* by R17. 12*a.* 3*p.*

8. ₨304. 15*a*. 9*p*. by ₨7. 8*a*. 9*p*.

9. £784. 17*s*. 11*d*. by £23. 19*s*. 2*d*.

10. £976 by £9. 9*s*. 9*d*.

11. Divide ₨994. 13*a*. 3*p*. into equal parts, each of which is equal to ₨17. 7*a*. 3*p*.

12. Divide £286. 3*s*. 2*d*. into parts, each equal to £1. 11*s*. 1½*d*.

13. How many maunds of flour, at ₨4. 8*a*. 3*p*. per maund, can be bought for ₨1354. 11*a*. ?

14. How many rupees of 1*s*. 6*d*. each are equivalent to £234 ?

15. A servant whose pay for a day is 2*a*. 6*p*., is fined 9*p*. if he comes in late, and at the end of 20 days he receives ₨2. 12*a*. 9*p*. ; how often was he late ?

16. Multiply ₨18957. 13*a*. by ₨189. 9*a*. 3*p*. ; and divide the same sum by the same sum. Shew that one of these operations is absurd and impossible, and perform the other.

XIII. MEASURES OF WEIGHT.

76. English Jewellers' or Troy Weight.

(*Chiefly used for weighing gold, silver and jewels.*)

24 Grains (gr.)	make	1	Pennyweight (1 dwt.)
20 Pennyweights	...	1	Ounce (1 oz.).
12 Ounces	...	1	Pound (1 lb.).

So that a pound Troy = 5760 Grains.

Diamonds and other precious stones are weighed by *carats*, each carat weighing about 3⅕ grains.

EXAMPLES. 34.

Reduce to grains :

1. 207 lb. 2. 29 lb. 8 oz. 3. 3 lb. 9 oz. 13 dwt. 15 gr.

4. 28 lb. 7 oz. 15 dwt. 5. 55 lb. 6 oz. 9 dwt. 6. 7 lb. 3 oz. 4 dwt. 9 gr.

Reduce to lb., etc. :

7. 7845 gr. 8. 8923 gr. 9. 57892 gr. 10. 100000 gr.

Addition.

11.

oz.	dwt.	gr.
3	17	23
9	12	7
7	7	15
6	3	2

12.

oz.	dwt.	gr.
11	13	21
9	2	19
8	17	13
6	15	4

13.

lb.	oz.	dwt.	gr.
3	10	7	9
4	3	9	3
7	7	8	12
8	9	3	13

14. Subtract 3 oz. 16 dwt. 14 gr. from 6 oz. 13 dwt. 12 gr.

15. Subtract 7 lb. 9 oz. 8 dwt. 20 gr. from 10 lb. 4 oz. 3 dwt. 4 gr.

16. Multiply 3 oz. 5 dwt. 16 gr. by 5, 32, 427.

17. Divide 15 lb. 11 oz. 13 dwt. 8 gr. by 23, and by 9 oz. 11 dwt. 16 gr.

18. Find the weight of 24 gold necklaces each weighing 2 oz. 7 dwt. 12 gr.

19. If 64 gold rings of equal weight are made of 1 lb. of gold, find the weight of each.

20. How many gold rings, each weighing 7 dwt. 12 gr., can be made out of 1 lb. 0 oz. 15 dwt. of gold ?

77. English Standard or Avoirdupois Weight.

16 Drams (dr.)	make	1 Ounce (1 oz.).
16 Ounces	...	1 Pound (1 lb.).
28 Pounds	...	1 Quarter (1 qr.).
4 Quarters	...	1 Hundredweight (1 cwt.).
20 Hundredweights	...	1 Ton (1 ton).

A stone (st.) = 14 lb.
A pound Avoir. = 7000 Grains Troy.

EXAMPLES. 35.

Reduce to drams :

1. 7 tons 13 cwt. **2.** 2 tons 2 cwt. 2 qr.

3. 3 tons 9 cwt. 3 qr. 21 lb. 9 oz. **4.** 9 tons 7 cwt.

5. 2 tons 3 cwt. 1 qr. **6.** 2 cwt. 3 qr. 20 lb. 11 oz. 12 dr.

Reduce to tons, etc.

7. 999999 dr. **8.** 123456 dr. **9.** 90000 gr. **10.** 1 billion gr.

Addition.

11.

lb.	oz.	dr.
7	7	10
9	9	7
12	15	6
3	12	12
4	4	3

12.

qr.	lb.	oz.
13	21	3
7	8	7
8	19	8
9	2	2
21	3	4

13.

tons	cwt.	qr.	lb.
1	16	3	19
2	8	3	0
	12	0	25
2	4	1	7
4	7	2	9

14. Subtract 7 lb. 8 oz. 9 dr. from 10 lb. 12 oz. 15 dr.

15. Subtract 2 tons 13 cwt. 3 qr. 12 lb. from 9 tons 2 cwt. 2 qr. 2 lb.

16. Multiply 7 cwt. 3 qr. 12 lb. 9 oz. 2 dr. by 7, 88, 329.

17. Divide 2 tons 10 cwt. 2 qr. 8 lb. 1 oz. by 29, and by 11 lb. 1 oz. 4 dr.

18. Find the weight of 625 iron balls, each weighing 7 lb. 8 oz.

19. The total weight of 56 bales of cotton is 7 tons 1 cwt.; what is the weight of each bale?

20. How many pick-axes, each weighing 4 lb. 6 oz., can be made from 1 ton 10 cwt. of iron?

21. Which is heavier, a pound of gold or a pound of feathers?

22. How many pounds Troy are equal to 144 pounds Avoir.?

78. Indian Bazar Weight.

4 Sikis	make	1 Tola.
5 Sikis	...	1 Kancha (Powa-chatak).
4 Kanchas or 5 Tolas	...	1 Chatak (1 ch.).
16 Chataks	...	1 Seer.
40 Seers	...	1 Maund (1 md.).

4 Chataks = 1 Powa. 4 Powas = 1 Seer.
5 Seers = 1 Punshury. 8 Punshuries = 1 Maund.

The weight of a rupee is called a tola. The Standard seer = 80 tolas. A tola = 180 grains Troy. 1 ton = 27 maunds (nearly) and 1 pound Avoir. = 39 tolas (nearly).

EXAMPLES. 36.

Reduce (i) to kanchas, (ii) to tolas:

1. 3 md. 7 seers 3 ch. **2.** 2 md. 20 seers 12 ch.

3. 1 md. 34 seers 15 ch. **4.** 2 md. 16 seers 2 powas.

5. 35 seers 3 powas. **6.** 2 md. 6 punshuries.

Reduce to md., etc.

7. 4664 kanchas. **8.** 3333 kanchas.

9. 39855 tolas. **10.** 100000 tolas.

Addition

11.

md.	seers	ch.
3	8	3
8	12	7
2	29	15
9	36	3
7	7	1

12.

md.	seers	ch.
13	22	7
7	36	13
12	21	8
4	32	9
2	20	2

13.

md.	seers	ch.	kanchas.
3	8	7	1
37	12	8	2
8	29	9	1
29	36	13	3
2	4	10	2

14. Subtract 3 md. 29 seers 7 ch. from 8 md. 17 seers 4 ch.

15. Subtract 2 md. 37 seers 12 ch. 2 kanchas from 10 md. 29 seers 7 ch.

16. Multiply 5 seers 10 ch. 3 kanchas by 9, 42, 2153.

17. Divide 71 md. 11 seers 9 ch. by 73, and by 2 md. 34 seers 1 ch.

18. Find the weight of 273 bags of rice, each bag weighing 2 md. 7 seers 3 ch.

19. If 44 bottles of equal size hold 1 md. 5 seers 6 ch. of ink, how much does one bottle hold?

20. 657 md. of flour are to be packed into bags holding 1 md. 1 seer 1 ch. each; how many bags will be required?

21. How many grains of gold are there in a plate weighing 1 seer 5 ch.?

22. If 3 chataks of gold be made into 36 equal rings, how many grains will each ring weigh?

79. **Madras Local Weight.**

3 Tolas	make	1 Pollum.
8 Pollums	...	1 Seer.
5 Seers or 40 Pollums	...	1 Viss.
8 Viss	...	1 Maund.
20 Maunds	...	1 Candy or Barum.

A Madras maund = 25 lb. Avoir.

80. **Bombay Local Weight.**

4 Dhans	make	1 Raktika.
8 Raktikas	...	1 Masha.
4 Mashas	...	1 Tank.
72 Tanks	...	1 Seer.
40 Seers	...	1 Maund.
20 Maunds	...	1 Candy.

A Bombay maund = 28 lb. Avoir.

XIV. MEASURES OF LENGTH.

81. English Linear Measure.

12 inches (in.)	make	1 Foot (1 ft.).
3 Feet	...	1 Yard (1 yd.).
$5\frac{1}{2}$ Yards	...	1 Pole, Rod or Perch (1 po.)
40 Poles or 220 yards	...	1 Furlong (1 fur.).
8 Furlongs or 1760 yards		1 Mile (1 mi.).
3 Miles	...	1 League (1 lea.).

1 Pole = 5 yd. 1 ft. 6 in.
9 Inches = 1 Span.
2 Spans or 18 Inches = 1 Cubit (*Hath*).
∴ 2 Cubits = 1 Yard.
6 Feet = 1 Fathom.
4 Poles or 22 Yards = 1 Chain } Used in land-surveying.
100 Links = 1 Chain }

Short lengths are usually expressed in *feet* and *inches* and *long distances* in *miles* and *yards*.

82. In reducing poles to yards, we multiply the number of poles by 11, and divide the product by 2. In the converse operation, we multiply the number of yards by 2, and divide the product by 11.

Example 1. Reduce 2 mi. 2 fur. 9 po. 3 yd. 1 ft. to inches.

```
           mi. fur. po. yd. ft.
Process :   2 . 2 . 9 . 3 . 1
            8
           18 fur.
           40
          729 po.
           11
     2 ) 8019 half-yd.
         4009 yd.+1 ft. 6 in. rem. [∵ a half-yd.=1 ft. 6 in.]
            3 yd.  1 ft.        added.
         4012 yd.  2 ft. 6 in.
            3
        12038 ft.
           12
       144462 in.  Ans.
```

Note. In reducing miles or furlongs to yards, it is convenient to reduce them at once to yards, unless we are prevented by the form of the question, as in the above example. Half-yards may be reduced directly to inches by multiplying the number of half-yards by 18 (∵ a half-yard = 18 inches).

Example 2. Reduce 201381 inches to miles.

Process :

```
12 ) 201381 in.
 3 )  16781 ft.    +9 in.
       5593 yd.    +2 ft.
          2
11 )  11186 half-yd.
40 )   1016 po.    +10 half-yd.
 8 )     25 fur.   +16 po.
          3 mi.    +1 fur.
```

∴ 201381 in. = 3 mi. 1 fur. 16 po. 10 half-yd. 2 ft. 9 in.
= 3 mi. 1 fur. 16 po. 5 yd. 2 ft. 9 in.
= 3 mi. 1 fur. 17 po. 1 ft. 3 in.
[∵ 5 yd. 1 ft. 6 in. = 1 po.]

If in a result the yd., ft. and inches exceed 5 yd. 1 ft. 6 in., we must substitute 1 po. for this.

EXAMPLES. 37.

Reduce to inches :

1. 125 yd. **2.** 5 fur. **3.** 3 mi. **4.** 2 lea.
5. 2 mi. 7 fur. 2 po. **6.** 3 mi. 2 fur. 20 po.
7. 3 lea. 5 fur. 11 po. **8.** 3 po. 4 yd. 2 ft.
9. 5 po. 3 yd. 1 ft. **10.** 7 po. 2 yd. 9 in.
11. 2 mi. 7 fur. 13 po. 4 yd. **12.** 2 lea. 6 fur. 20 po. 3 yd. 1 ft. 6 in.

Reduce to miles, furlongs, poles, etc. :

13. 156 yd. **14.** 202 yd. **15.** 107 yd. **16.** 196 yd.
17. 1234 in. **18.** 5890 ft. **19.** 73212 in. **20.** 80021 in.
21. 1000 in. **22.** 10000 ft. **23.** 234567 in. **24.** 987654 in.

XV. MEASURES OF AREA.

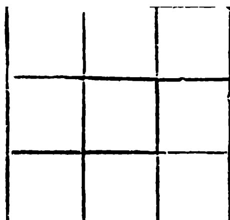
83. The **area** of any plane figure is the amount of surface included within its bounding lines.

A **square inch** is the amount of surface enclosed within a square of which each side is one inch in length.

A **square unit** (*i.e.*, a square inch, a square foot, etc.) is a square each of whose sides is one unit (*i.e.*, 1 inch, 1 foot, etc.) in length. It is used in measuring the area of a surface bounded by lines. Thus the area of any figure is measured by the number of square units (*i.e.*, square inches, square feet or other units of square measure) it contains.

1 Sq. in.

Let the adjoining figure represent a square the length of whose side is 1 yard. This is a "square yard". Divide each of the four sides into 3 equal parts and join the points which are just opposite to one another. The big square is now divided into smaller squares, each of which is a square foot. It is clear from the figure that there are three rows with three squares in each row, *i.e.*, in all 3×3 or 3^2 smaller squares; therefore 1 sq. yd. $=9$ sq. ft.



Similarly, if we took a square foot and divided each side into 12 parts and drew straight lines as above, we should see that a square foot is equal to 12×12, that is, 144 square inches.

Hence, from

12 in. = 1 ft., we get 12×12 or 144 sq. inches (sq. in.) = 1 square foot (1 sq. ft.)

3 ft. = 1 yd., „ „ 3×3 or 9 sq. ft. = 1 square yard (1 sq. yd.) etc. etc.

English Square Measure.

144	Square Inches (sq. in.) make		1 Square Foot (1 sq. ft.).
9	Square Feet	...	1 Square Yard (1 sq. yd.).
$30\frac{1}{4}$	Square Yards	...	1 Square Pole, Rod or Perch. [(1 sq. po.)].
40	Square Poles	...	1 Rood (1 ro.).
4 or 4840	Roods or sq. yards	...	1 Acre (1 ac.).
640	Acres	...	1 Square Mile (1 sq. mi.).

A square chain $=22 \times 22$ sq. yards or 484 sq. yards.

$\therefore$ 10 sq. chains = 1 acre.

1 sq. pole = 30 sq. yd. 2 ft. 36 in.

88. In reducing sq. poles to sq. yards, we multiply the number of sq. poles by 121, and divide the product by 4. In the converse operation, we multiply the number of sq. yards by 4, and divide the product by 121.

Example I. Reduce 2 ac. 1 ro. 13 sq. po. 12 sq. yd. 7 sq. ft. to sq. inches.

```
            ac. ro. po. yd. ft.
Process :    2 . 1 . 13 . 12 . 7
              4
              9 ro
             40
            373 sq. po.
             11
           4103
             11
  4 ) 45133 quarter sq. yd.
      11283 sq. yd.+2 sq. ft. 36 sq. in. [∵ a qr. sq. yd.=2 sq. ft. 36 sq. in.]
         12 sq. yd.   7 sq. ft.                 added.
      11295 sq. yd.   9 sq. ft. 36 sq. in.
          9
     101664 sq. ft.

    1219968
         13
   14639552 sq. in.  Ans.
```

[The learner should note that, 1 qr. sq. yd.=2 sq. ft. 36 sq. in. ; 2 qr. sq. yd.=4 sq. ft. 72 sq. in. ; and 3 qr. sq. yd.=6 sq. ft. 108 sq. in.].

Note. In reducing acres or roods to sq. yards, it is convenient to reduce them at once to sq. yards, unless we are prevented by the form of the question. Quarter sq. yards may be reduced directly to sq. inches by multiplying the number of quarter sq. yards by 18×18 (∵ a quarter sq. yd.=a sq. cubit$=18 \times 18$ sq. in.)

Example 2. Reduce 8753067 sq. inches to acres.

```
Process :        144 {12 ) 8753067 sq. in.
                     {12 )  729422...3 }
                          9 ) 60785...2 }   27 sq. in.
                               6753 sq. yd.+ 8 sq. ft.
                                  4
                 121 {11 )27012 quarter sq. yd.
                     {11 ) 2455...7 }
                        40 ) 223...2 }      29 qr. sq. yd.
                          4 )  5 ro.      +23 sq. po.
                               1 ac.      + 1 ro.
```

The result = 1 ac. 1 ro. 23 po. 29 qr. yd. 8 ft. 27 in.
= 1 ac. 1 ro. 23 po. 7 yd. 1 qr. yd. 8 ft. 27 in.
= 1 ac. 1 ro. 23 po. 7 yd. 10 ft. 63 in.
= 1 ac. 1 ro. 23 po. 8 yd. 1 ft. 63 in.

If in a result the sq. yd., ft. and inches exceed 30 sq. yd. 2 ft. 36 in. we must substitute 1 sq. po. for this.

EXAMPLES. 38.

Reduce to sq. inches :

1. 23 sq. yd. **2.** 3 roods. **3.** 120 ac. **4.** 2 sq. miles.
5. 7 ac. 2 ro. 8 po. **6.** 12 ac. 3 ro. 20 po. **7.** 1 ac. 1 ro. 1 po.
8. 3 sq. po. 7 yd. 7 ft. **9.** 5 sq. po. 3 yd. 2 ft.
10. 7 sq. po. 20 yd. 36 in. **11.** 2 ac. 3 ro. 7 po. 17 yd.
12. 3 ac. 2 ro. 17 po. 9 yd. 2 ft. 72 in.

Reduce to acres, roods, sq. poles, etc. :

13. 365 sq. yd. **14.** 740 sq. yd. **15.** 971 sq. yd.
16. 1000 sq. yd. **17.** 7824 sq. yd. **18.** 37821 sq. yd.
19. 93456 sq. ft. **20.** 87893 sq. ft. **21.** 7234 sq. in.
22. 78934 sq. in. **23.** 987650 sq. in. **24.** 9876543 sq. in.

Reduce

25. 7 sq. chains to sq. inches.
26. One million sq. links to sq. yards.

85. Land Measure of Bengal.

1 Square Cubit	makes	1 Ganda (1 ga.).
20 Gandas	make	1 Chatak (1 ch.).
16 Chataks	...	1 Cottah (1 cot.).
20 Cottahs	...	1 Bigha (1 bi.).

1 bigha = 1600 sq. yards.
121 bighas = 40 acres.
1936 bighas = 1 sq. mile.
1 acre = $\frac{121}{40}$ bighas = 3 bi. 8 ch.

86. Land Measure of the United Provinces.

20 Kachwansi	make	1 Biswansi.
20 Biswansi	...	1 Biswa.
20 Biswas	...	1 Bigha.

1 bigha = 3025 sq. yards.

87. Land Measure of the Punjab.

9 Square Karam or 9 Sarsai	make	1 Marla.
20 Marlas	...	1 Kanal.
4 Kanals	...	1 Bigha.
2 Bighas	...	1 Ghuma.

1 karam = 3 cubits. 1 bigha = 1620 sq. yards.

88. Land Measure of Madras.

144 Square Inches	make	1 Square foot.
2400 Square feet	...	1 Ground or Manai.
24 Grounds	...	1 Cawny.
484 Cawnies	...	1 Square Mile.

121 cawnies = 160 acres.

89. Land Measure of Bombay.

39½ Square Cubits	make	1 Kathi.
20 Kathis	...	1 Pand.
20 Pands	...	1 Bigha.
6 Bighas	...	1 Rukeh.
20 Rukehs	...	1 Chahur.

XVI. MEASURES OF SOLIDITY AND CAPACITY.

90. A **cube** is a solid figure contained by six equal squares. A **cubic inch** is a cube whose edge is an inch in length.

Measures of Solidity. (*English*).

1728 Cubic Inches	make	1 Cubic Foot (1 cu. ft.).
27 Cubic Feet	...	1 Cubic Yard (1 cu. yd.).

A ton of shipping = 42 cubic feet.

EXAMPLES. 39.

1. Reduce 3, 7, 12, 16, 20, 39 cu. yd. to cu. in.

2. Reduce 123456, 987654 cu. in. to cu. yd.

91. Measures of Capacity. (*English*).

4 Gills	make	1 Pint (1 pt.).	
2 Pints	...	1 Quart (1 qt.).	
4 Quarts	...	1 Gallon (1 gall.).	
2 Gallons	...	1 Peck (1 pk.).	
4 Pecks	...	1 Bushel (1 bus.).	
8 Bushels	...	1 Quarter (1 qr.).	For *dry* goods only.
5 Quarters	...	1 Load (1 ld.)	
2 Loads	...	1 Last (1 last).	

A *Barrel* contains 36 gallons.

Note. A gallon of distilled water weighs exactly 10 lb. Avoir. **A Pint of water weighs a pound and a quarter.** [A gallon contains 277·274 cubic inches.] A cubic foot of water weighs *about* 1000 oz. Avoir.

EXAMPLES. 40.

Reduce to gills :

1. 12 gall. 2 qt. 1 pt. **2.** 2 barrels 16 gall. **3.** 1 barrel 11 gall.

4. 6 bus. 2 pk. 1 gall. **5.** 4 qr. 4 bus. 2 pk. **6.** 1 ld. 3 qr. 7 bus.

7. 7 lasts 1 ld. 3 qr. **8.** 2 lasts 4 qr. 5 bus. **9.** 20 lasts 1 ld. 4 qr.

Reduce to barrels, gallons, etc. :

10. 1000 gills. **11.** 2073 gills. **12.** 3400 gills. **13.** 7225 gills.

Reduce to lasts, loads, quarters, etc. :

14. 3000 gills. **15.** 1500 gills. **16.** 25000 gills. **17.** 98765 gills.

18. What is the weight of 2 gall. 2 qt. of water ?

19. Give in pounds Avoir. the weight of 2 cu. yd. 2 cu. ft. of water.

XVII. MEASURES OF TIME, ANGLES, NUMBER, AND APOTHECARIES' WEIGHT.

92. Measures of Time. (*English.*)

60	Seconds (sec.)	make	1 Minute (1 min.).
60	Minutes	...	1 Hour (1 hr.).
24	Hours	...	1 Day (1 da.).
7	Days	...	1 Week (1 wk.).
365	Days	...	1 Year (1 yr.).
366	Days	...	1 Leap-year.
100	Years	...	1 Century.

Note 1. Each day is considered to commence at midnight.

Note 2. In rough calculations a month is taken to consist of 30 days. But the 12 months, called **Calendar** Months, into which the year is divided, are of variable length.

February has 28 days (and in Leap-year 29).

Thirty days have September,
April, June and November.

The other months have 31 days each.

Note 3. If the number of a particular year is divisible by 4 it is a Leap-year ; but centuries not divisible by 400 are not Leap-years. Thus 1888, 1732, 1600 are Leap-years ; 1887, 1739, 1800 are common years.

[The solar year consists of 365·242218 mean solar days (or 365 da. 5 hr. 48 min. 48 sec. very nearly) or nearly 365¼ days ; hence to make the civil year correspond with the solar, we take 3 consecutive years of 365 days and a fourth, called *leap-year*, of 366 days, those being leap-years of which the numbers are divisible by 4. But in this way we insert 100 days in 400 years, which is too much, for ·242218 × 400 is 96·8872 or 97 days nearly ; to make the necessary correction centuries not divisible by 400 are taken as common years.]

Note 4. The year contains 52 weeks and 1 day ($\because 52 \times 7 + 1 = 365$), but in calculating the income of men paid by the week, it is customary to consider the year to consist of 52 weeks.

EXAMPLES. 41.

Reduce to seconds :

1. 7 hr. 12 min. 3 sec. **2.** 7 da. 9 hr. 10 min. **3.** 2 wk. 3 da. 12 hr.

Reduce to weeks, days, hours, etc. :

4. 5000 sec. **5.** 98765 sec. **6.** One lac sec. **7.** One million sec.

Find the number of days (including one only of the two days named) from

8. 3rd Jan. to 7th April 1927. **9** 20th Jan. to 20th May 1928.

10. May 10th '27 to Jan. 9th '28. **11.** Aug. 1st '20 to March 1st '22.

12. 21st Feb. to 7th Dec. 1900. **13.** 30th Dec. '23 to 30th March '26.

14. The 1st January 1880 was on Monday ; what day of the week was June 20th of the same year ?

15. The 9th of December 1845 was on Sunday ; what day of the week was 1st January 1847 ?

93. Measures of Angles.

60 Seconds (60″) make		1 Minute (1′).
60 Minutes	...	1 Degree (1°).
90 Degrees	...	1 Right angle (1 rt. gle.).

EXAMPLES. 42.

Reduce to seconds :

1. 7°. 17′. 27″. **2.** 240°. 25′. 35″. **3.** 4 rt. gle.

Reduce to right angles, degrees, etc. :

4. 4000″. **5.** 37956″. **6.** 7000′. **7.** 8256′. **8.** 987654″.

94. Measures of Number.

	12 Units	make	Dozen.
	12 Dozen	...	Gross.
	12 Gross	...	Great Gross.
	20 Units	...	Score (*Kurri*).
Also	24 Sheets of paper	...	Quire.
	20 Quires	...	Ream.
	10 Reams	...	Bale.

EXAMPLES. 43.

1. In 50 reams of paper, how many sheets ?

2. How many reams, quires, etc. are there in fifty thousand sheets of paper ?

3. How many scores are there in 5 great gross ?

95. Apothecaries' Weight.

(i) *Measures of Weight.*

Druggists use the *grain* to weigh small quantities and the *pound* and *ounce* Avoir. to weigh large quantities. Some physicians in prescribing use the following table :

20 Grains	make	1 Scruple (1 scr.).
3 Scruples	...	1 Drachm (1 dr.).
8 Drachms	...	1 Ounce Troy.

(ii) *Measures of Capacity.*

60 Minims (m.) or drops	make	1 Fluid Drachm (fl. dr.).
8 Fluid drachms	...	1 Fluid Ounce (fl. oz.).
20 Fluid ounces	...	1 Pint (O.).
8 Pints	...	1 Gallon (C.).

A teaspoonful	=	1 Fluid drachm.
A dessertspoonful	=	$2\frac{1}{2}$ Fluid drachms.
A tablespoonful	=	4 Fluid drachms.

Note. Since a pint of water weighs a pound and a quarter, the weight of a fluid ounce of distilled water is an *ounce* Avoir.

EXAMPLES. 44.

Reduce

1. 2 oz. 2 dr. 2 scr. to grains. **2.** 3 oz. 3 dr. 12 gr. to grains.

3. 2 O. 12 fl. oz. to minims. **4.** 2 C. 4 O. to minims.

5. 7 C. 7 O. 15 fl. oz. 5 fl. dr. 9 m. to minims.

MISCELLANEOUS EXAMPLES. 45.

1. A girl can paper 2 pins in a second ; how many pins can she paper in a working day of 8 hours 30 minutes ?

2. Find the price of 3 md. 7 seers of milk at *2a.* *6p.* per seer.

3. Find the value of 12 lb. 7 oz. of gold at £3. 15*s.* 4½*d.* per oz.

4. A train travels 19 mi. 7 fur. 30 po. per hour ; how far will it travel in 24 hours ?

5. A fruit-seller sold 210 oranges at 1 pice each, 76 apples at 1 anna each, and 55 mangoes at *1a.* *6p.* each ; how much did he realise from the sale ?

6. How many cwt. of coal will supply 64 fires for 3 weeks, each fire consuming 1 cwt. 2 qr. 1 lb. per day ?

7. If the cost of 9 md. be R480, what is the cost of a chatak ?

8. If the cost of a ton be £203, what is the cost of a pound ?

9. How many shot, each weighing 2 oz. 3 dr., will make up a heap weighing one ton ?

10. How many parcels, each weighing 1 md. 10 seers, can be made up of goods weighing 132 md., and what weight will remain over ?

11. How many jars, each containing 2 gall. 3 qt. 1 pt. 3 gills, can be filled out of a cask containing 285 gallons ?

12. How many pieces of rope, each 2 ft. 9 in. long, can be cut off a length of 1760 yards, and what length will remain over ?

13. A train travels 45 miles in 2 hours ; how many yards does it travel in a second ?

14. A man gave R7. *9a.* *6p.* to each of 24 men, and then had R6. *7a.* *9p.* left ; how much had he at first ?

15. *A* has R3. *7a.* *9p.* more than *B*, *B* has R2. *8a.* *3p.* less than *C*, and *C* has R12 ; how much has *A* ?

16. If a man's net annual income be R17856. *4a.*, how much may he spend per day and per week to the nearest pie, so as not to run into debt ? [Reckon 52 weeks and 365 days to the year.]

17. If the daily income of a man be R3. *4a.* *9p.*, how much can he spend per day that he may save R239. *8a.* *6p.* in a year ?

18. If a man spends R5. *3a.* *3p.* daily, how much will he be able to save out of an annual income of R2400 ?

19. How much to the nearest farthing can a person spend daily if he wishes to save £300 out of an annual income of £700 ?

20. A gentleman's gross annual income is R3000 ; he pays R72. *3a.* annually in taxes : what must his daily expenses be that he may save R1080 in a year ?

21. A man spends R7. 8*a*. 9*p*. daily, and saves R1000 a year; what is his annual income?

22. A clerk received £114 . 7 . 6 as pay in 1928; how much is that per day?

23. A man was born on the 10th of January 1832; what was his age on the 17th of April 1888?

24. I distribute R300 among boys, giving a rupee, a half-rupee, a quarter-rupee and a two-anna piece to each; how many boys get a share?

25. Sound travels 1125 ft. in a second; if a gun is fired at a distance of 1875 yards, what time must elapse between the seeing of the flash and the hearing of the report?

26. How many steps does a soldier whose stride is 2 ft. 8 in. take in walking 2 miles?

27. If a soldier takes 3240 strides in walking 1 mile 1030 yards, what is the length of his stride?

28. The circumference of a bicycle wheel is 12 ft. 7 in.; how many complete revolutions does it make in going 10 miles?

29. A certain sum of money was divided into 18 equal parts; each part was R4. 8*a*. 3*p*. and there was R2. 7*a*. 6*p*. over: find the sum.

30. A man earned R35. 9*a*. 6*p*. in January and R49. 8*a*. 9*p*. in February; he spent R26. 3*a*. 3*p*. each month: how much did he save in the two months?

31. A man earns £1. 7*s*. 6*d*. per week, and pays 7*s*. 6*d*. every fourth week to his club; what is his net income in a year of 52 weeks?

32. How many complete yards are there in the united length of 7 benches, each 7 ft. 7 in. long?

33. A man spends in 4 months as much as he earns in 3 months; what does he save out of an annual income of R2750. 8*a*.?

34. *A* and *B* together have £56. 12*s*. 6*d*.; *A* has £3. 17*s*. 9*d*. more than *B*; how much has *A*?

35. The earnings of a man and his 2 sons amount to £600 a year, and their expenses to £400; if the balance be divided equally how much will each receive?

36. How many quart bottles can be filled out of a cask containing 2 cwt. 1 qr. 8 lb. of water?

37. The 1st of January 1881 was on Monday; find the number of Mondays in that year.

38. A vessel which holds 10 gallons weighs when empty 30 lb.; what is the weight of the vessel when full of water?

39. Your father was 25 yr. 7 mo. 10 da. old when you were born ; your sister was born when your father was 21 yr. 9 mo. 8 da. old : how old is your sister now if your age is 12 yr. 6 mo. ?

40. Four dollars, 3 half-guineas, 5 half-crowns and 6 florins amount to £3. 12*s.* 8*d.* ; what is the value of a dollar ?

41. Two pieces of cloth of equal length cost £3 . 0 . 9 and £2 . 5 . 0 respectively ; the price of the first was 3*s.* 4½*d.* per yd. ; what was the price of the second per yard ?

42. A merchant bought 350 lb. Avoir. of lead, and sold it by Troy weight ; how many pounds Avoir. did he gain ?

43. A shop-keeper's weight was deficient 3 tolas to a seer ; what quantity would he defraud his customers in selling 8 maunds ?

44. Fifty bags of rice are bought for R800. 12*a.* 6*p.* at R3. 3*a.* 3*p.* per maund ; what is the weight of each bag ?

45. Light travels 186500 miles in a second ; what time does it take in travelling from the sun to the earth, a distance of 92877000 miles ?

46. The small wheel of a tricycle makes 330 revolutions more than the large wheel in passing over a mile ; if the circumference of the large wheel be 8 ft., what is the circumference of the other ?

47. A weekly newspaper was numbered 4 on the 7th January 1925 ; when was it numbered 40 ?

48. A daily newspaper which is published on week days only was numbered 20 on Monday the 13th January 1884 ; on what date was it numbered 120 ?

49. A person travelled 120 miles by railway at 15 miles an hour, 120 miles by road at 8 miles an hour, and 60 miles by bullock cart at 2 miles an hour ; how long did he take ?

50. Supposing that the distance of the sun from the earth is 91776000 miles, and that light travels from the former to the latter in 7 min. 58 sec., find the velocity of light per second.

51. The value of a mark being 13*s.* 4*d.*, and that of a dollar 4*s.* 2*d.*, how many half-crowns are there in 9 marks + 12 dollars ?

52. A person laid out £43. 9*s.* 4*d.* in spirits at 5*s.* 4*d.* a gallon, some of which leaked out in the carriage ; he sold the remainder for £54, at the rate of 7*s.* 6*d.* a gallon : how many gallons leaked out ?

53. A wheel makes 600 revolutions in passing over 1 mile 40 yards ; what is its circumference ?

54. Divide R65. 10*a.* equally among 8 men, 12 women and 30 children ; supposing the children to have received their

shares, and the men to have given up their shares to the women, how much would each woman receive ?

55. How many times did a church-clock, which chimes the quarters, strike and chime in February 1900 ?

56. How many times does the 29th day of the month occur in 400 consecutive years ?

57. The circumferences of the large and small wheels of a tricycle are 13 ft. 9 in. and 3 ft. 4 in. respectively ; how many more turns will the latter have made than the former when the tricycle has gone a distance of 15 miles ?

58. If for every R1 rent paid to his landlord a man pays in addition 1 anna for gas, how much will he have left out of an annual income of R3000 if he lives in a house whose monthly rent is R20 ?

59. After measuring 40 yards of a rope it was discovered that the measuring yard was an inch too long ; what was the true length measured ?

60. One man is 30 years 17 weeks and 5 days old, and another is 26 years 9 weeks and 3 days old ; a third is just as much younger than the first as he is older than the second : what is the age of the third ?

XVIII. BARTER, GAIN AND LOSS, ETC.

96. Barter.—*Example.* How many seers of sugar at 4*a.* 6*p.* a seer must a grocer give in exchange for 9 lb. of tea at R1. 2*a.* a lb. ?

Cost of 9 lb. of tea = R1. 2*a.* × 9 = R10. 2*a.*

∴ The number of seers of sugar reqd. = R10. 2*a.* ÷ 4*a.* 6*p.* = 36.

EXAMPLES. 46.

1. How many pounds of tea at R1. 4*a.* a pound must be given in exchange for 40 yards of silk at R2. 10*a.* a yard ?

2. How many dollars of 4*s.* 2*d.* each can be obtained for 100 rupees of 1*s.* 10*d.* each ?

3. If 48 yards of ribbon be given in exchange for 2 maunds of brown-sugar at 3 annas a seer, what is the price of the ribbon per yard ?

4. A man exchanges 45 sheep at £2 . 5 . 9 each and 37 pigs at £3 . 13 . 6 each for 13 oxen at £17 . 6 . 6 each, the difference being paid or received in money ; how much does he pay or receive ?

5. Seven pounds of tea at R1 . 3 . 6 a pound and 13 pounds of coffee are given in exchange for 15 maunds of wheat at R1 . 13 . 3 a maund ; find the price of a pound of coffee.

97. Gain and loss.—*Example.* If 25 yards of cloth are bought at 7*s.* 6*d.* a yard and sold at 8*s.* 9*d.* a yard, how much is gained ?

		£	s.	d.
Profit on each yard = 8*s.* 9*d.* − 7*s.* 6*d.* = 1*s.* 3*d.*	1*s.* × 25 =	1	5	0
∴ Total profit = 1*s.* 3*d.* × 25 = £1. 11*s.* 3*d.*	3*d.* × 25 =		6	3
		£1	11	3

EXAMPLES. 47.

1. A man gives 15 maunds of rice worth R3. 8*a.* a maund, and receives in exchange 22 maunds of flour worth R2. 8*a.* a maund ; does he gain or lose, and by how much ?

2. A man buys 150 yards of cloth at R1. 1*a.* 3*p.* per yard, and sells at R1. 3*a.* 6*p.* per yard ; what does he gain altogether ?

3. A grocer bought a chest of tea containing 320 lb. for R405, and sold it at R1. 5*a.* 9*p.* per lb. ; what did he gain ?

4. Twenty-nine sheep are bought at R5. 8*a.* each ; 15 of them are sold at R6. 4*a.* each, and the rest at R5. 4*a.* each ; find the ain.

5. A grocer buys 15 maunds of sugar at 4*a.* 6*p.* a seer, and sells at R13. 4*a.* 6*p.* a maund ; what is his gain ?

6. Out of 2 md. 15 seers of milk, bought for R6. 9*a.* 9*p.*, 7 seers are lost by leakage ; what is gained by selling the remainder at 1*a.* 6*p.* a seer ?

7. A cwt. of sugar is bought for R14. 9*a.* 6*p.*, and is sold for R16. 5*a.* 6*p.* ; what is the gain per lb. ?

8. A grocer bought 1 cwt. 1 qr. of sugar for £1. 15*s.*, and gained 11*s.* 8*d.* by retailing it ; at what rate per lb. was it sold ?

9. A merchant bought 40 gallons of wine, and lost £5 by selling it for £37 ; at what rate per gallon did he buy it ?

10. A dealer bought wheat at 38*s.* 9*d.* per qr. ; he subsequently sold it at £2. 0*s.* 3*d.* per qr., and made a profit of £1. 16*s.* altogether ; how many quarters did he buy and sell ?

11. A man buys 45 yards of silk at 6*s.* 6*d.* per yard, 15 yards of which being damaged, he sells at 5*s.* per yard ; at what price must he sell the rest so as to gain £1 . 12 . 6 altogether ?

12. A grocer buys 200 lb. of tea at R1. 2*a.* per lb., and sells one-half of it at R1. 3*a.* per lb. ; at what rate must he sell the remainder so as to gain R25 on the whole ?

13. If 7*s.* 6*d.* be lost by selling an article for £3, what would have been gained or lost by selling it for £4 ?

14. I sold some goods weighing 13 cwt. 2 qr. 9 lb., for £72 . 17 . 7½, gaining thereby 3½*d.* per lb. How much should I have gained per lb. if I had sold them at £5 . 12 . 0 per cwt. ?

15 A tradesman buys a piece of cloth 50 yards in length for ₨40. 10*a.* ; at what price per yard must he sell it (i) that he may gain 5*a.* per yard, (ii) that he may gain ₨18. 12*a.* on the whole ?

98. Mixtures.—*Example* 1. If 3 maunds of rice at ₨2. 8*a.* per maund be mixed with 5 maunds at ₨3. 2*a.* per maund, find the price of the mixture.

Cost of 3 md. at ₨2. 8*a.*	=	₨2. 8*a.* × 3 = ₨7. 8*a.* ;
Cost of 5 md. at ₨3. 2*a.*	=	₨3. 2*a.* × 5 = ₨15. 10*a.* ;
∴ Cost of 8 md. of mixture	=	₨7. 8*a.* + ₨15. 10*a.*
	=	₨23. 2*a.*
∴ Cost of 1 md. of mixture	=	₨23. 2*a.* ÷ 8
	=	₨2. 14*a.* 3*p.*
∴ Price required	=	₨2. 14*a.* 3*p.* per maund.

Example 2. How much water must be added to 12 gallons of beer at 10*s.* a gallon, to reduce the price to 8*s.* a gallon ?

The price of the whole mixture at 8*s.* a gallon must be equal to the price of 12 gallons of beer at 10*s.* a gallon. Therefore, if we divide the price of 12 gallons of beer at 10*s.* a gallon by 8*s.* we shall get the number of gallons in the mixture.

Price of 12 gallons of beer = 10*s.* × 12 = 120*s.* ;
∴ number of gallons in the mixture = 120*s.* ÷ 8*s.* = 15 ;
∴ number of gallons of water added = 15 − 12 = 3.

EXAMPLES. 48.

1. A mixture is made of 7 seers of sugar at 4*a.* 6*p.* per seer, 2 seers at 4*a.* per seer and 3 seers at 3*a.* 0*p.* ; find the value per seer of the mixture.

2. A man bought 3 qr. of wheat at 30*s.* per qr. and 9 qr. at 26*s.* per qr. ; he mixed them, and sold the mixture at 3*s.* 7½*d.* per bushel ; how much did he gain ?

3. To 20 seers of milk, bought at 1*a.* 9*p.* a seer, 5 seers of water are added, and the mixture is sold at 2*a.* per seer ; how much is gained ?

4. A merchant buys 15 md. of sugar at ₨9. 8*a.* per md., 18 md. at ₨9. 4*a.* per md., and 10 md. at ₨9. per md., and pays ₨4. 2*a.* for carriage ; he mixes them : at what price per md. must he sell the mixed sugar so as not to lose by the sale ?

5. 10 lb. of coffee are mixed with 2 lb. of chicory; if the mixture be worth 1*s.* 11*d.* per lb., and the chicory 3*d.* per lb., what is the value per lb. of the pure coffee?

6. A grocer mixes 36 lb. of tea at 2*s.* $4\frac{1}{2}$*d.* per lb. with 48 lb. at 1*s.* $10\frac{1}{2}$*d.* per lb.; at what price per lb. must he sell the mixture so as to gain 13*s.* 6*d.* on his outlay?

7. A woman buys 8 dozen eggs at $2\frac{1}{2}$*d.* per dozen, and 12 dozen more at $1\frac{1}{4}$*d.* per dozen: at what price per dozen must she sell the whole so as to gain $\frac{1}{4}$*d.* per dozen?

8. How much water must be mixed with 36 seers of milk at 1*a.* 9*p.* per seer, so as to reduce the price to 1*a.* 6*p.* per seer?

9. How many pounds of tea-dust (worth nothing) must a grocer mix with 20 lb. of tea at 2*s.* 6*d.* per lb., to enable him to sell the mixture at 2*s.* per lb. and gain at the same time 8*s.* on the transaction?

10. A grocer buys 30 lb. of tea at 2*s.* a pound and 50 lb. of tea at 2*s.* 8*d.* a pound, and having mixed them sells 40 lb. of the mixture at 2*s.* 4*d.*; at what price per lb. must he sell the remainder that he may neither gain nor lose?

99. Division of Money.—*Example* 1. Divide R13. 9*a.* among *A*, *B* and *C*, so that *A* may have 12*a.* 3*p.* more than *B*, and *B* R1. 2*a.* 9*p.* more than *C*.

B is to have R1. 2*a.* 9*p.* more than *C*, and *A* is to have 12*a.* 3*p.* +R1. 2*a.* 9*p.* more than *C*; if we take away these sums to be subsequently given to *B* and *A* respectively, the remaining portions of their share will be each equal to the share of *C*.

R.	a.	p.
1	2	9
	12	3
1	2	9
R3	1	9

	R.	a.	p.
	13	9	0
	3	1	9
3)	10	7	3
	3	7	9 = *C*' share;
∴	4	10	6 = *B*' share;
and	5	6	9 = *A*'s share.

EXAMPLES. 49.

1. Divide R39. 7*a.* 9*p.* between *A* and *B*, so that *A* may get R7. 4*a.* 3*p.* more than *B*.

2. Divide £28. 7*s.* 6*d.* between *A* and *B*, so that *A* may receive £3. 14*s.* 3*d.* less than *B*.

3. Divide R357. 14*a.* 6*p.* among 15 men, giving R11. 14*a.* 9*p.* more to each of two of them than to each of the others.

4. Divide R679 among 27 men and 5 women, so that a man may get R6 less than a woman.

5. Divide R39. 4*a*. 6*p*. among *A*, *B* and *C*, so that *A* may receive R3 more than *B*, and *B* R4 more than *C*.

6. Divide R329. 7*a*. 9*p*. among *A*, *B* and *C*, so that *A* may get R7 more than *B*, and *B* R2 less than *C*.

7. £95. 10*s*. is divided among 8 men, 7 women and 6 boys, so that each man receives 10*s*. more than each woman, and each woman 10*s*. more than each boy; find how much the men receive.

Example 2. Divide R59. 6*a*. among 3 men, 5 women and 6 boys, so that each man may receive three times as much, and each woman twice as much, as a boy.

3 men	= 9 boys		R. *a*.	
5 women	= 10 ...	$25\begin{cases}5\\5\end{cases}$	5) 59 . 6	
6 boys	= 6 ...		5) 11 . 14	
	25		2 . 6	= each boy's share;
		∴	4 . 12	= ... woman's ...
		and	7 . 2	= ... man's ...

EXAMPLES. 50.

1. Divide R15. 9*a*. 6*p*. between a boy and a girl, so that the boy may receive twice as much as the girl.

2. Divide R31. 3*a*. between *A*, *B* and *C*, in such a manner that *A*'s share may be 3 times, and *B*'s twice, *C*'s.

3. Divide R100 among 3 men, 5 women and 10 boys, so that each man may receive 4 times as much as a boy, and each woman twice as much as a boy.

4. Divide £11 . 15 . 4½ among *A*, *B* and *C*, so that *A* may receive twice as much as *B*, and *B* twice as much as *C*.

5. Divide £10 . 7 . 6 among 3 persons, so that one may receive twice as much as each of the others.

6. Divide R39. 7*a*. 9*p*. between *A* and *B*, so that *A* may receive R1. 14*a*. 3*p*. more than twice the amount to be received by *B*.

Example 3. Divide R28 into an equal number of rupees, half-rupees and quarter-rupees.

Make a pile consisting of one of each of these coins. The value of the pile = a rupee + a half-rupee + a quarter-rupee = R1 + 8*a*. + 4*a*. = R1. 12*a*. There are, therefore, R28 ÷ R1. 12*a*., or 16 such piles.

∴ The number of each kind of coin = the number of piles = 16.

Example 4. A purse contains R27. 4a. in half-rupees and quarter-rupees. If the number of coins be 70, how many are there of each kind ?

If there were 70 quarter-rupees the total amount would have been only 4a. × 70 = R17. 8a. But the purse contains R27. 4a. Therefore the excess, (R27. 4a. − R17. 8a.), *i.e.*, R9. 12a. is due to the presence of half-rupees in the purse.

Now, for each half-rupee, the excess is 4a. ∴ R9. 12a. contains as many 4a. as there are half-rupee pieces in the purse.

∴ the number of half-rupees = R9. 12a. ÷ 4a.
= 156a. ÷ 4a. = 39.

∴ the number of quarter-rupees = 70 − 39 = 31.

EXAMPLES. 51.

1. Divide R22. 8a. into an equal number of rupees, half-rupees, quarter-rupees and two-anna pieces.

2. Divide £17 into an equal number of sovereigns, half-sovereigns, half-crowns, shillings and sixpences.

3. A box contains an equal number of crowns, shillings and pennies ; the total amount in the box is £3. 13s. : find the number of each.

4. R100 is divided among an equal number of men, women and boys ; each man receives R2. 8a., each woman R2 and each boy R1. 12a. : find the number of men, women or boys.

5. A bag contains a certain number of rupees, twice as many half-rupees, and 4 times as many quarter-rupees ; the whole sum amounts to R33 : find the number of each.

6. Among how many children may R60 be divided so that each child may receive a rupee, an eight-anna piece, a four-anna piece and a two-anna piece ?

7. A purse contains £2. 10s. 6d. in shillings and sixpences. The number of coins is 61 ; how many are there of each kind ?

8. A sum of R18. 4a. is made up of 50 coins which are either half-rupees or quarter-rupees ; how many are there of each ?

100. *Example.* *A* and *B* together have R13. 8a., *B* and *C* together have R8. 8a., *A* and *C* together have R11. 8a. ; how much has *A* ?

R13. 8a. + R11. 8a. = twice *A*'s money + *B*'s money + *C*'s money ;
but R8. 8a. = *B*'s money + *C*'s money.

∴ (R13. 8a. + R11. 8a. − R8. 8a.) or R16. 8a. = twice *A*'s money ;
∴ *A*'s money = R16. 8a. ÷ 2 = R8. 4a.

Or thus :

(R13. 8a. + R8. 8a. + R11. 8a.) or R33. 8a. = twice A's money + twice B's money + twice C's money ;

∴ (R33. 8a. ÷ 2) or R16. 12a. = A's money + B's money + C's money ;

but R8. 8a. = B's money + C's money ;

∴ A's money = R16. 12a. − R8. 8a. = R8. 4a.

EXAMPLES. 52.

1. *A* and *B* together have R6. 0a. 3p., *B* and *C* together have R4. 15a. 9p., *A* and *C* together have R5. 15a. ; how much has *A* ?

2. *A* and *B* together have R24. 1a., *B* and *C* together have R19. 15a., *A* and *C* together have R23. 12a. ; find how much *B* has.

3. A horse and a cow are together worth R101, a cow and a sheep are together worth R31, a horse and a sheep are together worth R81 ; find the price of a horse, of a cow and of a sheep.

4. A mark and a gulden are together worth 2s. $11\frac{1}{4}$d., a gulden and a rouble are together worth 5s. $1\frac{1}{2}$d., a rouble and a mark are together worth 4s. $1\frac{3}{4}$d. ; find the value of a mark, of a gulden and of a rouble.

5. A man and a woman together have R30. 7a. 6p., the woman and a boy together have R20. 8a., the man and the boy together have R25. 9a. 6p. ; find how much the man, the woman and the boy together have.

XIX. FACTORS AND PRIME NUMBERS.

101. If one number divides another *exactly*, the first is said to be a **factor** (or *sub-multiple*) of the second, and the second is said to be a **multiple** of the first. Thus 5 is a factor of 15, and 15 is a multiple of 5.

In speaking of the factors of a number we exclude the number *one* or *unity*, which may be said to be a factor of any number.

[*N. B.* In the present section the word *divisible* is used in the sense of *exactly divisible.*]

102. An **even** number is a number divisible by 2 so that it is a multiple of 2. An **odd** number is a number not divisible by 2.

103. Tests of Divisibility :

(1) *To ascertain whether a number is divisible by* 2, 4 *or* 8.

(i) Consider any number, say, 314.

We have $314 = 310 + 4 = 31$ tents $+ 4$.

Each ten can be split up into 2 *fives*. Therefore a number is divisible by 2 when its last figure is 0 or an even digit ; as 310, 54.

(ii) Consider any number, say, 4124.

We have $4124 = 4100 + 24 = 41$ hundreds $+ 24$.

Each hundred can be split up into 4 *twenty-fives*. Therefore a number is divisible by 4 when its last *two* figures are 00 or represent a number divisible by 4 ; as 500, 320, 324.

(iii) Consider any number, say, 5144.

We have $5144 = 5000 + 144 = 5$ thousands $+ 144$. But $1000 \div 8 = 125$.

Therefore a number is divisible by 8 when its last *three* figures are 000 or represent a number divisible by 8 ; as 3000, 3400, 3240, 3816.

(2) *To ascertain whether a number is divisible by* 5, 25 *or* 125.

Proceeding as before we can show that a number is divisible by

- 5 when its last figure is 0 or 5 ; as 370, 345 :
- 25 when its last *two* figures are 00 or represent a number divisible by 25 ; as 300, 625 :
- 125 when its last *three* figures are 000 or represent a number divisible by 125 ; as 6000, 76125.

(3) *To ascertain whether a number is divisible by* 9 *or* 3.

Consider any number, say, 531.

We have $531 = 500 + 30 + 1$.

And $500 = 5 \times 100 = 5 \times (99 + 1) = 5 \times 99 + 5$,

$30 = 3 \times 10 = 3 \times (9 + 1) = 3 \times 9 + 3$.

$$\begin{aligned} \therefore \quad 531 &= 5 \times 99 + 3 \times 9 + (5 + 3 + 1) \\ &= (5 \times 11 \times 9 + 3 \times 9) + (5 + 3 + 1) \\ &= (5 \times 11 + 3) \times 9 + (5 + 3 + 1) \\ &= 58 \times 9 + (5 + 3 + 1). \end{aligned}$$

Now, 58×9 is a multiple of 9. So 531 is divisible by 9 if the sum of its digits, *i.e.*, $(5 + 3 + 1)$ is divisible by 9. Therefore a number is divisible by 9 if the sum of its digits is divisible by 9.

Again, since $531=58\times9+(5+3+1)=58\times3\times3+(5+3+1)$, we can show that any number = a multiple of 3 + the sum of its digits; hence, a number is divisible by 3 when the sum of its digits is so divisible.

(4) *To ascertain whether a number is divsible by* 11.

Consider any number, say, 5732.

We have $5732=5000+700+30+2$.

Now,

$$2 = 2,$$
$$30=3\times10=3\times(11-1)=3\times11-3,$$
$$700=7\times100=7\times(99+1)=7\times99+7,$$
$$5000=5\times1000=5\times(1001-1)=5\times1001-5.$$
$$\therefore\ 5732=3\times11+7\times99+5\times1001+(2-3+7-5)$$
$$=3\times11+7\times9\times11+5\times91\times11+(2+7)-(3+5)$$
$$=(3+63+455)\times11+(2+7)-(3+5)$$
$$=\text{a multiple of } 11+(2+7)-(3+5).$$

Now, $2+7$ = the sum of the digits in the odd places;

and $3+5$ = the sum of the digits in the even places.

$\therefore$ 5732 = a multiple of 11
+(sum of digits in odd places)
−(sum of digits in even places).

We, therefore, conclude that a number is divisible by 11 when the difference between the sum of its digits in the odd places and the sum of its digits in the even places is either 0 or is divisible by 11; as 5731, 3927.

104. Criteria of Divisibility Summarised.

A number is divisible

by 2 when its last figure is 0, or an *even* digit; as 310, 54:

3 when the sum of its digits is divisible by 3; as 126, 402:

4 when its last *two* figures are 00 or represent a number divisible by 4; as 300, 320, 324:

5 when its last figure is 0 or 5; as 370, 345:

6 when it is divisible by 2 and 3; as 354, 972:

8 when its last *three* figures are 000 or represent a number divisible by 8; as 2000, 3400, 3240, 3816:

9 when the sum of its digits is divisible by 9; as 477, 801:

10 when its last figure is 0:

11 when the difference between the sum of its digits in the *odd* places and the sum of its digits in the *even* places is either 0, or divisible by 11 ; as 34672, 582934 :

12 when it is divisible by 4 and 3 ; as 612, 2532 :

15 „ „ „ „ „ 5 and 3 ; as 2235 :

Other tests of Divisibility—To determine whether a number is divisible by 7, 11, or 13 we have the following rule :

Divide the figures of the number into groups containing *three* each, as far as possible, counting from right to left. Add the alternate groups, and subtract the smaller sum from the greater ; then if the remainder is 0, or is divisible by 7, 11, or 13, the number itself is also divisible by 7, or by 11, or by 13.

Thus 98126 is divisible by 7, but not by 11 or by 13 : for $126-98=28$ which is divisible by 7, but not by 11 or by 13.

105. Important Propositions :

(i) If a number is divisible separately by two numbers which have no common factor, it is also divisible by their product.

(ii) If a number is divisible by 3 (or 9), any other number expressed by the *same digits* is also divisible by 3 (or 9).

(iii) If each of two numbers is divisible by a third number, their sum (and difference) is also divisible by the third.

(iv) If a number is divisible by another, any multiple of the first is also divisible by the second.

(v) If each of two numbers is divisible by a third number, then the sum (and difference) of any multiple of the first and any multiple of the second is also divisible by the third number.

EXAMPLES. 53.

Determine whether the following numbers are divisible by 2, 3, 4, 5, 8, 9, 10 or 11 :

1. 138. **2.** 945. **3.** 684. **4.** 420. **5.** 8844.
6. 7942. **7.** 1230. **8.** 1772. **9.** 2311. **10.** 3475.
11. 8976. **12.** 7128. **13.** 12345. **14.** 98765. **15.** 35600.
16. 23000. **17.** 709281. **18.** 777777. **19.** 989898. **20.** 1234567890.

Determine whether the following numbers are divisible by 7, 11 or 13 :

21. 99120. **22.** 89133. **23.** 67119. **24.** 555555.
25. 433378. **26.** 4123210. **27.** 55734545. **28.** 123789666.

Determine whether the following numbers are divisible by 6, 12 or 30 :

29. 372. **30.** 948. **31.** 7740. **32.** 3725.

33. What is the least number which being added to 2311 will make the sum divisible (i) by 3, (ii) by 4 ?

34. What is the least number which being subtracted from 70031 will make the remainder divisible (i) by 5, (ii) by 8, (iii) by 9 ?

35. What number is the same multiple of 11 as 3705 is of 15 ?

36. Find without actual division the remainder when each of the following numbers is divided by 9 :

(i) 34678. (ii) 73877. (iii) 897654.

37. Prove that the difference between two numbers expressed by the same digits in different orders is divisible by 9.

106. A **prime number** or a **prime** is a number which is not divisible by any number (except itself and unity).

1, 2, 3, 5, 7, 11, 13, etc. are *prime* numbers.

A **composite** number is a number which has factors each greater than 1.

4, 6, 8, 9, 10, 12, etc, are *composite* numbers

107. To ascertain what numbers are primes.

(i) To find the prime numbers in a series of numbers 1, 2, 3,... cancel every second number after 2, every third number after 3, every fifth number after 5, and so on ; the remaining numbers will be primes. [*In finding the primes in any series of numbers, we need not divide by any prime number whose square is greater than the largest number in the series.*]

(ii) To determine whether a given number is a prime, divide the number successively by the primes 2, 3, 5, 7, 11, etc. ; if there is a remainder in each case the given number is a prime. [*It is not necessary to try a divisor whose square is greater than the given number.*]

Note. From Art. 104 it will appear that the units' figure of every prime number (except 2 and 5) must be 1, 3, 7 or 9. Hence any given number (not being 2 or 5) need only be examined when its units' figure is 1, 3, 7 or 9 ; and in such a case we need not try the divisors 2 and 5.

108. The following is a list of PRIME NUMBERS between 1 and 1009.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

109. Every composite number can be resolved into factors which are all primes.

Note. A number has only one set of prime factors.

Example 1. Find the prime factors of 4452.

We divide the number successively (and in each case as often as possible) by those of the primes 2, 3, 5, 7, 11, 13,......, *that can be used as divisors*, until we come to a quotient which is a prime number.

$$\begin{array}{r} 2\,)\,4452 \\ 2\,)\,2226 \\ 3\,)\,1113 \\ 7\,)\,\;371 \\ 53 \end{array}$$

Thus $4452 = 2 \times 2 \times 3 \times 7 \times 53$.

Example 2. Find whether the number 547 is prime or not.

We find by trial that the number is not divisible by any of the primes 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Now it is not necessary to try other primes, for $547 \div 29$ gives a quotient less than 29; if, therefore, 547 were divisible by a prime greater than 29, the quotient would be less than 29, and 547 would be divisible by this quotient, that is, by a number less than 29, which we know is not the case. Hence 547 is a prime number.

EXAMPLES. 54.

Find the prime factors of

1. 8. **2.** 12. **3.** 18. **4.** 24. **5.** 27.

6. 32. **7.** 48. **8.** 50. **9.** 63. **10.** 64.

11. 80.	**12.** 88.	**13.** 99.	**14.** 100.	**15.** 108.
16. 176.	**17.** 117.	**18.** 288.	**19.** 495.	**20.** 625.
21. 999.	**22.** 1050.	**23.** 1296.	**24.** 1760.	**25.** 2000.
26. 3650.	**27.** 5760.	**28.** 2457.	**29.** 13824.	**30.** 200100.

Determine which of the following numbers are primes, and find the prime factors of those which are composite :

31. 29.	**32.** 61.	**33.** 81.	**34.** 79.	**35.** 97.
36. 107.	**37.** 113.	**38.** 207.	**39.** 227.	**40.** 349.
41. 3751.	**42.** 507.	**43.** 4573.	**44.** 619.	**45.** 713.
46. 997.	**47.** 6539.	**48.** 1793.	**49.** 509.	**50.** 1363.

Find the number of primes between

51. 1 and 30. **52.** 10 and 50. **53.** 20 and 70.

54. By what prime numbers may 37 be divided, so that the remainder may be 2 ?

55. By what prime numbers may 109 be divided, so that the remainder may be 4 ?

56. By what numbers may 29 be divided, so that the remainder may be 5 ?

XX. HIGHEST COMMON FACTOR.

110. A **common factor** of two or more numbers is a number which divides each of them exactly.

Thus, each of the numbers 2, 3 and 6, is a *common factor* of 12 and 18.

The **Highest Common Factor** (H. C. F.) of two or more numbers is the *highest* number which divides each of them exactly.

Thus, 6 is the H. C. F. of 12 and 18.

Note. Two numbers are said to be *prime to each other* when they have no common factor.

N. B. The term *measure* is often used as synonymous with *factor*, and **Greatest Common Measure** instead of *highest common factor*.

111. *The H. C. F. of two or more numbers is the highest number which divides each of them exactly and is therefore the product of all their common prime factors.*

Example 1. Find the H. C. F. of 18 and 30.

$$18=2\times3\times3\,; \qquad 30=2\times3\times5.$$

The factors common to the two numbers are 2 and 3 ; hence the H. C. F. required $=2\times3=6$.

Note. In finding the H. C. F. it is not necessary to find the prime factors of all the numbers. It is sufficient to find the prime factors of *one* of the numbers, and to form the product of those that divide each of the remaining numbers exactly.

Example 2. Find the H. C. F. of 84, 140 and 168.

Now, $84 = 2 \times 2 \times 3 \times 7$; and we find that each of the remaining numbers is divisible by $2 \times 2 \times 7$, but not by 3 ; therefore the H. C. F. required $= 2 \times 2 \times 7 = 28$.

EXAMPLES. 55.

(*Examples* 1—16 *should be taken orally.*)

Read off the H. C. F. of

1.	4, 6.	**2.**	9, 24.	**3.**	6, 30.	**4.**	16, 20.
5.	20, 48.	**6.**	28, 35.	**7.**	21, 63.	**8.**	22, 33.
9.	24, 40.	**10.**	35, 80.	**11.**	13, 78.	**12.**	4, 6, 18.
13.	5, 10, 15.	**14.**	21, 28, 35.	**15.**	9, 24, 36.	**16.**	17, 51, 68.

Find, by the method of factors, the H. C. F. of

17.	126 and 144.	**18.**	90 and 325.	**19.**	252 and 348.
20.	150 and 375.	**21.**	256 and 788.	**22.**	480 and 792.
23.	15, 35, 120.	**24.**	16, 24, 140.	**25.**	90, 125, 342.
26.	224, 336, 728.	**27.**	625, 750, 1225.	**28.**	868, 3164, 4228.

112. When the numbers cannot be easily resolved into prime factors, a different method has to be used in finding their H. C. F. It depends on the following principles :

(1) If a number has a certain factor, any multiple of the number contains that factor and is, therefore, divisible by that factor.

Thus 5 is a factor of 15, and, therefore, of any multiple of 15, *e.g.*, 30, 45, 60, 75, etc.

(2) If two numbers have a common factor, their *sum* and their *difference* and also the *sum* and *difference* of any multiples of them contain that factor and are, therefore, divisible by that factor.

Thus 5 is a factor of 25 and 15, and is also a factor of $25+15$ and of $25-15$; and also of $(25 \times 3)+(15 \times 4)$ and $(25 \times 5)-(15 \times 2)$.

(3) If a greater number is divided by a smaller number, then

(i) the remainder is equal to the difference between the greater number and some multiple of the smaller and hence it follows that *any common factor of the original numbers* is also a factor of the remainder, and, therefore, *is a common factor of the remainder and the smaller number;* and

(ii) the greater number is equal to the sum of the remainder and some multiple of the smaller, and hence it follows that *any common factor of the remainder and the smaller number* is a factor of the greater number also, and therefore, *is a common factor of the two original numbers.*

The H. C. F. of two numbers is, therefore, the same as the H. C. F. of the smaller number and the remainder left after dividing the greater number by the smaller number.

Hence the following rule is deduced for finding the H. C. F. of two numbers :

Divide the greater number by the less, the divisor by the remainder, then the second divisor by the second remainder, and so on, until there is no remainder ; the *last divisor* is the H. C. F. required.

Example 1. Find the H. C. F. of 384 and 1296.

Process :

```
384 ) 1296 ( 3
      1152
       144 ) 384 ( 2
             288
              96 ) 144 ( 1
                    96
                    48 ) 96 ( 2
                         96
```

∴ The H. C. F. required is 48.

Explanation. The H. C. F. of 1296 and 384 is also a factor of $(1296-384\times3)$ or 144 ; therefore a common factor of 384 and 144. Again the H. C. F. of these numbers is a factor of $(384-144\times2)$ or 96 and therefore a common factor of 144 and 96 ; also of their difference 48 ; and also of 96 and 48. And the *highest* common factor of these is evidently 48.

To save space and avoid zig-zag operation the above work may be compressed between parallel columns, placing the quotients alternately to right and left as shewn below on the left. Further, to shorten work the Italian method of division may be used as shewn on the right. But the student is again reminded

that this method is much more liable to error and an error is difficult to detect. It is not recommended to beginners.

(i)

2	384	1296	3
	288	1152	
	96	144	1
	96	96	
		48	

(ii)

2	384	1296	3
2	96	144	1
		48	

(i) First 1296 is divided by 384 and the quotient 3 is written to the right of 1296. The remainder is 144. The first divisor 384 on the left is now divided by the remainder 144 without rewriting either, and the quotient 2 is shewn to the left of 384.

When this division is performed the new remainder 96 is just opposite the number 144 which it has next to divide.

Finally, 48 divides 96 exactly, and therefore the H. C. F. required is 48

(ii) In the Italian Method the partial products of the divisor are omitted, and the operations of finding the partial product and of subtracting it from the dividend are, in all successive steps, performed mentally in one.

Note 1. If in any example the last divisor is 1, the numbers have no common factor except unity, *i.e.*, they are prime to each other.

Note 2. When the H. C F. of three or more numbers is required, we first find the H. C. F. of any two of them and then find the H. C. F. of this result and another number, and so on, through all the given numbers; the last result is the H. C. F. required.

Example 2. Find the greatest number that will divide 50 and 60 leaving the remainders 8 and 4 respectively.

When the remainder is subtracted from the dividend, the difference is the product of the divisor and the quotient. Hence the number required is the greatest number which will divide $(50-8)$, *i.e.*, 42 and $(60-4)$, *i.e.*, 56 exactly.

∴ The number required = the H. C. F. of 42 and 56 = 14.

Example 3. Find the three numbers which are prime to one another such that the product of the first two is 437 and that of the last two is 551.

From the question we see that the second number is a common factor of the two products, and since the numbers are prime to one another, it is their H. C. F. and is, therefore, 19.

3	437	551	1
	342	437	
5	95	114	1
	95	95	
		19	

∴ The first number $=437 \div 19 = 23$, and the third number $= 551 \div 19 = 29$.

Hence the numbers are 23, 19 and 29.

Example 4. Two bills, one of £3. 17*s*. 6*d*. and the other of £5. 7*s*. 6*d*., are to be paid in coins of one kind; find the largest coin that can be used.

∴ The largest coin is the G. C. M. of the two quantities. To find the G. C. M. of two concrete quantities, they must first be reduced to the same unit.

£5. 7*s*. 6*d*. = 1290*d*.;

£3. 17*s*. 6*d*. = 930*d*.

```
(i) 930 ) 1290 ( 1
          930
          360 ) 930 ( 2
                720
                210 ) 360 ( 1
                      210
                      150 ) 210 ( 1
                            150
                             60 ) 150 ( 2
                                  120
                                   30 ) 60 ( 2
                                        60
```

or (ii) $1290 = 10 \times 3 \times 43$;

and $930 = 10 \times 3 \times 31$.

∴ G.C.M. of 1290 and 930 $= 10 \times 3 = 30$.

∴ The largest coin that can be used = (30*d*.)

= 2*s*. 6*d*. or a half-crown.

Or thus:

£5. 7*s*. 6*d*. = 215 sixpences;

and £3. 17*s*. 6*d*. = 155 „

Now, $215 = 5 \times 43$;

and $155 = 5 \times 31$.

∴ The G. C. M. of 215 and 155 = 5.

Hence the largest coin that can be used = one containing 5 sixpences = 2*s*. 6*d*. = a half-crown.

N. B. Here 2*s*. 6*d*. is the G. C. M. of £3. 17*s*. 6*d*. and £5. 7*s*. 6*d*. It cannot be spoken of as a factor of £3. 17*s*. 6*d*. or £5. 7*s*. 6*d*. The word factor is used only in the case of abstract numbers.

Example 5. What is the greatest length which can be used to measure exactly 5 ft, 12 ft. 6 in., 20 ft., and 22 ft. 6 in.?

Expressed in inches the given lengths are 60 in., 150 in., 240 in., and 270 in. respectively and the G. C. M. of these is 30 in., or 2 ft. 6 in.

Hence the greatest length to measure exactly the given lengths is 2 ft. 6 in.

EXAMPLES. 56

Find the H. C. F. of

1. 48 and 144. **2.** 76 and 238. **3.** 92 and 772.
4. 252, 348. **5.** 493, 899. **6.** 620, 2108.
7. 2121, 1313. **8** 429, 715. **9.** 377, 1131.
10. 1379, 2401. **11.** 266, 2793. **12.** 3775, 10000.
13. 6023, 15466. **14.** 5865, 69180. **15.** 4081, 5141.
16. 3556, 3444. **17.** 5187, 5850. **18.** 6441, 10283
19. 13667, 14186. **20.** 43365, 44688. **21.** 11050, 35581.
22. 12321, 54345. **23.** 6327, 23997. **24.** 13202, 146083.
25. 5325, 8307. **26.** 9945, 50609. **27.** 4155, 24720.
28. 109056, 179712. **29.** 218707, 826769. **30.** 123456, 987654.

Are the following prime to each other?

31. 403 and 527. **32.** 3370, 2703. **33.** 387, 9234.
34. 1726, 1623. **35.** 3890, 8275. **36.** 3486, 9448.
37. 211, 2701. **38.** 5789, 7337. **39.** 9367, 14501.

Find the G. C. M. of

40. 703037 and 5134083. **41.** 271469, 30599.
42. 805, 1311, 1978. **43.** 204, 1190, 1445. **44.** 1617, 123, 789.
45. 1300, 725, 870. **46.** 723, 807, 735. **47.** 504, 2394, 4835.
48. 1190, 1445, 2006. **49.** 13338, 14136, 15903.
50. 314, 570, 618, 720. **51.** 602, 7394, 876, 92458.

52. What is the largest sum of money which is contained in R6. 4*a*. and R7. 8*a*. exactly?

53. What is the largest sum of money which will divide £7. 7*s*. 6*d*. and £13. 17*s*. 9*d*. exactly?

54. What is the greatest length which can be used to measure exactly 4 ft. 6 in., 6 ft. 9 in., 11 ft. 3 in. and 15 ft. 9 in.?

55. Two bills, one of £6. 7*s*. 6*d*. and the other of £9. 17*s*. 6*d*., are to be paid in coins of the same kind. Find the largest coin that can be used.

56. Find the greatest number that will divide 728 and 900, leaving the remainders 8 and 4 respectively.

57. Find the greatest number that will divide 261, 933 and 1381, leaving the remainder 5 in each case.

58. Is there any number that will divide 620 and 730, leaving the remainders 3 and 7 respectively?

59. Two vats contain respectively 540 and 720 gallons; find the vessel of greatest capacity that will empty off both vats.

60. Two masses of gold weighing 4427 and 7219 tolas respectively are each to be made into coins of the same size; what is the weight of the largest possible coin?

61. Find the three numbers which are prime to one another such that the product of the first two is 91 and the last two is 143.

62. Of the three numbers prime to one another, the product of the first two is 187, and that of the last two is 451. Find the numbers.

63. A labourer was engaged for a certain number of days for Rs. 2. 8*a.*, but being absent on some of those days he was paid only Rs. 1. 12*a.*; prove that his daily wages could not be more than 4 annas.

64. A woman bought a certain number of eggs for 15*a.* 6*p.*, and sold some of them without profit for 5*a.* 6*p.*; shew that she had still left *at least* 20 eggs.

65. In finding the G. C. M. of two numbers, the last remainder is 35 and the quotients are 1, 2, 1, 3. Find the numbers.

XXI. LOWEST COMMON MULTIPLE.

113. **A common multiple** of two or more numbers is a number which is exactly divisible by each of them.

The **Lowest Common Multiple (L. C. M.)** of two or more numbers is the *lowest* number which is exactly divisible by each of them, or, in other words, it is the lowest number which contains each of them as a factor.

Thus, each of the numbers 12, 24 and 36, is a common multiple of 3, 4 and 6; but 12 is their *lowest* common multiple.

Example. Find the L. C. M. of 42 and 72.

$$42 = 2 \times 3 \times 7 ;$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$

The L. C. M. must contain every prime factor of each of the numbers and each of these factors must occur at least as often in the L. C. M. as it occurs in *either* of the numbers themselves.

$\therefore$ The L. C. M. required $= 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 504.$

114. Again, if we compare the factors of the L. C. M., 504, with the total number of the prime factors of 42 and 72, we see that the factors which are common to both of them have been taken only once. But the product of these common factors would give the H. C. F. of the numbers. Hence

The product of two numbers is equal to the product of their H. C. F. and L. C. M. Thus, 2 is the H. C. F. and 12 is the L. C. M. of 4 and 6; and $4 \times 6 = 2 \times 12$.

Hence we have the following rule for finding the L. C. M. of two numbers :

Divide one of the numbers by the H. C. F. and multiply the quotient thus obtained by the other.

Example. Find the L. C. M. of 38 and 57.

The H. C. F. of 38 and $57 = 19$; $38 \div 19 = 2$.

$\therefore$ The L. C. M. required $= 2 \times 57 = 114$.

Note. When the L. C. M. of three or more numbers is required, we find the L. C. M. of any two of the numbers, and then find the L. C. M. of this result and a third number, and so on ; the last result being the L. C. M. required.

EXAMPLES. 57.

(*Examples* 1—16 *should be taken orally.*)

Read off the L. C. M. of

1. 2, 8. **2.** 3, 6. **3.** 10, 15. **4.** 5, 7.

5. 6, 10. **6.** 4, 16. **7.** 6, 16. **8.** 15, 25.

9. 8, 4. **10.** 12, 16. **11.** 22, 33. **12.** 2, 4, 6.

13. 2, 5, 9. **14.** 3, 5, 12. **15.** 7, 14, 21. **16.** 4, 9, 18.

Find the L. C. M. of

17. 117, 192. **18.** 1224, 1696. **19.** 224, 336.

20. 754, 806. **21.** 957, 1001. **22.** 845, 899.

23. 779, 1197. **24.** 1287, 6281. **25.** 76, 95, 106.

26. 629, 851, 253. **27.** 265, 385, 495. **28.** 300, 906, 708.

29. Resolve 210 and 385 into their prime factors, and hence obtain their L. C. M.

30. Find the L. C. M. of 44, 54 and 72 by resolving them into their prime factors.

31. Find the L. C. M. of R3. 9*a*. 4*p*. and R7. 10*a*. 3*p*.

32. The H. C. F. and L. C. M. of two numbers are 16 and 192 respectively ; one of the numbers is 48 ; find the other.

33. The H. C. F. and L. C. M. of two numbers are 10 and 30030 respectively ; one of the numbers is 770 : what is the other ?

115. The following rule gives the most convenient method of finding the L. C. M. of several small numbers.

Place the numbers side by side in a line ; divide by any one of the prime numbers, 2, 3, 5, 7, 11,...... which will divide any two at least of the given numbers exactly ; set down the quotients thus obtained and the undivided numbers side by side ; and proceed in this way until you get a line of numbers which are prime to one another. The continued product of all the divisors and the numbers in the last line will be the L. C. M. required.

Example 1. Find the L. C. M. of 12, 18, 20 and 105.

Process :

```
2 ) 12, 18, 20, 105
2 )  6,  9, 10, 105
3 )  3,  9,  5, 105
5 )  1,  3,  5,  35
     1,  3,  1,   7
```

L. C. M. $=2\times2\times3\times5\times3\times7$
$=1260$.

```
2 ) 12, 18, 20, 105
2 )  6,  9, 10, 105
3 )  3,  9,  5, 105
         3,      35
```

$\therefore$ L. C. M. $=2\times2\times3\times3\times35$
$=1260$.

Note. Work may be shortened by rejecting, at *any* stage, from the line any one of the numbers, which is a factor of any other number in the same line.

Thus, if it is required to find the L. C. M. of 6, 12, 15, 30 and 40, it will be sufficient to find the L. C. M. of 12, 30 and 40.

Example 2. Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

The L. C. M. of 12, 16 and $18=144$.

$\therefore$ The number required $=144+5=149$.

EXAMPLES. 58.

Find the L. C. M. of

1. 6, 8, 16.
2. 12, 16, 24.
3. 5, 18, 16, 9.
4. 9, 4, 18, 6.
5. 12, 15, 18, 24, 56.
6. 15, 16, 20, 28, 42.
7. 22, 17, 33, 25, 85.
8. 8, 9, 12, 18, 30.
9. 6, 15, 27, 35, 45.
10. 28, 36, 54, 72, 90.
11. 24, 10, 32, 45, 25.
12. 9, 18, 24, 72, 144.
13. 51, 187, 153, 165.
14. 33, 55, 60, 80, 90.
15. 22, 88, 132, 198.
16. 17, 51, 119, 210.
17. 50, 338, 675, 702, 975.
18. 24, 35, 52, 60, 91, 108.

19. 315, 156, 126, 108, 91. **20.** 27, 87, 203, 261, 189.

21. 126, 145, 87, 210, 585. **22.** 2, 3, 4, 5, 6, 7, 8, 9, 10.

23. 2, 4, 6, 8, 10, 12, 14, 16. **24.** 15, 16, 18, 20, 24, 25, 27, 30.

25. 24, 35, 52, 60, 91, 108, 126, 156, 315.

26. Find the least number which when divided by 12, 18 and 30, gives the same remainder 9 in each case.

27. Find the least number which when divided by 128 and 96 will leave in each case the same remainder 5.

28. Find the least number which being increased by 3, will be exactly divisible by 24, 36 and 48.

29. Find the smallest number of sq. inches which contains an exact number of sq. feet or of sq. cubits.

30. What is the smallest sum of money that can be paid in pounds, or in guineas, or in moidores ?

31. Five bells toll at intervals of 3, 5, 7, 8 and 10 seconds respectively, beginning together ; after what interval of time will they again toll together ?

32. Three men journey 10, 15 and 18 miles a day respectively ; find the least distance which would occupy each of them a complete number of days.

33. Two round pillars are 14 yd. 1 ft. 9 in. and 18 yd. 2 ft. 3 in. respectively in circumference ; find the shortest rope that can be wrapped round each an exact number of times.

34. A heap of shot when made up into groups of 28, 32 and 42, leaves always a remainder 5 ; find the least number of shot such heap can contain.

35. Find the least number which is divisible by all the numbers from 1 to 20 inclusive.

36. The circumferences of the wheels of a carriage are 6 ft. 3 in. and 9 ft. ; what is the least distance in which both the wheels will make an exact number of revolutions ?

116. Additional Examples on H. C. F. and L. C. M.

Example. Find the number lying between 900 and 1000 which, when divided by 38 and 57, leaves in each case a remainder 23.

The least common multiple of 38 and 57 is 114 and the multiple which is between 900 and 1000 is 912. Now 912 + 23, *i.e.*, 935 lies between 900 and 1000 and when divided by 38 and 57 leaves in each case 23 as the remainder. Therefore 935 is the number required.

```
114 ) 1000 ( 8
      912
       88
```

EXAMPLES. 59.

1. Find the two numbers lying between 100 and 200, of which the H. C. F. is 48.

2. Find the two numbers which lie between 100 and 200 and which have 36 as their H. C. F.

3. Find the two numbers lying between 200 and 300, of which the H. C. F. is 37.

4. Find the numbers lying between 400 and 500, which are divisible by 12, 15 and 20.

5. Find the numbers lying between 200 and 300, which when divided by 6, 8 or 9 will leave a remainder 5 in each case.

6. Find the greatest number and the least number which being subtracted from 3000 will make the result divisible by 7, 11 and 13.

7. What is the least number that must be added to and what is the greatest number that must be subtracted from 90900 that the results may be divisible by 777, 819 and 4329 ?

8. Find the greatest and the least numbers of six digits which are divisible by 27, 45, 60, 72 and 96.

9. The H. C. F. of two numbers is 21 and the L. C. M. is 4641 ; one of the numbers lies between 200 and 300 ; find the numbers.

10. Find the three largest numbers such that their H. C. F. is 7 and their L. C. M. is 1155.

11. What numbers of four digits each can have 143 as their H. C. F. and 25025 as their L. C. M. ?

12. Find the least number which when divided by 12 and 16 will leave the remainders 5 and 9 respectively. [Here, 12 − 5 = 7, and 16 − 9 = 7 ; therefore if 7 be added to the number required the sum will be divisible by both the numbers 12 and 16. Now the L. C. M. of 12 and 16 = 48 ; ∴ the number required = 48 − 7 = 41.]

13. Find the least number which when divided by 24 and 36 will leave the remainders 14 and 26 respectively.

14. Find the least number which when divided by 48, 64, 72, 80, 120 and 140 will leave the remainders 38, 54, 62, 70, 110 and 130 respectively.

15. Among how many children may 429 mangoes and also 715 oranges be equally divided ?

[The number of children required must be a common factor of 429 and 715. Now the H. C. F. of 429 and 715 = 143 : ∴ the number of children required must be 143 or a factor of 143. But 143 = 13 × 11 ; ∴ the number of children required is 143, 13 or 11.]

16. Among how many children may 175 mangoes and also 105 oranges be equally divided ?

17. Among a certain number of children 1001 mangoes and also 910 oranges may be equally divided. How many are the children ?

18. Find the number of children among whom 1596 mangoes and also 1428 oranges may be equally divided.

19. When 325 and 535 are divided by a certain number there is left the same remainder 10 in each case. Find the number.

20. A man bought two heaps of mangoes, one for R10. 5*a*. and the other for R18. 0*a*. 9*p*. If the price of each mango be the same, and not less than three and not more than four annas, find the total number of mangoes he bought.

21. What two numbers both greater than 29 have 29 for their H. C. F. and 4147 for their L. C. M. ?

22. A heap of pebbles can be made up exactly into groups of 25, but when made up into groups of 18, 27 and 32, there is in each case a remainder of 11 ; find the least number of pebbles such a heap can contain.

23. Find the least number which when divided by 35 leaves a remainder 25, when divided by 45 leaves 35, when divided by 55 leaves 45.

24. Find the greatest number of 4 digits divisible by 15, 25, 40 and 75.

25. Find the greatest number of 5 digits which can be added to 8321 so that the sum may be exactly divisible by 15, 20, 24, 27, 32 and 36.

26. The G. C. M. of two numbers is 7 and their L. C. M. is 140. Find the possible values of the numbers.

27. What greatest number and what least number can be subtracted from 23759143 that the remainders may be divisible by 24, 35, 91, 130 and 150 ?

XXII. FRACTIONS.

117. When a quantity is composed solely of entire units, its measure is called a **whole number** or an **integer**.

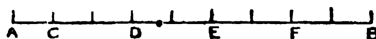
[In Sections II—XXI the word *number* has been used in the sense of a *whole number*.]

When a quantity is composed of one or more equal parts of the unit, its measure is called a **fractional number** or simply a **fraction**.

[The word *fraction* is derived from the Latin word *fractus*, which means "broken." The word, thus, refers to the *breaking up* of the unit into a number of parts which for the mathematical definition must be *equal* parts.]

Example. If we divide ("break") a foot into *four* equal parts, there will be 3 inches in each part. Each of these parts is called one *fourth-part*, or briefly, **one-fourth** of the whole foot and its relation to the foot is expressed by the symbol $\frac{1}{4}$.

Again, let AB represent the unit and let it be divided into 9 equal parts.



Then AC, AD, AE and AF respectively contain 1, 3, 5 and 7 such parts and represent *one ninth-part* (or **one-ninth**), *three ninth-parts* (or **three-ninths**), *five ninth-parts* (or **five-ninths**) and *seven ninth-parts* (or **seven-ninths**) of the unit. These fractions are expressed by the symbols $\frac{1}{9}$, $\frac{3}{9}$, $\frac{5}{9}$ and $\frac{7}{9}$.

Next consider $\frac{9}{9}$. What does it mean? It means that AB has been divided into 9 equal parts and all the 9 parts have been taken. This, of course, is the same as taking the whole line AB, that is, the unit itself.

$$\therefore \quad \tfrac{9}{9} = 1.$$

In the same way

1 pie = *one-twelfth*	of an anna	$= \frac{1}{12} a.$,
5 pies = *five-twelfths*	" " "	$= \frac{5}{12} a.$,
12 pies = *twelve-twelfths*	" " "	$= \frac{12}{12} a. = 1 a.$,
1 foot = *one-third*	of a yard	$= \frac{1}{3}$ yd.,
2 feet = *two-thirds*	" " "	$= \frac{2}{3}$ yd.,
3 feet = *three-thirds*	" " "	$= \frac{3}{3}$ yd. = 1 yd.

We thus see that if a unit be divided into any integral number of equal parts, and one or more of these parts be taken, a **fraction** is obtained expressing the relation which such parts bear to the whole.

The fraction $\frac{1}{9}$ is read as *one-ninth* or "one over nine". Similarly, $\frac{3}{9}$ is read as *three-ninths* or "three over nine" and so on. The fraction $\frac{1}{2}$ is read as *one-half* or "one over two".

A fraction is thus expressed by two figures, one over the other, separated by a horizontal line. The lower figure which indicates (or *names*) the number of equal parts into which the unit has been divided is called the *naming* figure, Latin **denominator**; and the upper figure which indicates the

number of these equal parts actually taken to make up the quantity is called the *numbering* figure, Latin **numerator.** The numerator and denominator are called **terms** of the fraction.

Thus, $\frac{3}{16}$ represents the fraction of which the numerator is 3 and the denominator is 16.

Note 1. Sometimes it is convenient to write fractions in the form 3/16, 7/16, 13/16, the numerator and denominator being separated by a slanting line called the "solidus." But they are read as stated above.

A fraction expressed in the above notation is called a **Vulgar** or **Common Fraction,** as distinguished from a **Decimal Fraction** (or **Decimals**) which will be explained later on.

Example. '$\frac{2}{3}$ of a yard' indicates a quantity which is composed of *two* equal parts, *three* of which make up one yard; that is, '$\frac{2}{3}$ of a yard' = 2 feet.

Note 2. We should get the same result whether we divide a yard (or any other unit) into 3 equal parts and take 2 such parts, or divide 2 yards (or twice that other unit) into 3 equal parts and take one of these parts. *A fraction may thus be considered to express the quotient of the numerator by the denominator.* Hence $\frac{2}{3}$ is often read '2 divided by 3'.

EXAMPLES. 60.

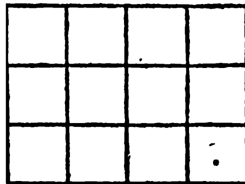
(*Oral.*)

1. What fractions of 1 anna are 1 pice, 2 pice, 3 pice?
2. What fractions of 1 foot are 1 inch, 6 inches, 4 inches?
3. (*a*) What fractions of 1 yard are 1 foot, 2 feet?
   (*b*) What fraction of a seer is a chatak?

Read off the value of

4. $\frac{1}{4}$ of R1. 5. £$\frac{1}{3}$. 6. $\frac{1}{2}$*d.* 7. $\frac{1}{40}$ of a md.
8. $\frac{5}{16}$ of R1. 9. $\frac{9}{20}$ of £1. 10. $\frac{7}{12}$ of a ft. 11. $\frac{5}{12}$ of an anna.
12. $\frac{19}{36}$ of a yd. 13. $\frac{4}{12}$ of 1*s.* 14. $\frac{3}{64}$ of R1. 15. $\frac{3}{20}$ ton.

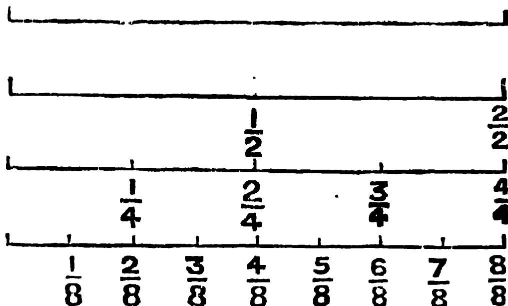
16. Draw the adjoining diagram and by means of letters indicate the portions that will represent $\frac{1}{12}$, $\frac{3}{12}$, $\frac{1}{3}$, $\frac{1}{4}$ of the whole area.



Write down the value of

17. $\frac{10}{110}$ mile.	**18.** $\frac{3}{8}$ seer.	**19.** $\frac{1}{16}$ sq. ft.
20. $\frac{7}{112}$ cwt.	**21.** $\frac{2}{5}$ of 15*a*.	**22.** $\frac{3}{7}$ of Re 1. 5*a*.
23. $\frac{4}{13}$ of 3 ft. 3 in.	**24.** $\frac{8}{15}$ of $7\frac{1}{2}$*d*.	**25.** $\frac{3}{13}$ of 1 hr. 5 min.

118. Let us consider the following four equal straight lines of unit length. Let the second of them be divided into two equal



parts, the third into four and the fourth into eight equal parts. It is clear that $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ represent one-half of the unit line.

$$\therefore \quad \frac{1}{2}=\frac{2}{4}=\frac{4}{8}.$$

But $\frac{2}{4}=\frac{1\times 2}{2\times 2}$ and $\frac{4}{8}=\frac{1\times 4}{2\times 4}$.

Again $\frac{1}{2}=\frac{2\div 2}{4\div 2}$ and also $=\frac{4\div 4}{8\div 4}$.

We thus see that

(i) if the numerator and denominator of a fraction are each multiplied by the same number, the value of the fraction is not altered and

(ii) if the numerator and denominator of a fraction are each divided by the same number, the value of the fraction is not altered.

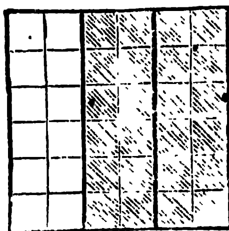
Again, let us consider the fractions $\frac{2}{3}$ and $\frac{24}{36}$: the first indicates that the unit is divided into 3 equal parts and 2 of these parts are taken ; the second indicates that the unit is divided into 36 equal parts and 24 of these parts are taken. Now, a part in the former case is obviously equal to 12 parts in the latter case : consequently 2 parts (taken) in the former case = 24 parts (taken) in the latter case.

$$\therefore \quad \frac{2}{3}=\frac{24}{36} \text{ ; and } \frac{24}{36}=\frac{2\times 12}{3\times 12}.$$

Illustration : $\frac{2}{3}$ of a yard = 2 ft. ; and $\frac{24}{36}$ of a yard = 24 in. = 2 ft.

119. The following graphical illustration will enable the student to visualise the truth of the statements made in the preceding Article.

Let the adjoining square, regarded as a unit, be divided into 36 equal squares. The shaded portion contains 24 of these squares and therefore it represents the fraction $\frac{24}{36}$. Again, the figure may also be regarded as being divided into 3 equal rectangles by thick vertical lines and of these parts the shaded portion contains 2. Therefore the shaded portion also represents the fraction $\frac{2}{3}$.



$\therefore \quad \frac{2}{3} = \frac{24}{36}.$

120. A whole number may be expressed as a fraction with *any* given denominator.

Thus, for example, $3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} =$ etc.

121. A given fraction can be transformed into another fraction of which the denominator is any multiple of the given denominator.

Example. Transform $\frac{2}{3}$ into a fraction with the denominator 12.

$12 = 3 \times 4$; hence $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. *Ans.*

EXAMPLES. 61.

(*Examples 1—9 should be taken orally.*)

Read off the number of

1. Annas in $\frac{1}{2}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{5}{16}$, $\frac{13}{16}$ of a rupee.
2. Pence in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{12}$ of a shilling.
3. inches in $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{11}{18}$, $\frac{1}{36}$ of a yard.
4. Chataks in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$ of a seer.
5. Hours in $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{12}$, $\frac{1}{24}$ of a day.

Fill in the blanks :

6. $\frac{2}{7}$ yd. $= \frac{\ }{14}$ yd. 7. in. $= \frac{3}{\ }$ in. 8.

9. £$\frac{5}{8}$ = £$\frac{15}{\ }$.

Show by drawings that

10. $\frac{1}{3} = \frac{2}{6}$. 11. $\frac{1}{3} = \frac{3}{9}$. 12. $\frac{1}{5} = \frac{3}{15}$.

13. $\frac{2}{7} = \frac{8}{28}$. 14. $\frac{1}{2} = \frac{3}{6} = \frac{\ }{18}$. 15. $\frac{2}{3}$ $\frac{4}{6} = \frac{8}{12}$.

Work with rulers :

16. Draw a line 6 in. long. How many times can $1\frac{1}{2}$ in. be marked along it ?

17. Draw a line $6\frac{3}{4}$ in. long and mark off portions each $1\frac{1}{3}$ in. long.

Use your rulers to show what fraction of

18. 1*a.* is 1 pie ? **19.** £1 is 1*s.* ? **20.** 1 ft. is 1 in. ?

21. 1 lb. is 1 oz. ?

EXAMPLES. 62.

1. Express each of the whole numbers 2, 5, 7, 10 as a fraction with denominator 9.

2. Change 11 to fractions having 2, 9, 11, 25 and 35 for their denominators.

3. Express 21, 76 and 159 as fractions with denominators 5, 9 and 75 respectively.

4. Express $\frac{5}{6}$ and $\frac{7}{8}$ each as a fraction with denominators 12, 18, 96 and 600.

5. Find fractions equal to $\frac{1}{3}$, $\frac{3}{5}$, $\frac{7}{9}$, $\frac{2}{15}$, $\frac{13}{30}$, having 90 for their denominator.

6. Transform $\frac{22}{121}$, $\frac{39}{65}$ and $\frac{54}{90}$ into equivalent fractions whose denominators shall be 11, 5 and 10 respectively.

7. Express $\frac{35}{42}$, $\frac{50}{60}$, $\frac{60}{72}$ and $\frac{77}{66}$, each as a fraction with the denominator 6.

122. A fraction is said to be in its **lowest terms** when its numerator and denominator have no *common* factor.

Example 1. Reduce $\frac{630}{840}$ to its lowest terms.

We divide the numerator and denominator by their H. C. F. which is 210.

Thus $\frac{630}{840}=\frac{630\div210}{840\div210}=\frac{3}{4}$. *Ans.*

Note. In reducing a fraction to its lowest terms, it is convenient first to remove any factors common to both numerator and denominator, that can be found by inspection or by the application of the tests of divisibility. (Art. 104).

Example 2. Reduce $\frac{78}{84}$ to its lowest terms.

Process : $\frac{78}{84}=\frac{13}{14}$. *Ans.*

Here, first 78 and 84 are divided by 2, giving quotients 39 and 42 ; next 39 and 42 are divided by 3, giving quotients 13 and 14 which are prime to each other ; hence the answer is $\frac{13}{14}$.

Example 3. Reduce by cancelling to their lowest terms

(i) $\frac{2\times15}{35\times8}$, (ii) $\frac{3\times4\times5}{6\times20}$.

(i) $\frac{\cancel{2}\times\cancel{15}^{3}}{\cancel{35}_{7}\times\cancel{8}_{4}}=\frac{3}{28}$. *Ans.* (ii) $\frac{\cancel{3}\times\cancel{4}\times\cancel{5}}{\cancel{6}\times\cancel{20}_{2}}=\frac{1}{2}$. *Ans.*

It should be borne in mind that when a factor is *cancelled*, it is replaced by 1 and not by 0.

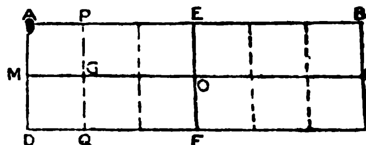
EXAMPLES. 63.

(*Oral.*)

Reduce to their lowest terms :

1. $\frac{3}{6}$. 2. $\frac{4}{12}$. 3. $\frac{15}{30}$. 4. $\frac{12}{16}$. 5. $\frac{14}{21}$.
6. $\frac{12}{27}$. 7. $\frac{21}{28}$. 8. $\frac{18}{30}$. 9. $\frac{16}{24}$. 10. $\frac{25}{75}$.
11. $\frac{64}{80}$. 12. $\frac{49}{56}$. 13. $\frac{15}{40}$. 14. $\frac{33}{44}$. 15. $\frac{60}{80}$.
16. $\frac{19}{57}$. 17. $\frac{26}{78}$. 18. $\frac{51}{68}$. 19. $\frac{16}{100}$ 20. $\frac{34}{85}$.

21. Use the adjoining diagram to show that
(i) $\frac{1}{2}=\frac{2}{4}=\frac{6}{12}=\frac{1\times6}{2\times6}$;
(ii) $\frac{1}{6}=\frac{2}{12}$; (iii) $\frac{1}{3}=\frac{4}{12}=\frac{1\times4}{3\times4}$.



EXAMPLES. 64.

Reduce to their lowest terms :

1. $\frac{12}{18}$. 2. $\frac{16}{36}$. 3. $\frac{21}{70}$. 4. $\frac{10}{15}$. 5. $\frac{57}{95}$.
6. $\frac{64}{128}$. 7. $\frac{171}{189}$. 8. $\frac{63}{144}$. 9. $\frac{147}{189}$. 10. $\frac{555}{999}$.
11. $\frac{875}{1000}$. 12. $\frac{720}{864}$. 13. $\frac{495}{1210}$. 14. $\frac{665}{1881}$. 15. $\frac{990}{1035}$.
16. $\frac{3094}{3042}$. 17. $\frac{1708}{1830}$. 18. $\frac{1485}{2160}$. 19. $\frac{5239}{6076}$. 20. $\frac{3300}{4235}$.
21. $\frac{1261}{1649}$. 22. $\frac{3276}{4914}$. 23. $\frac{5544}{6552}$. 24. $\frac{1632}{2976}$. 25. $\frac{2172}{3204}$.
26. $\frac{2755}{3515}$. 27. $\frac{3366}{4488}$. 28. $\frac{9648}{10224}$. 29. $\frac{30429}{50841}$. 30. $\frac{3200}{5640}$.
31. $\frac{7568}{9504}$. 32. $\frac{8398}{20995}$. 33. $\frac{48510}{49005}$. 34. $\frac{714285}{999999}$. 35. $\frac{135795}{222210}$.

FXAMPLES. 65.

Reduce by cancelling to their simplest forms :

1. $\frac{3\times4}{8\times9}$.	2. $\frac{7\times15}{5\times14}$.	3. $\frac{15\times4}{12\times5}$.
4. $\frac{6\times10\times15}{3\times8\times15}$.	5. $\frac{8\times14\times24}{48\times35\times3}$.	6. $\frac{7\times8\times24}{56\times21\times16}$.
7. $\frac{3\times25\times11}{55\times30\times4}$.	8. $\frac{5\times81\times77}{22\times33\times75}$.	9. $\frac{80\times30\times12}{100\times72\times10}$.
10. $\frac{17\times63\times4}{35\times34\times6}$.	11. $\frac{7\times9\times10}{24\times28\times5}$.	12. $\frac{15\times21\times32}{35\times48\times7}$.

123. A **mixod number** is composed of a whole number and a fraction, as $3\frac{2}{5}$. This stands for $3+\frac{2}{5}$, and is read '*three and two-fifths*'.

A mixed number can be expressed as a fraction.

Example. Express $4\frac{2}{3}$ as a fraction.

$$4\tfrac{2}{3}=4+\tfrac{2}{3}=\tfrac{12}{3}+\tfrac{2}{3}=\tfrac{14}{3}.$$

For, 12 thirds of the unit and 2 thirds of the unit make (12 + 2) or 14 thirds of the unit.

Hence the rule : Multiply the whole number by the denominator of the fractional part ; add the result to the numerator of that part for the new numerator, and retain the same denominator.

EXAMPLES. 66.

(*Examples* 1—8 *should be taken orally.*)

Express the following mixed numbers as fractions :

1. $3\frac{1}{3}$.	2. $7\frac{2}{3}$.	3. $9\frac{3}{11}$.	4. $8\frac{4}{15}$.	5. $5\frac{1}{3}$.
6. $7\frac{1}{100}$.	7. $12\frac{3}{75}$.	8. $20\frac{3}{25}$.	9. $39\frac{2}{11}$.	10. $90\frac{1}{35}$.
11. $29\frac{7}{200}$.	12. $76\frac{13}{96}$.	13. $25\frac{23}{25}$.	14. $111\frac{110}{111}$.	15. $99\frac{19}{99}$.
16. $7\frac{43}{100}$.	17. $8\frac{320}{1000}$.	18. $22\frac{33}{131}$.	19. $40\frac{4}{201}$.	20. $4\frac{4}{59}$.

124. A **proper** fraction is one, of which the numerator is less than the denominator, as $\frac{2}{3}$.

An **improper** fraction is one, of which the numerator is equal to or greater than the denominator, as $\frac{3}{3}$, $\frac{7}{3}$.

An improper fraction is either equal to an integer or a mixed number.

Example. Reduce $\frac{21}{7}$ and $\frac{29}{6}$ to whole or mixed numbers.

$$\tfrac{21}{7}=\tfrac{3\times7}{1\times7}=\tfrac{3}{1}=3. \qquad \tfrac{29}{6}=\tfrac{24}{6}+\tfrac{5}{6}=4+\tfrac{5}{6}=4\tfrac{5}{6}.$$

Hence the rule : Divide the numerator by the denominator ; the quotient will be the integral part of the mixed number ; the

remainder will be the numerator, and the denominator of the given fraction the denominator, of the fractional part.

(i)	(ii)
7) 21	6) 29
3, rem. 0.	4, rem. 5.
Hence $\frac{21}{7}=3$.	Hence $\frac{29}{6}=4\frac{5}{6}$.

125. The **reciprocal** of a fraction is a fraction formed by interchanging its terms ; thus the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, of 4 (or $\frac{4}{1}$) is $\frac{1}{4}$.

EXAMPLES. 67.

(*Examples* 1—15 *should be taken orally.*)

Express as whole or mixed numbers :

1. $\frac{7}{2}$. 2. $\frac{9}{4}$. 3. $\frac{13}{5}$. 4. $\frac{21}{4}$. 5. $\frac{23}{7}$.
6. $\frac{30}{8}$. 7. $\frac{45}{5}$. 8. $\frac{70}{12}$. 9. $\frac{99}{16}$. 10. $\frac{85}{17}$.
11. $\frac{39}{14}$. 12. $\frac{56}{11}$. 13. $\frac{80}{13}$. 14. $\frac{89}{17}$. 15. $\frac{69}{19}$.
16. $\frac{300}{75}$. 17. $\frac{304}{101}$. 18. $\frac{702}{99}$. 19. $\frac{850}{15}$. 20. $\frac{879}{125}$.

Express the reciprocals of the following fractions as whole or mixed numbers. .

21. $\frac{130}{3715}$. 22. $\frac{1}{325}$. 23. $\frac{76}{7695}$. 24. $\frac{75}{735}$. 25. $\frac{90}{4395}$.
26. $\frac{3003}{7007}$. 27. $\frac{425}{4975}$. 28. $\frac{123}{1231}$. 29. $\frac{60}{605}$. 30. $\frac{1885}{9995}$.

126. Two or more given fractions may be reduced to equivalent fractions having the lowest common denominator.

Example. Reduce $\frac{2}{9}$, $\frac{5}{12}$ and $\frac{3}{10}$ to equivalent fractions having the lowest common denominator.

The denominators are 9, 12 and 10 ; their L. C. M. is 180.

$$180 \div 9 = 20, \quad \therefore \quad \frac{2}{9} = \frac{2 \times 20}{9 \times 20} = \frac{40}{180} ;$$
$$180 \div 12 = 15, \quad \therefore \quad \frac{5}{12} = \frac{5 \times 15}{12 \times 15} = \frac{75}{180} ;$$
$$180 \div 10 = 18, \quad \therefore \quad \frac{3}{10} = \frac{3 \times 18}{10 \times 18} = \frac{54}{180}.$$

Hence $\frac{2}{9}$, $\frac{5}{12}$ and $\frac{3}{10} = \frac{40}{180}$, $\frac{75}{180}$ and $\frac{54}{180}$ respectively ; and these latter have the lowest common denominator.

EXAMPLES. 68.

Reduce to equivalent fractions having the least common denominator :

1. $\frac{1}{4}$ and $\frac{5}{6}$. 2. $\frac{3}{10}$ and $\frac{4}{15}$. 3. $\frac{7}{18}$ and $\frac{3}{25}$. 4. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
5. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. 6. $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$. 7. $\frac{3}{8}$, $\frac{7}{9}$, $\frac{5}{12}$. 8. $\frac{4}{15}$, $\frac{7}{18}$, $\frac{21}{35}$.

9. $\frac{5}{16}, \frac{7}{24}, \frac{11}{40}$. 10. $\frac{2}{15}, \frac{3}{20}, \frac{1}{100}$. 11. $\frac{21}{50}, \frac{7}{80}, \frac{81}{90}$.
12. $\frac{1}{25}, \frac{2}{45}, \frac{3}{60}$. 13. $3\frac{1}{3}, 4\frac{1}{4}, 6\frac{1}{6}$. 14. $2, \frac{1}{5}, \frac{1}{8}$.
15. $3, \frac{1}{5}, \frac{7}{12}$. 16. $\frac{1}{7}, 3\frac{1}{3}, 2, \frac{3}{5}$. 17. $3, \frac{1}{3}, 4, \frac{1}{4}$.
18. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$. 19. $\frac{1}{11}, \frac{3}{22}, \frac{4}{77}, \frac{8}{14}, 1$.
20. $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}$. 21. $\frac{73}{60}, \frac{11}{15}, \frac{13}{24}, \frac{31}{40}, \frac{117}{100}$.
22. $2, 2\frac{1}{3}, \frac{7}{12}, \frac{4}{9}, \frac{11}{16}$. 23. $\frac{1}{10}, \frac{1}{16}, \frac{1}{60}, \frac{1}{32}, \frac{1}{100}$.
24. $2, 3\frac{1}{3}, 7\frac{1}{4}, \frac{5}{16}, \frac{1}{3}$. 25. $\frac{4}{9}, \frac{2}{51}, \frac{1}{85}, \frac{7}{60}, \frac{1}{40}$.
26. $3, 7\frac{1}{2}, 2\frac{1}{3}, \frac{5}{6}, \frac{7}{8}$. 27. $1\frac{1}{7}, \frac{5}{7}, \frac{1}{2}, \frac{4}{4}, \frac{5}{16}, 2\frac{1}{14}$.

127. Of two fractions having a common denominator the *greater* is that which has the *greater* numerator.

Thus, of the fractions $\frac{7}{12}$ and $\frac{5}{12}$, the former is obviously greater.

Of two fractions having a common numerator the *greater* is that which has the *less* denominator.

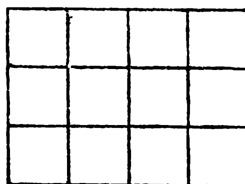
Thus, of the fractions $\frac{5}{3}$ and $\frac{5}{4}$ the former is greater.

Note. In comparing values of fractions, they must be reduced to equivalent fractions having the L. C. D. or L. C. N.

EXAMPLES. 69.

Which is greater,

1. $\frac{2}{3}$ or $\frac{3}{4}$? 2. $\frac{7}{11}$ or $\frac{9}{13}$? 3.
4. $\frac{15}{19}$ or $\frac{16}{23}$? 5. $\frac{7}{5}$ or $\frac{8}{7}$? 6. $\frac{29}{27}$



7. Use the adjoining diagram and say which is greater,

$\frac{2}{3}$ or $\frac{3}{4}$; $\frac{5}{6}$ or $\frac{3}{4}$; $\frac{1}{2}$ or $\frac{7}{12}$; $\frac{4}{6}$ or $\frac{3}{4}$.

Find the greatest and the least of the following fractions :

8. $\frac{15}{[illegible]}, \frac{19}{25}$, 9. $\frac{9}{10}, \frac{10}{11}$. 10. $\frac{20}{31}, \frac{30}{55}, \frac{10}{34}$.
11. $\frac{2}{5}, \frac{13}{18}, \frac{12}{17}, \frac{1}{2}$. 12. $\frac{3}{7}, \frac{5}{13}, \frac{7}{16}, \frac{9}{20}$. 13. [illegible]

Arrange in order of magnitude :

14. $\frac{3}{4}, \frac{5}{9}, \frac{7}{12}$. 15. $\frac{7}{16}, \frac{13}{33}, \frac{4}{45}$. 16. $\frac{3}{5}, \frac{7}{9}, \frac{11}{13}$.
17. $\frac{15}{4}, 3\frac{2}{3}, \frac{27}{8}$. 18. $\frac{21}{25}, \frac{26}{[illegible]}, \frac{31}{36}$. 19. $\frac{27}{87}, \frac{118}{341}, \frac{145}{428}$.
20. $\frac{6}{7}, \frac{13}{28}, \frac{4}{9}, \frac{5}{8}$. 21. $\frac{51}{76}, \frac{113}{152}, \frac{7}{12}, \frac{2}{3}$. 22. $\frac{3}{4}, \frac{9}{20}, \frac{11}{15}, \frac{17}{12}$.

ADDITION AND SUBTRACTION OF FRACTIONS.

128. Addition.—Just as concrete quantities which have the same denomination or name can be added together, similarly fractions having the same denominator can also be added; for just as

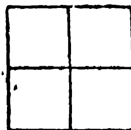
$$2 \textit{ annas} + 5 \textit{ annas} = 7 \textit{ annas},$$

$$\text{so} \quad 2 \textit{ ninths} + 5 \textit{ ninths} = 7 \textit{ ninths};$$

$$\textit{i.e.,} \quad \tfrac{2}{9} + \tfrac{5}{9} = \tfrac{7}{9}.$$

Again, just as rupees cannot be added to annas without changing rupees into annas,* so fractions having different dinominators cannot be added without reducing them to fractions of the same denominator. Thus it is clear that the sum of fractions having a common denominator is a fraction whose numerator is the sum of the numerators, and whose denominator is the common denominator, of the original fractions. When fractions to be added have different denominators, they must be reduced to equivalent fractions having the L. C. D. (see Art. 126.)

Example 1. Use the adjoining diagram to find the sum of $\frac{1}{2}$ and $\frac{1}{4}$ and test the truth of the principle enunciated above.



Example 2. Add together $\frac{7}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$.

Process : $\frac{7}{5} + \frac{3}{5} + \frac{4}{5} = \frac{7+3+4}{5} = \frac{14}{5} = 2\frac{4}{5}$. *Ans.*

Example 3. Add together $\frac{1}{2}$, $\frac{5}{6}$ and $\frac{4}{9}$.

The L. C. M. of 2, 6 and 9 = 18.

$\therefore \quad \frac{1}{2} + \frac{5}{6} + \frac{4}{9} = \frac{9}{18} + \frac{15}{18} + \frac{8}{18} = \frac{9+15+8}{18} = \frac{32}{18} = \frac{16}{9} = 1\frac{7}{9}$. *Ans.*

Note. The sum should always be expressed in its lowest terms : and if it is an improper fraction, it should be reduced to a mixed number.

EXAMPLES. 70.

(*Examples* 1—12 *may be taken orally.*)

Add together

1. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$. 2. $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$. 3. $\frac{1}{9}$, $\frac{2}{9}$, $\frac{5}{9}$.

4. $\frac{7}{11}$, $\frac{2}{11}$, $\frac{8}{11}$. 5. $\frac{7}{15}$, $\frac{12}{15}$, $\frac{8}{15}$. 6. $\frac{21}{50}$, $\frac{30}{50}$, $\frac{49}{50}$.

* We can no doubt write the sum of ₨5 and 6*a.* as ₨5. 6*a.* ; but this only means ₨5 *and* 6*a.* and does not combine them. If we change ₨5 into 80*a.*, we can combine 80*a.* and 6*a.*, and write their sum as 86*a.*

7. $\frac{8}{48}, \frac{19}{48}, \frac{21}{48}$. 8. $\frac{3}{100}, \frac{721}{100}, \frac{6}{100}$. 9. $\frac{12}{63}, \frac{18}{63}, \frac{51}{63}$.
10. $\frac{1}{2}, \frac{1}{3}$. 11. $\frac{2}{3}, \frac{3}{4}$. 12. $\frac{7}{9}, \frac{5}{12}$.
13. $\frac{3}{16}, \frac{7}{20}, \frac{8}{15}$. 14. $\frac{9}{20}, \frac{12}{27}, \frac{5}{12}$. 15. $\frac{81}{30}, \frac{9}{10}, \frac{7}{12}$.

Simplify

16. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$. 17. $\frac{5}{12}+\frac{5}{16}+\frac{5}{20}$. 18. $\frac{2}{27}+\frac{8}{15}+\frac{9}{20}$.
19. $\frac{1}{9}+\frac{1}{12}+\frac{1}{15}$. 20. $\frac{9}{7}+\frac{7}{5}+\frac{5}{3}$. 21. $\frac{4}{5}+\frac{5}{4}+\frac{2}{15}$.
22. $\frac{1}{6}+\frac{1}{9}+\frac{1}{18}+\frac{1}{27}$. 23. $\frac{7}{8}+\frac{7}{12}+\frac{7}{15}+\frac{7}{28}$. 24. $\frac{3}{5}+\frac{5}{3}+\frac{9}{15}+\frac{15}{9}$.
25. $\frac{3}{51}+\frac{21}{17}+\frac{7}{9}+\frac{1}{2}$. 26. $\frac{15}{13}+\frac{14}{39}+\frac{1}{9}+\frac{1}{52}$. 27. $\frac{7}{12}+\frac{11}{18}+\frac{27}{20}+\frac{14}{15}$.
28. $\frac{113}{226}+\frac{220}{330}+\frac{565}{678}$. 29. $\frac{31}{144}+\frac{329}{128}+\frac{70}{64}$. 30. $\frac{7}{10}+\frac{10}{17}+\frac{17}{20}+\frac{20}{51}$.

129. In adding mixed numbers it is convenient to proceed as in the following example :

Example. Add together $2\frac{1}{2}$, $3\frac{1}{4}$ and $7\frac{5}{6}$.

Process : $2\frac{1}{2}+3\frac{1}{4}+7\frac{5}{6}=2+3+7+\frac{1}{2}+\frac{1}{4}+\frac{5}{6}$

$=12+\frac{6}{12}+\frac{3}{12}+\frac{10}{12}$

$=12+\frac{6+3+10}{12}$

$=12+\frac{19}{12}=12+1\frac{7}{12}=13\frac{7}{12}$. *Ans.*

N. B. It is also convenient to reduce improper fractions to mixed numbers.

EXAMPLES. 71.

1. $3\frac{1}{2}+4\frac{1}{4}$. 2. $7\frac{1}{3}+6\frac{5}{6}$. 3. $5\frac{1}{10}+7\frac{1}{4}$. 4. $13\frac{1}{6}+2\frac{3}{5}$.
5. $3\frac{1}{3}+5\frac{1}{6}+15\frac{1}{18}$. 6. $7\frac{1}{7}+8\frac{1}{8}+14\frac{1}{14}$. 7. $2\frac{1}{2}+\frac{1}{8}+3$.
8. $31+9\frac{2}{5}+\frac{11}{15}$. 9. $1\frac{1}{11}+2\frac{2}{33}+\frac{99}{11}$. 10. $7\frac{3}{17}+2+\frac{100}{51}$.
11. $3\frac{1}{7}+4\frac{1}{21}+6\frac{1}{28}+1\frac{1}{14}$. 12. $2\frac{1}{5}+3\frac{1}{3}+4\frac{1}{2}+1\frac{7}{15}$.
13. $37\frac{1}{7}+\frac{900}{7}+\frac{330}{11}$. 14. $\frac{333}{36}+\frac{721}{36}+4$.
15. $2\frac{1}{4}+3+1\frac{1}{2}+\frac{53}{8}$. 16. $1+\frac{99}{4}+2\frac{1}{2}+3\frac{1}{10}$.
17. $\frac{1000}{7}+\frac{1000}{2}+\frac{1000}{3}$. 18. $\frac{9}{2}+\frac{99}{4}+\frac{999}{8}$.
19. $10+3\frac{1}{7}+\frac{222}{49}+\frac{1}{21}$. 20. $\frac{7}{3}+\frac{11}{9}+\frac{17}{15}+\frac{29}{18}$.

21.

R.	a.	p.
7	9	$2\frac{1}{2}$
5	10	$7\frac{11}{12}$
13	14	$6\frac{2}{3}$
2	7	$0\frac{13}{18}$

22.

£.	s.	d.
1	9	$2\frac{3}{8}$
2	0	$5\frac{3}{4}$
3	7	$0\frac{15}{16}$
1	0	$3\frac{1}{2}$

23.

yd.	ft.	in.
7	1	$3\frac{3}{4}$
2	2	$2\frac{1}{2}$
3	0	$7\frac{3}{8}$
2	1	$5\frac{5}{16}$

	lb.	oz.	dr.		oz.	dwt.	gr.		hr.	min.	sec.
24.	1	7	$7\frac{1}{2}$	25.	3	10	$7\frac{3}{4}$	26.	3	20	$9\frac{3}{4}$
	2	9	$3\frac{3}{4}$		7	0	$8\frac{5}{6}$		7	22	$19\frac{7}{8}$
	3	13	$0\frac{1}{3}$		8	3	$0\frac{4}{9}$		4	7	$29\frac{9}{16}$
	4	3	$7\frac{3}{16}$		2	7	$2\frac{1}{2}$		5	34	$34\frac{5}{12}$

130. Subtraction.—The method of subtraction of fractions is similar to that of addition.

Example 1. Subtract $\frac{3}{7}$ from $\frac{5}{7}$.

Process : $\frac{5}{7}-\frac{3}{7}=\frac{5-3}{7}=\frac{2}{7}$. *Ans.*

Example 2. Subtract $\frac{3}{8}$ from $\frac{5}{6}$.

The L. C. M. of 8 and 6 = 24.

$\therefore\ \frac{5}{6}-\frac{3}{8}=\frac{20}{24}-\frac{9}{24}=\frac{20-9}{24}=\frac{11}{24}$. *Ans.*

EXAMPLES. 72.

(*Examples* 1—10 *should be taken orally.*)

Perform the following subtractions :

1. $\frac{4}{5}-\frac{2}{5}$.	2. $\frac{11}{17}-\frac{5}{17}$.	3. $\frac{12}{19}-\frac{9}{19}$.	4. $\frac{30}{49}-\frac{23}{49}$.
5. $\frac{77}{83}-\frac{23}{83}$.	6. $1-\frac{3}{7}$. $(1=\frac{7}{7})$.	7. $1-\frac{5}{12}$.	8. $1-\frac{7}{13}$.
9. $1-\frac{8}{15}$.	10. $1-\frac{19}{30}$.	11. $1-\frac{25}{37}$.	12. $1-\frac{49}{68}$.
13. $\frac{1}{2}-\frac{1}{3}$.	14. $\frac{1}{3}-\frac{1}{4}$.	15. $\frac{7}{8}-\frac{3}{10}$.	16. $\frac{7}{18}-\frac{7}{24}$.
17. $\frac{8}{15}-\frac{8}{25}$.	18. $\frac{3}{10}-\frac{1}{30}$.	19. $\frac{29}{48}-\frac{31}{64}$.	20. $\frac{5}{14}-\frac{3}{21}$.
21. $\frac{13}{20}-\frac{7}{50}$.	22. $\frac{101}{100}-\frac{101}{800}$.	23. $\frac{9}{5}-\frac{5}{3}$.	24. $7\frac{1}{2}-2\frac{1}{7}$.
25. $1\frac{7}{15}-1\frac{1}{5}$.	26. $\frac{8}{9}-\frac{11}{18}$.	27. $\frac{5}{3}-1\frac{1}{3}$.	28. $2\frac{2}{3}-2\frac{1}{3}$.
29. $7\frac{5}{8}-7\frac{3}{16}$.	30. $\frac{17}{88}-\frac{2}{77}$.		

131. The following examples are important.

Example 1. Subtract $3\frac{2}{5}$ from $7\frac{5}{8}$.

Process : $7\frac{5}{8}-3\frac{2}{5}=7\frac{25}{40}-3\frac{16}{40}=7-3+\frac{25}{40}-\frac{16}{40}=4+\frac{9}{40}=4\frac{9}{40}$. *Ans.*

Example 2. Subtract $2\frac{2}{3}$ from $4\frac{1}{5}$.

Process : $4\frac{1}{5}-2\frac{2}{3}=4\frac{3}{15}-2\frac{10}{15}=3\frac{18}{15}-2\frac{10}{15}=3-2+\frac{18}{15}-\frac{10}{15}$

$=1+\frac{8}{15}=1\frac{8}{15}$. *Ans.*

Example 3. Subtract $1\frac{5}{12}$ from 7.

Process : $7-1\frac{5}{12}=6+1-1\frac{5}{12}=(6-1)+(1-\frac{5}{12})=5+\frac{7}{12}=5\frac{7}{12}$. *Ans.*

Example 4. Subtract $3\frac{1}{4}$ from 9.

Process : $9-3\frac{1}{4}=6-\frac{1}{4}=5+1-\frac{1}{4}=5+\frac{3}{4}=5\frac{3}{4}$. *Ans.*

EXAMPLES. 73.

Perform the following subtractions :

1. $8\frac{1}{2}-5\frac{1}{3}$. 2. $9\frac{3}{5}-7\frac{1}{4}$. 3. $3\frac{1}{3}-\frac{2}{9}$. 4. $5\frac{7}{8}-\frac{1}{4}$.
5. $12\frac{3}{5}-7\frac{1}{3}$. 6. $17\frac{6}{17}-12\frac{12}{51}$. 7. $8\frac{13}{21}-2\frac{7}{28}$. 8. $10\frac{18}{39}-2\frac{1}{13}$.
9. $5\frac{1}{3}-2\frac{1}{2}$. 10. $7\frac{3}{8}-3\frac{5}{6}$. 11. $8\frac{1}{12}-7\frac{3}{16}$. 12. $23\frac{1}{21}-17\frac{1}{7}$.
13. $5\frac{29}{64}-2\frac{39}{48}$. 14. $12\frac{13}{28}-3\frac{30}{49}$. 15. $34\frac{1}{3}-24\frac{1}{7}$. 16. $50\frac{7}{81}-40\frac{8}{27}$.
17. $39\frac{10}{63}-28\frac{20}{21}$. 18. $9\frac{5}{72}-2\frac{41}{45}$. 19. $7\frac{1}{8}-\frac{5}{6}$. 20. $10\frac{1}{6}-\frac{3}{8}$.
21. $3-\frac{1}{2}$. 22. $7-\frac{3}{8}$. 23. $9-\frac{11}{12}$. 24. $10-\frac{10}{17}$.
25. $12-3\frac{4}{5}$. 26. $17-4\frac{3}{11}$. 27. $18-4\frac{1}{18}$. 28. $20-9\frac{20}{21}$.

Simplify

29. $2\frac{1}{2}+3\frac{1}{3}-4\frac{1}{4}$. 30. $7\frac{1}{3}+9\frac{1}{15}-10\frac{1}{6}$. 31. $3\frac{1}{7}+4\frac{1}{3}-\frac{11}{14}$.
32. $17\frac{1}{2}-3\frac{1}{4}-7\frac{1}{6}$. 33. $9\frac{3}{16}-8\frac{1}{2}+3\frac{1}{4}$. 34. $12\frac{1}{15}-7\frac{1}{3}-2\frac{1}{3}$.
35. $8-2\frac{1}{2}+7\frac{1}{3}-3\frac{7}{12}$. 36. $7-3\frac{15}{16}-2\frac{7}{8}+\frac{9}{32}$.
37. $7-\frac{100}{7}+\frac{200}{21}+\frac{300}{28}$. 38. $7-\frac{3}{8}+8-\frac{4}{7}$.
39. $1\frac{3}{14}-7\frac{1}{2}+9-2\frac{1}{7}$. 40. $3\frac{1}{3}+4\frac{1}{4}-5\frac{1}{6}-2\frac{1}{20}$.
41. Subtract R2. 13*a*. $4\frac{3}{7}$*p*. from R13. 9*a*. 6*p*.
42. Subtract R7. 10*a*. $5\frac{6}{7}$*p*. from R10. 7*a*. 3*p*.
43. Subtract R2. 13*a*. $11\frac{3}{8}$*p*. from R7. 2*a*. $3\frac{1}{2}$*p*.
44. Subtract £3. 17*s*. $9\frac{7}{8}$*d*. from £14. 7*s*. $3\frac{5}{16}$*d*.
45. Subtract £4. 7*s*. $3\frac{1}{3}$*d*. from £10. 0*s*. $2\frac{1}{16}$*d*.
46. Subtract 7 yd. 2 ft. $9\frac{3}{4}$ in. from 14 yd. 0 ft. $3\frac{1}{8}$ in.

MULTIPLICATION AND DIVISION OF FRACTIONS.

132. Multiplication by an Integer.—Just as 2 *annas* × 3 = 2 *annas* + 2 *annas* + 2 *annas* = 6 *annas*, so 2 *seventeenths* × 3 = 2 *seventeenths* + 2 *seventeenths* + 2 *seventeenths* = 6 *seventeenths*.

Thus $\frac{2}{17}\times 3=\frac{2}{17}+\frac{2}{17}+\frac{2}{17}=\frac{2+2+2}{17}=\frac{2\times 3}{17}=\frac{6}{17}$.

RULE. *To multiply a fraction by a whole number, multiply the numerator by that number, leaving the denominator unchanged.*

Example 1. $\frac{4}{21}\times 14=\frac{4\times 14}{21}=\frac{4\times 2}{3}=\frac{8}{3}=2\frac{2}{3}$.

Example 2. $23\frac{2}{3}\times 5=23\times 5+\frac{2}{3}\times 5=115+\frac{10}{3}=115+3\frac{1}{3}=118\frac{1}{3}$.

Example 3. Multiply $\frac{99}{100}$ by 57.

Since $\frac{99}{100}=1-\frac{1}{100}$,

$\frac{99}{100}\times 57=57-\frac{57}{100}=56+1-\frac{57}{100}=56+\frac{43}{100}=56\frac{43}{100}$.

Example 4. Multiply $99\frac{99}{100}$ by 7.

Since $99\frac{99}{100}=100-\frac{1}{100}$,

$$99\tfrac{99}{100}\times 7=700-\tfrac{7}{100}=699+1-\tfrac{7}{100}=699+\tfrac{93}{100}=699\tfrac{93}{100}.$$

EXAMPLES. 74.

(*Examples* 1—10 *may be taken orally.*)

Multiply

1. $\frac{2}{3}$ by 7. **2.** $\frac{7}{8}$ by 8. **3.** $\frac{11}{12}$ by 6. **4.** $\frac{3}{7}$ by 9.

5. $\frac{8}{15}$ by 10. **6.** $\frac{13}{15}$ by 15. **7.** $\frac{17}{35}$ by 30. **8.** $\frac{100}{101}$ by 303.

9. $\frac{2}{15}$ by 21. **10.** $\frac{7}{24}$ by 36. **11.** $\frac{15}{34}$ by 51. **12.** $\frac{18}{35}$ by 70.

13. $\frac{70}{111}$ by 110. **14.** $\frac{47}{112}$ by 144. **15.** $\frac{43}{38}$ by 570.

16. $\frac{13}{100}$ by 91. **17.** $3\frac{1}{3}$ by 4. **18.** $6\frac{4}{5}$ by 7.

19. $7\frac{3}{7}$ by 9. **20.** $8\frac{5}{13}$ by 12. **21.** $2\frac{27}{35}$ by 12.

22. $5\frac{7}{50}$ by 12. **23.** $29\frac{7}{8}$ by 11. **24.** $9\frac{22}{45}$ by 21.

25. $3\frac{17}{18}$ by 54. **26.** $4\frac{11}{12}$ by 249. **27.** $3\frac{43}{112}$ by 144.

28. $2\frac{3}{220}$ by 88. **29.** $\frac{99}{100}$ by 29. **30.** $\frac{98}{100}$ by 39.

31. $\frac{999}{1000}$ by 19. **32.** $\frac{789}{790}$ by 45. **33.** $99\frac{99}{100}$ by 9.

34. $9\frac{9}{10}$ by 39. **35.** $999\frac{999}{1000}$ by 23. **36.** $99\frac{7}{10}$ by 32.

37. $9\frac{97}{100}$ by 21. **38.** $319\frac{729}{730}$ by 20.

39. 7*s*. $7\frac{7}{12}$*d*. by 5. **40.** 9*s*. $11\frac{6}{13}$*d*. by 9.

41. R7. 3*a*. $3\frac{3}{4}$*p*. by 7. **42.** R8. 3*a*. $4\frac{7}{8}$*p*. by 6.

43. 4*s*. $0\frac{5}{11}$*d*. by 11. **44.** £3. 0*s*. $7\frac{3}{16}$*d*. by 12.

45. 2 yd. $1\frac{3}{4}$ ft. by 8. **46.** 10 seers $2\frac{7}{18}$ ch. by 32.

47. 1 ton $1\frac{7}{12}$ cwt. by 6.

133. Division by an Integer.

Just as 12 *annas* ÷ 4 = 3 *annas*,

so 12 *seventeenths* ÷ 4 = 3 *seventeenths*, that is,

$$\frac{12}{17}\div 4=\frac{3}{17}\ \text{ or }\ \frac{12\div 4}{17}.$$

Again, suppose $\frac{2}{3}$ is to be divided by 5. Here since the number of thirds cannot be divided exactly by 5, the numerator and denominator of the fraction should each be expressed as a multiple of 5.

It has already been explained that if the numerator and

denominator of a fraction are each multiplied by the same number, the value of the fraction is not altered.

$$\text{Now}\quad \frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15}$$

$$\therefore\quad \frac{2}{3}\div 5=\frac{10}{15}\div 5$$

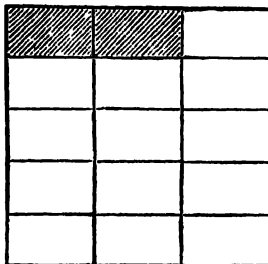
$$=10\ \textit{fifteenths}\div 5$$

$$=2\ \textit{fifteenths}$$

$$=\frac{2}{15}.$$

This operation can be illustrated graphically as follows :

Let the adjoining square represent a certain unit. Divide it into three equal parts by vertical lines. Then two-thirds of the unit is represented by the portion bounded by thick lines. Again divide the square into five equal parts by horizontal lines. The unit square is now divided into 15 small equal rectangles. Then $\frac{2}{3}\div 5$ is clearly represented by the shaded portion of the unit square which contains two out of fifteen small rectangles ; hence $\frac{2}{3}\div 5=\frac{2}{15}$.



Since $\frac{2}{15}$ can also be written as $\frac{2}{3\times 5}$, hence *in order to divide a fraction by a whole number, we multiply the denominator by the whole number, leaving the numerator unchanged.*

Example 1. $7\frac{1}{2}\div 10=\frac{15}{2}\div 10=\frac{15}{2\times 10}=\frac{3}{2\times 2}=\frac{3}{4}$.

Example 2. Divide $3759\frac{2}{3}$ by 5.

Process : $5\,)\,3759\frac{2}{3}$

$751,\ 4\frac{2}{3}$ rem.

Now $4\frac{2}{3}\div 5=\frac{14}{3}\div 5=\frac{14}{15}$; $\therefore\ 3759\frac{2}{3}\div 5=751\frac{14}{15}$.

Note. In the division of integers by integers, the complete quotients can always be obtained by the aid of fractions. Thus for example, $320\div 9=\frac{320}{9}=35\frac{5}{9}$.

EXAMPLES. 75.

Divide

1. $\frac{1}{2}$ by 4. **2.** $\frac{2}{3}$ by 5. **3.** $\frac{3}{4}$ by 7. **4.** $\frac{7}{9}$ by 7.

5. $\frac{9}{10}$ by 12. **6.** $\frac{21}{20}$ by 28. **7.** $\frac{55}{72}$ by 22. **8.** $\frac{99}{710}$ by 11.

9. $\frac{700}{746}$ by 5. **10.** $\frac{70}{145}$ by 42. **11.** $\frac{121}{135}$ by 88. **12.** $\frac{63}{121}$ by 54.
13. $\frac{54}{65}$ by 135. **14.** $\frac{64}{71}$ by 160. **15.** $\frac{128}{125}$ by 95. **16.** $\frac{145}{312}$ by 87.
17. $7\frac{1}{3}$ by 4. **18.** $3\frac{3}{4}$ by 9. **19.** $3\frac{2}{5}$ by 85. **20.** $4\frac{5}{6}$ by 11.
21. $16\frac{2}{3}$ by 15. **22.** $4\frac{2}{3}$ by 57. **23.** $3\frac{7}{8}$ by 21. **24.** $2\frac{2}{7}$ by 40.
25. $213\frac{1}{3}$ by 5. **26.** $73\frac{5}{6}$ by 6. **27.** $713\frac{3}{8}$ by 4. **28.** $100\frac{1}{13}$ by 15.
29. $333\frac{19}{20}$ by 21. **30.** $356\frac{1}{32}$ by 33. **31.** $999\frac{7}{18}$ by 16.
32. $729\frac{13}{14}$ by 19. **33.** $324\frac{2}{7}$ by 15. **34.** $39\frac{1}{3}$ by 24.
35. R10. 12*a*. $2\frac{1}{2}$*p*. by 8. **36.** R22. 13*a*. $3\frac{3}{4}$*p*. by 9.
37. £20. 7*s*. $6\frac{3}{8}$*d*. by 11. **38.** £99. 19*s*. $11\frac{11}{12}$*d*. by 13.

Obtain the complete quotient in the division of

39. 720 by 9. **40.** 1345 by 7. **41.** 1000 by 23. **42.** 1234 by 11.
43. R20. 8*a*. 3*p*. by 8. **44.** R13. 12*a*. 6*p*. by 11.
45. R420. 7*a*. 9*p*. by 13. **46.** R100. 3*a*. 11*p*. by 16.
47. £17. 17*s*. 7*d*. by 5. **48.** £59. 19*s*. 11*d*. by 15.

134. Multiplication by a Fraction.—The definition of multiplication which we have given in Art. 34 implies that the multiplier is a whole number, and it is not applicable when the multiplier is a fraction, for as we have seen 4 multiplied by 3 means 4 repeated 3 times *i.e.*, 4+4+4. According to this definition it is absurd to say that multiplication by a fraction, say $\frac{1}{2}$, means that the number is to be repeated half a-time. We, therefore, give below the general definition of multiplication.

Def. To multiply one given number by another is to perform upon the number multiplied that operation which is performed upon unity to obtain the multiplier.

Since 1 is repeated five times to obtain the number 5, so to multiply a number by 5 is to repeat that number five times.

Again, since 1 is divided into 5 equal parts and three of these parts are taken to obtain the number $\frac{3}{5}$, to multiply a number by $\frac{3}{5}$ is to divide that number into 5 equal parts and take three of these parts ; that is, to multiply a number by $\frac{3}{5}$ we have to divide the number by 5 and multiply the result by 3.

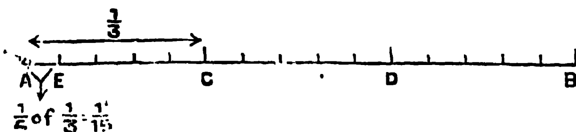
Similarly, when we want to find the value of $\frac{3}{5}$ of $\frac{2}{7}$ we divide $\frac{2}{7}$ by 5, and multiply the result by 3.

Now $\frac{2}{7}\div 5=\frac{2}{7\times 5}$ and multiplying this by 3, we get $\frac{2\times 3}{7\times 5}$. Hence $\frac{3}{5}$ of $\frac{2}{7}=\frac{2\times 3}{7\times 5}$. We thus see that multiplying any quantity by $\frac{3}{5}$ is the same as taking $\frac{3}{5}$ of that quantity.

Example 1. Simplify $\frac{1}{5}$ of $\frac{1}{3}$.

$$\text{Since } \tfrac{1}{3}\div 5=\tfrac{1}{3\times 5},$$
$$\therefore\ \tfrac{1}{5}\text{ of }\tfrac{1}{3}=\tfrac{1}{3\times 5}=\tfrac{1\times 1}{5\times 3}=\tfrac{1}{15}$$

This operation can be illustrated graphically as follows :



Let the straight line AB, regarded as unit, be divided into 3 equal parts at the points C and D.

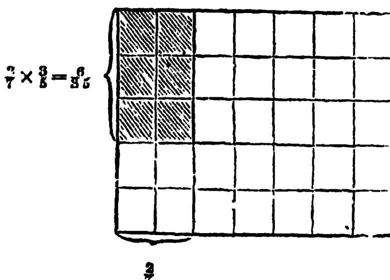
Then AC, CD and DB each represent $\frac{1}{3}$ of the unit. Let each of these parts be again subdivided into 5 equal parts. Then the unit line is divided into 15 equal parts. Now $\frac{1}{5}$ of $\frac{1}{3}$ means $\frac{1}{5}$ of AC, *i.e.*, AE. Since AE represents *one-fifteenth* of AB, (*i.e.*, the unit),

we have $\frac{1}{5}$ of $\frac{1}{3}=\frac{1}{15}$.

Example 2. Multiply $\frac{2}{7}$ by $\frac{3}{5}$.

$$\text{Since } \tfrac{2}{7}\div 5=\tfrac{2}{7\times 5}\ ;\ \text{and } \tfrac{2}{7\times 5}\times 3=\tfrac{2\times 3}{7\times 5}\ ;$$
$$\therefore\ \tfrac{2}{7}\times\tfrac{3}{5}=\tfrac{2\times 3}{7\times 5}=\tfrac{6}{35}.\quad \textit{Ans.}$$

The diagram given below illustrates graphically the above solution.



Hence the rule : *To multiply one fraction by another, multiply the numerators for the numerator of the product, and multiply the denominators for its denominator.*

[*N. B.* This rule holds good for the continued product of three or more fractions.]

Note 1. From the above diagram it is clear that $\frac{2}{7}\times\frac{3}{5}=\frac{3}{5}\times\frac{2}{7}$.

Thus, as in the case of integers, the multiplier and multiplicand are interchangeable.

Note 2. In the case of multiplication of an integer by another integer the product can never be less than either of the factors. But the student should here observe that the product of the two fractions $\frac{2}{7} \times \frac{3}{5}$ is *smaller* than either of them. The diagram given above will enable him to understand this clearly.

135. A **compound** fraction is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{4}{5}$.

The compound fraction, $\frac{2}{3}$ of $\frac{4}{5}$, means that we are to divide $\frac{4}{5}$ (regarded as a whole) into 3 equal parts and take 2 of these parts. Hence $\frac{2}{3}$ of $\frac{4}{5}$ is equivalent to $\frac{4}{5} \times \frac{2}{3}$, *i.e.*, to $\frac{2}{3} \times \frac{4}{5}$.

Example. Simplify $3\frac{7}{9}$ of $9\frac{3}{8}$.

$3\frac{7}{9}$ of $9\frac{3}{8} = 3\frac{7}{9} \times 9\frac{3}{8} = \frac{34}{9} \times \frac{75}{8} = \frac{34 \times 75}{9 \times 8} = \frac{17 \times 25}{3 \times 4} = \frac{425}{12} = 35\frac{5}{12}$. *Ans.*

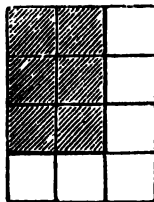
N. B. Before effecting the multiplication, common factors should be removed from the numerator and denominator.

EXAMPLES. 76.

Multiply

1. $\frac{2}{3}$ by $\frac{5}{6}$.	2. $\frac{7}{8}$ by $\frac{3}{4}$.	3. $\frac{8}{9}$ by $\frac{3}{4}$.
4. $\frac{12}{25}$ by $\frac{10}{9}$.	5. $\frac{39}{44}$ by $\frac{66}{13}$.	6. $\frac{18}{65}$ by $\frac{39}{80}$.
7. $\frac{27}{51}$ by $\frac{34}{45}$.	8. $\frac{36}{85}$ by $\frac{34}{81}$.	9. $5\frac{3}{4}$ by $\frac{3}{46}$.
10. $3\frac{3}{4}$ by $\frac{8}{9}$.	11. $\frac{25}{39}$ by $2\frac{3}{5}$.	12. $\frac{80}{49}$ by $3\frac{1}{2}$.
13. $4\frac{3}{11}$ by $7\frac{1}{2}$.	14. $7\frac{3}{4}$ by $3\frac{1}{3}$.	15. $2\frac{3}{8}$ by $1\frac{2}{13}$.
16. $4\frac{4}{9}$ by $3\frac{3}{10}$.	17. $2\frac{4}{5}$ by $3\frac{4}{7}$.	18. $3\frac{7}{11}$ by $2\frac{1}{3}$
19. $5\frac{5}{12}$ by $5\frac{1}{4}$.	20. $3\frac{3}{8}$ by $4\frac{5}{6}$.	21. $2\frac{1}{9}$ by $4\frac{1}{2}$.

22. Use the adjoining diagram to prove that $\frac{2}{3} \times \frac{3}{4} = \frac{3}{4} \times \frac{2}{3}$.



Simplify

23. $3\frac{1}{2}$ of $2\frac{2}{7}$. 24. $\frac{1}{4}$ of $4\frac{1}{2}$ of $3\frac{1}{3}$. 25. $2\frac{1}{2}$ of $3\frac{1}{3}$ of $4\frac{1}{5}$

26. $\frac{4}{7}$ of $1\frac{2}{3}\times 7\frac{1}{5}$. 27. $4\frac{1}{3}\times\frac{27}{32}$ of $4\frac{7}{25}$. 28. $1\frac{3}{4}\times 2\frac{1}{7}\times 3\frac{2}{5}$.

29. $\frac{1}{2}$ of $2\frac{1}{2}\times 3\frac{3}{5}$ of 9. 30. $3\frac{2}{3}$ of $2\frac{3}{4}\times 4\times 7\frac{1}{9}$.

31. $\frac{2}{3}$ of $\frac{6}{13}$ of $\frac{39}{10}$. 32. $3\frac{4}{5}\times 5\frac{3}{4}$ of $\frac{2}{19}\times 1\frac{1}{45}$.

33. $4\frac{1}{2}\times 2\frac{1}{3}\times 1\frac{1}{5}$ of $2\frac{1}{2}$. 34. $\frac{1}{3}$ of $\frac{3}{4}$ of $2\frac{1}{7}$.

35. $\frac{3}{4}\times\frac{7}{8}\times\frac{10}{3}\times\frac{4}{21}\times 1\frac{1}{2}$. 36. $\frac{1}{2}$ of $\frac{4}{3}$ of $\frac{1}{4}$ of $\frac{3}{5}$ of $\frac{5}{6}$.

37. $2\frac{1}{2}$ of $3\frac{1}{4}\times 1\frac{1}{5}$ of $2\frac{2}{13}\times 1\frac{1}{3}$. 38. $\frac{4}{5}$ of $9\times 7\frac{1}{2}\times 4\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{4}{3}$.

136. *Example.* Reduce 29 poles to inches.

Process :

```
  29   po.
   5½
 145   = 29 × 5.
  14½ = 29 ÷ 2, i.e., 29 × ½.
 159½ yd.
    3
 478½ ft.
   12
 5742 in.
```

EXAMPLES. 77.

Reduce to inches :

1. 7 po. 2. 13 po. 3. 29 po. 4. 39 po. 5. 49 po.
6. 4 fur. 39 po. 5 yd. 7. 10 mi. 5 fur. 0 po. 3 yd.

Reduce to sq. inches :

8. 7 sq. po. 9. 13 sq. po. 10. 29 sq. po. 11. 39 sq. po.
12. 49 sq. po. 13. 9 ac. 2 ro. 7 po. 14. 1 sq. mi. 3 ac. 10 po

137. Division by a fraction is the *inverse* of multiplication.

To divide $\frac{5}{8}$ by $\frac{3}{4}$ is to find that number which being multiplied by $\frac{3}{4}$ gives $\frac{5}{8}$ as the product. But $\frac{5}{8}\times\frac{4}{3}$ being multiplied by $\frac{3}{4}$ gives $\frac{5}{8}$ as the product ($\because$ $\frac{4}{3}\times\frac{3}{4}=1$) ; therefore $\frac{5}{8}\div\frac{3}{4}=\frac{5}{8}\times\frac{4}{3}$; and hence we have the rule : Multiply the dividend by the reciprocal of the divisor.

Example 1. $8\frac{1}{4}\div 3\frac{2}{3}=\frac{33}{4}\div\frac{11}{3}=\frac{33}{4}\times\frac{3}{11}=\frac{3\times 3}{4}=\frac{9}{4}=2\frac{1}{4}$. *Ans.*

Example 2. If $\frac{5}{8}$ of a number is 4, what is the number ?
Here the product of the number (required) by $\frac{5}{8}$ is 4 ;
$\therefore$ the number required $=4\div\frac{5}{8}=\frac{4}{1}\times\frac{8}{5}=\frac{32}{5}=6\frac{2}{5}$.

EXAMPLES. 78.

Divide

1. $\frac{3}{4}$ by $\frac{2}{5}$. 2. $\frac{3}{7}$ by $\frac{8}{9}$. 3. $\frac{7}{12}$ by $\frac{21}{36}$. 4. $\frac{56}{99}$ by $\frac{42}{77}$.
5. $3\frac{1}{2}$ by $2\frac{1}{3}$. 6. $7\frac{7}{8}$ by $\frac{9}{10}$. 7. $\frac{22}{7}$ by $1\frac{1}{21}$. 8. $\frac{75}{18}$ by $\frac{50}{33}$.
9. $11\frac{4}{25}$ by $\frac{3}{5}$. 10. $16\frac{2}{3}$ by $12\frac{1}{2}$. 11. $\frac{32}{75}$ by $\frac{8}{15}$. 12. $11\frac{1}{11}$ by $12\frac{1}{3}$.
13. $12\frac{2}{3}$ by $1\frac{1}{9}$. 14. $13\frac{1}{5}$ by $2\frac{1}{10}$. 15. $10\frac{1}{5}$ by $\frac{10}{17}$. 16. 9 by $\frac{24}{23}$.
17. $14\frac{3}{7}$ by $5\frac{3}{5}$. 18. $11\frac{1}{4}$ by $7\frac{1}{2}$. 19. 10 by $7\frac{1}{2}$. 20. 76 by $28\frac{1}{2}$.
21. $\frac{3}{4}$ of $4\frac{1}{2}$ by $7\frac{1}{2}$ of $3\frac{3}{4}$. 22. $3\frac{1}{2}\times 6\frac{3}{4}$ by $1\frac{1}{5}\times 14$.
23. $4\frac{1}{5}+7\frac{1}{10}$ by $4\frac{3}{4}-2\frac{7}{8}$. 24. $3\frac{3}{4}$ of $3\frac{1}{3}$ by $7-3\frac{1}{3}$.
25. $\frac{7}{8}$ of a number is 14 ; what is the number ?
26. $3\frac{3}{7}$ of a number is $2\frac{1}{2}$; what is the number ?
27. Find the number, $\frac{3}{8}$ of which is $\frac{4}{5}$ of $\frac{7}{9}$.
28. $3\frac{1}{2}$ of $4\frac{1}{3}$ of a number is 7 ; find the number.
29. $\frac{1}{2}$ of $\frac{3}{4}$ of a number is $3\frac{1}{2}$ of 10 ; what is the number ?
30. Which is greater, the quotient of $3\frac{1}{2}$ by $6\frac{1}{4}$ or the continued product of $\frac{3}{5}$, $\frac{5}{6}$ and $\frac{6}{7}$?

H. C. F. AND L. C. M. OF FRACTIONS.

138. The definitions which we have given of the H. C. F. and L. C. M. of two or more whole numbers will also be applicable when the given numbers are fractions, provided that we understand by *exact division*, that the complete quotients must be *integers*.

RULE. To find the H. C. F. or the L. C. M. of fractions, reduce them to their least common denominator ; then find the H. C. F. or the L. C. M. of the new numerators, and write it over the common denominator.

Example 1. Find the H. C. F. and L. C. M. of $\frac{3}{4}$, $2\frac{1}{2}$ and $\frac{15}{16}$

The given fractions are equivalent to $\frac{12}{16}$, $\frac{40}{16}$, $\frac{15}{16}$;

the H. C. F. of 12, 40 and $15=1$, and their L. C. M. $=120$;

$\therefore$ the H. C. F. required $=\frac{1}{16}$;

and the L. C. M. required $=\frac{120}{16}=\frac{15}{2}=7\frac{1}{2}$.

The following rules will be found practically more convenient.

(i) The H. C. F. of two or more fractions *in their lowest terms* is a fraction whose numerator is the H. C. F. of their numerators, and whose denominator is the L. C. M. of their denominators.

(ii) The L. C. M. of two or more fractions *in their lowest terms* is a fraction whose numerator is the L. C. M. of their numerators, and whose denominator is the H. C. F. of their denominators.

Example 2. Find the H. C. F. and L. C. M. of $\frac{3}{12}$, $2\frac{2}{3}$ and $\frac{4}{5}$.

The given fractions when reduced to their lowest terms are equal to $\frac{1}{4}$, $\frac{8}{3}$ and $\frac{4}{5}$.

(i) H.C.F. of numerators $= 1$, and L.C.M. of denominators $= 35$;

$\therefore$ the H. C. F. required $= \frac{1}{35}$.

(ii) L.C.M. of numerators $= 8$, and H.C.F. of denominators $= 1$;

$\therefore$ the L. C. M. required $= \frac{8}{1} = 8$.

EXAMPLES. 79.

Find the H. C. F. and L. C. M of

1. $\frac{1}{3}$ and $\frac{1}{4}$.
2. $\frac{9}{10}$ and $\frac{3}{4}$.
3. $\frac{8}{21}$ and $\frac{16}{49}$.
4. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{8}{9}$.
5. $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{15}$.
6. $3\frac{1}{2}$, $5\frac{1}{4}$, $\frac{1}{12}$.
7. $\frac{39}{52}$, $\frac{9}{8}$, $3\frac{1}{2}$.
8. $\frac{7}{8}$, $8\frac{2}{3}$, $\frac{77}{55}$.
9. $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$.

0. 3, $\frac{9}{2}$, $10\frac{1}{2}$.

11. $\frac{100}{25}$, $\frac{88}{33}$, 4.

12. $1\frac{37}{78}$, $2\frac{54}{65}$, $5\frac{35}{39}$

13. What is the greatest length which is contained a whole number of times exactly in both $7\frac{1}{2}$ feet and $4\frac{1}{4}$ feet ?

14. Find the least number which, when divided by each of the fractions $\frac{4}{5}$, $\frac{3}{10}$ and $\frac{12}{35}$, gives a whole number as quotient in each case.

15. Four bells commence tolling together ; they toll at intervals of 1, $1\frac{1}{4}$, $1\frac{1}{2}$ and $1\frac{3}{4}$ seconds respectively ; after what interval will they toll together again ?

MISCELLANEOUS EXAMPLES. 80.

1. What number must be added to $3\frac{1}{2}$ of $\frac{4}{5}$ that the sum may be 9 ?
2. What must we take from $3\frac{1}{2}$ to leave $2\frac{1}{3}$?
3. From what must $4\frac{1}{2}$ be taken to leave $\frac{8}{9}$ of $\frac{3}{4}$?
4. What number multiplied by $\frac{3}{4}+\frac{5}{8}$ gives the product $\frac{3}{4}-\frac{5}{8}$?
5. By what do we divide $\frac{4}{5}$, if the quotient is 8 ?
6. How many times does $\frac{1}{2}+\frac{1}{3}$ contain $\frac{1}{2}-\frac{1}{3}$?
7. What number do we divide by $7\frac{1}{2}$, if the quotient is $2\frac{1}{3}$?
8. If the divisor be $\frac{4}{5}$, and quotient $\frac{5}{8}$ of the divisor, what is the dividend ?

9. Find the price of 217 lb. at $5\frac{1}{2}d.$ per lb.

10. Find the cost of 325 maunds at R2. 9*a.* $4\frac{1}{2}p.$ per maund.

11. Find the weight of 125 boxes, each $7\frac{3}{8}$ lb.

12. R720 is $\frac{9}{10}$ of what amount?

13. Find the sum of money, $\frac{3}{8}$ of which is £30.

14. Which is the greatest, $4\frac{1}{2} \div 3\frac{1}{3}$, $4\frac{1}{2} \times 3\frac{1}{3}$, $4\frac{1}{2} - 3\frac{1}{3}$ or $4\frac{1}{2} + 3\frac{1}{3}$?

15. What number is that from which if we subtract $\frac{1}{2} - \frac{1}{3}$, and to the remainder add $\frac{1}{2}$ of $\frac{1}{3}$, the sum will be $\frac{1}{2} + \frac{1}{3}$?

16. Find the least fraction which being added to $\frac{4}{5}$ shall make the result an integer.

17. *A* gives *B* $\frac{1}{3}$ of his money; *B* gives *C* $\frac{1}{4}$ of what he receives; and *C* gives *D* $\frac{1}{5}$ of what he receives; what fraction of *A*'s money does *D* receive?

18. If I lose $\frac{3}{5}$ of my money, what fraction of it have I left?

[The fraction $= 1 - \frac{3}{5} = \frac{2}{5}$.]

18a. $\frac{2}{5}$ of a post are imbedded in mud, $\frac{3}{10}$ are in the water, and 6 ft. are above the surface; what is the length of the post?

[$\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$; $1 - \frac{7}{10} = \frac{3}{10}$. $\therefore$ $\frac{3}{10}$ of the post $= 6$ ft.;
and $\therefore$ the length of the post $= 6 \times \frac{10}{3}$ ft. $= 20$ ft.]

19. A book contains 25 pages, and a boy has read 15 of them; what fraction of the whole has he yet to read?

20. A sum of money is divided among three persons, *A*, *B* and *C*. *A* receives $\frac{3}{8}$ of it, and *B* receives $\frac{2}{5}$. How much does *C* get?

21. A man owns $\frac{5}{16}$ of an estate, and sells $\frac{1}{4}$ of his share; what fraction of the estate does he still own?

22. A merchant owned $\frac{12}{25}$ of a ship, and sold $\frac{5}{16}$ of his share: what part of the whole ship had he left?

23. If I give away $\frac{3}{10}$ of my money, and then $\frac{2}{5}$ of what remains, how much of the whole is left?

24. One-fifth of an estate is left to the eldest son, $\frac{1}{4}$ to the second, and $\frac{5}{6}$ of the remainder to the third; how much was over?

25. At his first game a person loses $\frac{1}{4}$ of his money, at the second $\frac{1}{3}$ of the remainder, at the third $\frac{4}{5}$ of the rest; what fraction of his original money has he left?

26. When $1\frac{1}{2}$ of $\frac{2}{3}$ of a loaf of bread has been eaten, how much of the loaf will be left?

27. After payiny $\frac{3}{8}$ of a bill, R24 is still due ; what was the amount of the bill ?

28. A person expends $\frac{1}{3}$ of his income in board and lodging, $\frac{1}{8}$ in clothing and $\frac{1}{10}$ in charity, and saves £318. What is his income ?

29. A boy after giving away $\frac{1}{3}$ of his pocket-money to one companion, and $\frac{4}{5}$ of the remainder to another, has 2*s*. left. How much had he at first ?

30. A man travelled $\frac{3}{11}$ of his journey by coach, $\frac{7}{22}$ by rail, and walked the remaining 9 miles ; how far did he go ?

31. One-tenth of a rod is coloured red, one-twentieth orange, one-thirtieth yellow, one-fortieth green, one-fiftieth blue, one-sixtieth indigo, and the remainder which is 302 inches long, violet. Find the length of the rod.

32. Of a certain dynasty $\frac{1}{3}$ of the kings were of the same name, $\frac{1}{4}$ of another, $\frac{1}{8}$ of another, $\frac{1}{12}$ of a fourth, and there were 5 besides. How many kings were there of each name ?

33. How many whole cakes would be wanted for 100 children if each has a third of a cake ?

34. By what number should $\frac{3}{5}\frac{1}{8}\frac{1}{12}$ be multiplied so as to produce the least possible integer ?

35. Simplify $\dfrac{£7. \quad 5s.}{£14. \quad 5s.} - \dfrac{1 \text{ ton} \quad 5 \text{ cwt.}}{4 \text{ tons} \quad 15 \text{ cwt.}}$.

36. How often may $\frac{7}{9}$ be subtracted from 7, so as to leave a remainder not less than 3 ?

37. From a rope 20 ft. long, as many pieces as possible are cut off, each $2\frac{3}{5}$ ft. long ; what fraction of the latter length will be left ?

38. A cistern has two pipes attached to it, one to supply and one to draw off. The first can supply $\frac{3}{4}$ of a gallon, and the second can draw off $\frac{4}{5}$ of a gallon, per minute. If both the pipes are opened when the cistern contains 81 gallons, how soon will the cistern be empty ?

39. The double and fourth part of a number, added together, give $7\frac{1}{2}$ as the result ; what is the number ?

40. Find the number, of which the eighth part exceeds the tenth part by $7\frac{3}{4}$.

41. What are the nearest integers to $12\frac{5}{9}$ and $17\frac{3}{7}$? Give reasons for your answer.

42. A number of mangoes is to be divided amongst 3 persons so that one may get $\frac{5}{12}$ of it, another $\frac{7}{18}$, and the third the remainder ; what must the number *at least* be that this may be done without cutting any of the mangoes ?

XXIII. COMPLEX FRACTIONS.

139. A **simple** fraction is one, in which the numerator and denominator are both whole numbers ; as $\frac{3}{7}$, $\frac{9}{8}$.

A **complex** fraction is one, in which the numerator or denominator or both are not whole numbers ; as

$$\frac{\frac{2}{3}}{5},\quad \frac{7}{2\frac{1}{2}},\quad \frac{3\frac{2}{5}}{4\frac{2}{3}},\quad \frac{\frac{2}{3}+1\frac{1}{2}}{3\frac{1}{4}\text{ of }2\frac{2}{3}}.$$

Note. $\frac{3\frac{2}{5}}{4\frac{2}{3}}$ is read '$3\frac{2}{5}$ divided by $4\frac{2}{3}$', or '$3\frac{2}{5}$ by $4\frac{2}{3}$'.

140. Complex fractions can always be *simplified* as in the following examples :

Example 1. $\frac{\frac{2}{3}}{5}\left[=\frac{2}{3}\div 5=\frac{2}{3}\div\frac{5}{1}\right]=\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}.$

Example 2. $\frac{7}{2\frac{1}{2}}\left[=7\div 2\frac{1}{2}=\frac{7}{1}\div\frac{5}{2}\right]=\frac{7}{1}\times\frac{2}{5}=\frac{14}{5}=2\frac{4}{5}.$

Example 3. $\frac{3\frac{2}{5}}{4\frac{2}{3}}\left[=3\frac{2}{5}\div 4\frac{2}{3}=\frac{17}{5}\div\frac{14}{3}\right]=\frac{17}{5}\times\frac{3}{14}=\frac{51}{70}.$

Example 4. $\frac{\frac{2}{3}+1\frac{1}{2}}{3\frac{1}{4}\text{ of }2\frac{2}{3}}=\frac{\frac{2}{3}+\frac{3}{2}}{\frac{13}{4}\times\frac{8}{3}}=\frac{\frac{13}{6}}{\frac{26}{3}}=\frac{13}{6}\times\frac{3}{26}=\frac{1}{4}.$

N. B. The work within the brackets may be omitted in practice.

Note. There is another method of simplifying complex fractions, which is explained by the following example.

Example 5. Simplify $\frac{4\frac{1}{2}-3\frac{1}{3}}{\frac{3}{4}+\frac{5}{6}}.$

We multiply the terms of the fraction by 12, the L. C. M. of the denominators 2, 3, 4 and 6.

Thus the fraction $=\frac{\frac{9}{2}-\frac{10}{3}}{\frac{3}{4}+\frac{5}{6}}=\frac{54-40}{9+10}=\frac{14}{19}.$

EXAMPLES. 81.

Simplify

1. $\frac{3\frac{1}{3}}{5}.$ 2. $\frac{13}{8\frac{1}{2}}.$ 3. $\frac{\frac{21}{8}}{\frac{35}{48}}.$ 4. $\frac{7\frac{1}{2}}{\frac{5}{8}}.$ 5. $\frac{\frac{3}{10}}{4\frac{1}{2}}.$

6. $\frac{10\frac{2}{3}}{5\frac{1}{3}}.$ 7. $\frac{9\frac{5}{14}}{13\frac{4}{21}}.$ 8. $\frac{99\frac{1}{10}}{24\frac{7}{11}}.$ 9. $\frac{\frac{1}{2}+\frac{1}{3}}{2\frac{1}{7}}.$ 10. $\frac{3-\frac{3}{4}}{3+\frac{3}{4}}.$

11. $\frac{\frac{3}{4}}{\frac{5}{6}\times\frac{3}{10}}.$ 12. $\frac{3\frac{3}{7}+1\frac{1}{14}}{4\frac{1}{2}-3\frac{1}{3}}.$ 13. $\frac{7\frac{1}{2}-3\frac{1}{4}}{\frac{2}{3}\div\frac{4}{5}}.$ 14. $\frac{10\frac{1}{10}\text{ of }7\frac{1}{7}}{\frac{1}{3}+\frac{1}{4}}.$

15. $\dfrac{1\frac{1}{2}+2\frac{1}{3}+3\frac{1}{4}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}$. 16. $\dfrac{5\frac{1}{6} \text{ of } 2\times\frac{2}{7}}{7\frac{1}{7}-3\frac{1}{4}}$. 17. $\dfrac{7}{4\frac{2}{3}}\div\dfrac{2\frac{1}{3}}{7}$. 18. $\dfrac{3\frac{1}{2}}{1\frac{2}{5}}-1\frac{3}{4}$.

19. $\dfrac{5\frac{1}{2}}{1\frac{1}{2}}+\dfrac{2\frac{1}{3}}{\frac{1}{3}}-\dfrac{4\frac{1}{2}}{7}$. 20. $\dfrac{3}{14}$ of $\dfrac{4\frac{5}{9}}{6\frac{1}{6}}$ of $\dfrac{6\frac{8}{11}}{11\frac{5}{7}}$. 21. $\dfrac{7}{\frac{14}{15} \text{ of } \frac{1}{4}}\times\dfrac{2\frac{1}{10}}{3\frac{1}{2}}$.

22. $\dfrac{2\frac{1}{4}}{2\frac{2}{3}}+\dfrac{5\frac{1}{6}-2\frac{1}{2}}{3\frac{1}{3}\div 9\frac{1}{2}}$. 23. $\dfrac{\frac{2\frac{1}{2}}{54}+\frac{1}{8}-\frac{1}{216}+\frac{1}{12}}{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}$. 24. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$.

141. Fractions of the form $\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{4+\frac{1}{5}}}}$

are called **Continued Fractions.**

A continued fraction is also written as $\dfrac{1}{2+}\;\dfrac{1}{3+}\;\dfrac{1}{4+}\;\dfrac{1}{5}$.

It is necessary that the sign '+' should be written in the denominator.

Example. Simplify $3+\dfrac{1}{7-\dfrac{2}{4+\dfrac{5}{6+\frac{1}{2}}}}$.

Process : $3+\dfrac{1}{7-\dfrac{2}{4+\dfrac{5}{6+\frac{1}{2}}}}=3+\dfrac{1}{7-\dfrac{2}{4+\frac{10}{13}}}=3+\dfrac{1}{7-\frac{13}{31}}$

$=3+\dfrac{31}{204}=3\dfrac{31}{204}$. *Ans.*

Begin at the bottom. First take up the lowest complex fraction, namely, $\dfrac{5}{6+\frac{1}{2}}$. Multiply the numerator and denominator by 2, and we get $\frac{10}{13}$. Next multiply the numerator and denominator of the fraction $\dfrac{2}{4+\frac{10}{13}}$ by 13, and we get $\frac{13}{31}$. Then multiply the numerator and denominator of $\dfrac{1}{7-\frac{13}{31}}$ by 31, and we get $\frac{31}{204}$. Hence the fraction is reduced to $3+\frac{31}{204}$ or $3\frac{31}{204}$.

Note. In simplifying the several complex fractions we have here adopted the method of *Example* 5 of the preceding Article. The method explained before in the same Article may also be used.

EXAMPLES. 82.

Simplify

1. $\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}$.

2. $\dfrac{2}{2-\dfrac{2}{2+\frac{1}{2}}}$.

3. $\dfrac{3}{3+\dfrac{3}{3-\frac{1}{3}}}$.

4. $2+\dfrac{3}{4+\dfrac{5}{7+\frac{5}{6}}}$.

5. $3\div\dfrac{1}{4+\dfrac{7}{2-\frac{2}{5}}}$.

6. $7+\dfrac{8}{3-\dfrac{9}{4+\frac{1}{3}}}$.

7. $1+\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{4+\frac{1}{5}}}}$.

8. $6+\dfrac{1}{6-\dfrac{1}{6+\dfrac{1}{6-\frac{1}{6}}}}$.

9. $2\frac{1}{2}+\dfrac{5}{2+\dfrac{5}{2+\dfrac{5}{2+\frac{2}{5}}}}$.

10. $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\dfrac{1}{3+\frac{1}{3}}}}}$.

11. $\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\frac{1}{2}}}}}$.

12. $\dfrac{1}{2-\dfrac{2}{2+\dfrac{4}{2-\dfrac{5}{2+\frac{6}{7}}}}}$.

13. $4+\dfrac{1}{4+}\;\dfrac{1}{1+}\;\dfrac{1}{2}$.

142. The following examples of simplification are important.

Example 1. $\frac{1}{2}\div\frac{3}{4}\div\frac{2}{7}=\frac{1}{2}\times\frac{4}{3}\times\frac{7}{2}=\frac{7}{3}=2\frac{1}{3}$.

Example 2. $\frac{4}{5}\div\frac{2}{3}\times\frac{3}{4}=\frac{4}{5}\times\frac{3}{2}\times\frac{3}{4}=\frac{9}{10}$.

Example 3. $\frac{3}{5}\times\frac{1}{2}\div\frac{3}{8}=\frac{3}{5}\times\frac{1}{2}\times\frac{8}{3}=\frac{4}{5}$.

Example 4. $2\times\frac{1}{3}\div\frac{1}{4}\times\frac{3}{5}\div\frac{8}{5}\div\frac{5}{6}=\frac{2}{1}\times\frac{1}{3}\times\frac{4}{1}\times\frac{3}{5}\times\frac{5}{8}\times\frac{6}{5}=$ etc.

In the above examples the operations of division are converted into those of multiplication by inverting the fractions which are preceded by the sign of division ; since division by a fraction is equivalent to multiplication by its reciprocal.

Note. In simplifying an expression, a *compound fraction* must be treated as a **single number**. The difference in meaning between $\frac{1}{2}\div\frac{1}{3}$ of $\frac{1}{4}$ and $\frac{1}{2}\div\frac{1}{3}\times\frac{1}{4}$ should be noticed.

$$\tfrac{1}{2}\div\tfrac{1}{3}\text{ of }\tfrac{1}{4}=\tfrac{1}{2}\times\tfrac{3}{1}\times\tfrac{4}{1}=6\ ;$$

$$\text{but}\quad\tfrac{1}{2}\div\tfrac{1}{3}\times\tfrac{1}{4}=\tfrac{1}{2}\times\tfrac{3}{1}\times\tfrac{1}{4}=\tfrac{3}{8}.$$

EXAMPLES. 83.

Simplify

1. $\frac{7}{8}\div5\frac{1}{3}\div2\frac{2}{5}$.
2. $1\frac{1}{3}\div1\frac{1}{2}\div1\frac{1}{9}$.
3. $\frac{3}{5}\div2\frac{1}{2}\times2\frac{7}{10}$.
4. $2\frac{3}{4}\div\frac{1}{2}\times1\frac{1}{11}$.
5. $2\frac{1}{3}\times\frac{3}{4}\div1\frac{3}{4}$.
6. $2\frac{1}{4}\times1\frac{1}{2}\div2\frac{2}{5}$.
7. $1\frac{1}{2}\div1\frac{1}{8}\times2\frac{1}{4}\div2\frac{2}{5}$.
8. $\frac{1}{9}\times\frac{7}{8}\div\frac{1}{3}\times\frac{4}{7}\div\frac{2}{3}\div\frac{1}{2}$.
9. $3\frac{1}{2}\div1\frac{3}{4}\div\frac{4}{5}\times\frac{1}{2}$.
10. $\frac{1}{9}\div\frac{7}{8}\times\frac{1}{3}\div\frac{4}{9}\div\frac{2}{3}\times\frac{1}{2}$.
11. $3\frac{1}{4}\div2\frac{1}{2}$ of $6\frac{1}{2}$.
12. $2\frac{1}{2}\div3\frac{1}{3}$ of $4\frac{1}{4}$.
13. $2\frac{1}{2}\div3\frac{1}{3}\times4\frac{1}{4}$.
14. $2\frac{1}{2}\times\frac{3}{4}\div3\frac{1}{3}$ of $1\frac{1}{3}$.
15. $4\frac{2}{3}\times2\frac{1}{2}\div1\frac{3}{4}$ of $3\frac{1}{3}$.
16. $2\frac{1}{2}$ of $\frac{3}{4}\div3\frac{1}{3}\times1\frac{1}{8}$.
17. $4\frac{2}{3}$ of $2\frac{1}{2}\div1\frac{3}{4}\times3\frac{1}{3}$.
18. $2\frac{1}{2}$ of $\frac{3}{4}\div3\frac{1}{3}$ of $1\frac{1}{8}$.
19. $4\frac{2}{3}$ of $2\frac{1}{2}\div1\frac{3}{4}$ of $3\frac{1}{3}$.
20. $2\frac{1}{2}\times\frac{3}{4}\div3\frac{1}{3}\times1\frac{1}{8}$.
21. $4\frac{2}{3}\times2\frac{1}{2}\div1\frac{3}{4}\times3\frac{1}{3}$.
22. $1\frac{1}{2}\div2\frac{1}{3}$ of $3\frac{1}{4}\times1\frac{1}{3}$.
23. $1\frac{1}{2}\div2\frac{1}{3}\times3\frac{1}{4}$ of $1\frac{1}{3}$.
24. $1\frac{1}{2}\times2\frac{1}{3}\times3\frac{1}{4}\div1\frac{1}{3}$ of $2\frac{1}{4}$ of $3\frac{1}{3}\times\frac{9}{13}$.

143. Convention of Signs :—When an expression contains all (or some of) the signs $+$, $-$, $\times$ and $\div$, *the multiplication and division are to be worked before the addition and subtraction* in order from left to right. Quantities connected by the word '*of*' should be considered as composing *one single quantity* and should be treated first before performing the other operations. (See Art. 142).

Example 1. $\frac{21}{28}\div\frac{2}{7}$ of $\frac{3}{4}=\frac{21}{28}\div\frac{2\times3}{7\times4}=\frac{21}{28}\times\frac{7\times4}{2\times3}=\frac{7}{2}=3\frac{1}{2}$.

Example 2. $\frac{5}{2}+2\times\frac{1}{4}\div\frac{1}{3}-\frac{1}{2}=\frac{5}{2}+\frac{2}{1}\times\frac{1}{4}\times\frac{3}{1}-\frac{1}{2}=\frac{5}{2}+\frac{3}{2}-\frac{1}{2}$
$=4-\frac{1}{2}=3\frac{1}{2}$.

EXAMPLES. 84.

Simplify

1. $1\frac{1}{7}$ of $3\frac{1}{2}-\frac{4}{15}$ of $3\frac{3}{4}$.
2. $2\frac{1}{2}\times\frac{3}{5}+7\frac{1}{2}\times\frac{4}{15}$.
3. $\frac{2}{3}\div1\frac{5}{7}-\frac{5}{8}\div3\frac{2}{11}$.
4. $17\frac{1}{2}-3\frac{1}{4}\times4\frac{3}{13}+\frac{1}{12}$.
5. $3\frac{1}{13}+4\frac{1}{2}\div\frac{2}{3}-\frac{1}{2}$.
6. $2\frac{3}{4}+1\frac{3}{4}$ of $\frac{3}{14}-1\frac{1}{2}$.
7. $5\frac{1}{2}+3\frac{1}{3}\times4\frac{1}{8}-7$ of $1\frac{1}{14}$.
8. $3\frac{1}{2}+4\frac{1}{3}-\frac{5}{6}$ of $\frac{3}{4}$.
9. $2\frac{1}{2}$ of $3\frac{1}{3}-1\frac{1}{2}+\frac{3}{8}$ of $\frac{2}{3}$.
10. $3\frac{3}{7}$ of $4\frac{4}{7}\div5\frac{5}{7}-2\frac{2}{7}$.
11. $\frac{3}{4}$ of $4\frac{1}{4}+\frac{7}{8}\div\frac{5}{12}-\frac{2}{3}$.
12. $3\frac{1}{4}\div4\frac{1}{2}$ of $\frac{7}{8}+\frac{1}{4}$.
13. $\frac{3}{4}+\frac{2}{3}$ of $\frac{1}{7}\div\frac{1}{8}$ of $\frac{3}{14}$.
14. $\frac{3}{4}\div1\frac{1}{2}\times2\frac{1}{3}-\frac{1}{2}$ of $\frac{1}{3}$.
15. $\frac{2}{3}$ of $1\frac{1}{8}-\frac{1}{8}$ of $\frac{1}{4}-\frac{1}{3}\div5$.
16. $7\frac{1}{2}+\frac{1}{3}\div\frac{1}{4}$ of $\frac{1}{6}-\frac{3}{2}\times1\frac{4}{5}$.
17. $1\frac{1}{7}$ of $3\frac{1}{2}+\frac{4}{15}$ of $3\frac{3}{4}$ of $3\frac{1}{4}\div4\frac{1}{2}$ of $\frac{7}{8}-1\frac{3}{4}\times\frac{3}{14}$.
18. $4\frac{4}{7}+5\frac{5}{7}\div8-20\frac{1}{4}\times3\frac{1}{3}$ of $\frac{1}{25}\div\frac{3}{16}$ of $2\frac{2}{3}$.

144. Simplification of Fractions involving Brackets.

Example. Simplify $7-[\frac{3}{4}+\{2\frac{1}{2}-(1\frac{1}{2}-\frac{1}{3})\}]$.

The expression

(i) $=7-[\frac{3}{4}+\{2\frac{1}{2}-1\frac{1}{2}+\frac{1}{3}\}]$
$=7-[\frac{3}{4}+2\frac{1}{2}-1\frac{1}{2}+\frac{1}{3}]$
$=7-\frac{3}{4}-2\frac{1}{2}+1\frac{1}{2}-\frac{1}{3}$
$=$etc.

or (ii) $=7-[\frac{3}{4}+\{2\frac{1}{2}-\frac{7}{6}\}]$
$=7-[\frac{3}{4}+\frac{4}{3}]$
$=7-\frac{25}{12}$
$=$etc.

EXAMPLES. 85.

Simplify

1. $3-(\frac{1}{2}+1\frac{1}{2})$.
2. $4-(3\frac{1}{2}-\frac{3}{4})$.
3. $(3-1\frac{1}{12})$ of $3\frac{5}{7}$.
4. $(3-1\frac{1}{12})\times 3\frac{5}{7}-1\frac{1}{14}$.
5. $3-1\frac{1}{12}(3\frac{5}{7}-1\frac{1}{14})$.
6. $(3-1\frac{1}{12})(3\frac{5}{7}-1\frac{1}{14})$.
7. $(3+1\frac{1}{12})\div 3\frac{5}{7}-1\frac{1}{14}$.
8. $3+1\frac{1}{12}\div(3\frac{5}{7}-1\frac{1}{14})$.
9. $(3+1\frac{1}{12})\div(3\frac{5}{7}-1\frac{1}{14})$.
10. $7\frac{1}{2}+2\frac{1}{2}\div(\frac{3}{4}\times 1\frac{1}{2})$.
11. $6+\{1\frac{1}{2}+(\frac{3}{4}-\frac{1}{2})\}$.
12. $6-\{1\frac{1}{2}+(\frac{3}{4}-\frac{1}{2})\}$.
13. $6-\{1\frac{1}{2}-(\frac{3}{4}-\frac{1}{2})\}$.
14. $6-\{1\frac{1}{2}-(\frac{3}{4}+\frac{1}{2})\}$.
15. $17\frac{1}{2}-[8\frac{1}{4}+\frac{3}{4}(2\frac{1}{2}-1\frac{3}{4})\}$.
16. $17\frac{1}{2}-\{8\frac{1}{4}-\frac{3}{4}(2\frac{1}{2}+1\frac{3}{4}\}$.
17. $9\frac{1}{2}-[7\frac{1}{2}+\{4-(5-2)\}]$.
18. $9\frac{1}{2}+[7\frac{1}{2}-\{4+(5-2)\}]$.
19. $3\div[2+3\div\{4+5\div(2-\frac{1}{3})\}]$.
20. $(2-\frac{1}{2}$ of $\frac{1}{3})\div(7\frac{1}{2}\div 2\frac{1}{2})$.
21. $5\frac{1}{2}-[2\frac{1}{3}\div\{\frac{3}{4}-\frac{1}{2}(\frac{2}{3}-\overline{\frac{1}{6}-\frac{1}{8}})\}]$.
22. $6-[4-\frac{1}{8}\{7-(3\div\overline{2-\frac{1}{2}})\}]$.

145. *Example.* Simplify

$$\frac{\frac{3}{7}-\frac{2}{9}}{\frac{3}{7}+\frac{2}{9}} \text{ of } 2\frac{11}{26}\div\frac{4}{13-3\frac{8}{9}}+3\frac{11}{16}-\frac{3}{3-1\frac{10}{13}}.$$

The expression $=\frac{27-14}{27+14}$ of $\frac{63}{26}\div\frac{4}{9\frac{1}{9}}+3\frac{11}{16}-\frac{3}{1\frac{3}{13}}$

$$=\frac{13}{41} \text{ of } \frac{63}{26}\div\frac{4}{\frac{82}{9}}+3\frac{11}{16}-\frac{3}{\frac{16}{13}}$$

$$=\frac{13}{41}\times\frac{\overset{7}{63}}{26}\times\frac{1}{4}\times\frac{\overset{2}{82}}{9}+3\frac{11}{16}-\frac{3}{1}\times\frac{13}{16}$$

$$=\frac{7}{4}+\frac{59}{16}-\frac{39}{16}$$

$$=\frac{28+59-39}{16}$$

$$=\frac{48}{16}$$

$=3.$ *Ans.*

EXAMPLES. 86.

Simplify

1. $\dfrac{3\frac14 - 2\frac13 \text{ of } 1\frac27 - \frac17}{(3\frac14 - 2\frac13) \text{ of } (1\frac27 - \frac17)}$.

2. $\dfrac{7\frac12}{6\frac12} + \dfrac{11\frac12 - 2\frac25}{11\frac12 + 2\frac25} \times 10\frac{9}{13} - 6\frac{42}{273}$.

3. $\dfrac{2\frac14}{2\frac23} + \dfrac{2\frac12 + 5\frac13}{3\frac13 + 9\frac12} + \frac12 + \frac38 \text{ of } \frac{3}{26}$.

4. $\dfrac{5\frac67 - 3\frac23 + 4\frac{1}{14}}{3\frac13 + \dfrac{1 + \frac45}{2 - \frac58}}$.

5. $\dfrac{17}{7 + \dfrac{3}{4 - 2\frac34}} \times \dfrac{2021}{2193} \div \left(1\dfrac{37}{48} - \dfrac{15}{16}\right) + \dfrac34 \text{ of } \dfrac{3\frac{1\frac12}{2}}{2\frac12}$.

6. $\{(1\frac{2}{15} + \frac13) \times (3 - \frac12)\} \div (\frac13 + \frac15) + \dfrac{1}{2 + \dfrac{1}{2 + \frac12}} \text{ of } \frac34 \text{ of } \frac23 \div 2$.

7. $\dfrac{3\frac12 - 2\frac16}{\frac14 \text{ of } (\frac15 + \frac17)} \div 15\frac59$.

8. $\dfrac{1 + 5\frac45(1 + 5\frac45)}{1 + 2\frac13(1 + 2\frac13)} \text{ of } 3\frac12$.

9. $\frac12 \text{ of } \frac13 \times \frac34 \text{ of } \frac43 \div (\frac73 + \frac13 \text{ of } 20)$.

10. $\dfrac{\frac34 \div \frac34 \text{ of } \frac34}{\frac34 \div \frac34 \times \frac34}$.

11. $\dfrac{1\frac79 \text{ of } \frac{27}{64}}{1\frac12 \times 9\frac{9}{11}} \div \dfrac{4\frac47 \text{ of } \frac{21}{160}}{2\frac56 \div 2\frac{2}{15}}$.

12. $\dfrac{1}{\frac12 + \frac13 + \dfrac{\frac16}{\frac34 - \frac13}}$.

13. $\dfrac{6\frac{41}{88}}{6 - 4\frac{4}{11}} + \frac79 \times 1\frac{22}{27} \text{ of } \frac{8}{35} - \dfrac{6 + \dfrac{1}{6 + \dfrac{1}{6 - \frac16}}}{6}$.

14. $\dfrac{5 + \dfrac{1}{3 - \frac13}}{5 + \dfrac{1}{5 - \frac15}} \times 7\frac56$.

15. $\dfrac{8\frac35 + 7\frac34 + 5\frac23 - 4\frac12}{13 - 11\frac{9}{10} + 10\frac79 - 9\frac{17}{20}} \text{ of } \frac{2}{11} \text{ of } 365$.

16. $\left(\dfrac{9-4}{5} \div \dfrac17\right)^2$.

17. $\dfrac{7\frac12}{6\frac12} + \dfrac{1\frac12 - \frac23}{1\frac12 + \frac23} \div \frac{1}{13} \text{ of } \dfrac{1}{1 + \dfrac{2}{4\frac12}}$.

18. $(4\frac13 - 3\frac34) \times (7\frac25 - 5\frac{3}{14}) \div \frac35 \text{ of } \frac57 \text{ of } \frac79 \text{ of } 2\frac{109}{175}$.

19. $1\dfrac{4\frac12}{32} + \dfrac{5\frac56 \div \frac23}{1\frac15 \text{ of } \frac59 \div 10\frac13} \times \frac23 \text{ of } \dfrac{1\frac12 \text{ of } 4\frac16}{13\frac78 \times 5\frac13}$.

20. $\dfrac{1\frac13 \div 1\frac15}{\frac23 \text{ of } \frac56 \div 10\frac13} \times \dfrac{1\frac12 \text{ of } 4\frac16}{5\frac{1}{16} \text{ of } 5\frac13}$.

21. $\dfrac{\dfrac{\frac25}{1 - \frac{1}{25}} + \frac13 + \frac17}{1 - \frac17 \text{ of } \left(\dfrac{\frac25}{1 - \frac25} + \frac13\right)}$.

22. $\left\{\dfrac{2}{3-\dfrac{1}{1-\frac{1}{2}}}-\frac{1}{3}\text{ of }\left(5-\dfrac{2}{\frac{3}{2}-\frac{1}{6}}\right)\right\}\div\dfrac{\frac{1}{2}+\frac{5}{4}}{1\frac{1}{2}}.$

23. $\dfrac{7}{5-\frac{8}{3}}\div\dfrac{3-\dfrac{2}{3-\frac{3}{2}}}{4-\frac{3}{2}}-\frac{5}{7}\text{ of }\left\{\dfrac{1}{1\frac{3}{7}}+\frac{6}{5}\text{ of }\dfrac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}.$

24. $8-8\times\dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\dfrac{1}{6-\frac{1}{6}}}.$

25. $\dfrac{1+2\frac{1}{2}+3\frac{1}{3}}{\dfrac{1}{1\frac{1}{2}}+\dfrac{2}{2\frac{1}{3}}+\dfrac{3}{3\frac{1}{4}}}\times\dfrac{55\frac{2}{3}\div 11}{1\frac{2}{11}\text{ of }13\frac{2}{3}}.$

26. $\left(\frac{1}{3}+\dfrac{\frac{3}{7}}{1\frac{3}{5}}+\frac{11}{14}+\frac{5}{9}-1\right)\div\frac{5}{9}\text{ of }\frac{5}{8}\text{ of }2\frac{5}{7}.$

27. $\dfrac{\dfrac{2}{2+\dfrac{2}{2+\frac{1}{2}}}\text{ of }\frac{1}{2}\div(1+\frac{1}{2})}{4\left(1+\dfrac{2}{4\frac{1}{2}}\right)-3}.$

28. $1\dfrac{3\frac{1}{2}}{14}\div\frac{3}{5}\text{ of }\dfrac{2\frac{2}{3}\text{ of }9}{2+\dfrac{4}{6-\dfrac{8}{10+\frac{1}{12}}}}.$

29. $\dfrac{3+\dfrac{1}{3+\dfrac{1}{3+\frac{1}{3}}}}{1\frac{3}{4}\text{ of }\frac{1}{3}\div 3\frac{1}{2}}\text{ of }\dfrac{5}{9}\div\dfrac{\frac{1}{3}\text{ of }\frac{1}{4}}{\frac{1}{2}\text{ of }\dfrac{3}{4-1\dfrac{3\frac{1}{3}}{5}}}.$

30. $3+3\div\dfrac{3-3\text{ of }\frac{4}{5}\div 7\times 3}{1+\frac{1}{2}+3+\frac{1}{3}}.$

31. $\left\{\frac{1}{8}\text{ of }(\frac{1}{10}-\frac{1}{11})\div\dfrac{\frac{1}{7}-\frac{1}{9}\div(\frac{4}{9}+\frac{4}{11})}{\frac{1}{7}+\frac{1}{9}\div(\frac{4}{9}-\frac{4}{11})}\right\}\times\dfrac{\frac{1}{9}+\frac{1}{7}\div(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{3}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{9}}.$

32. $\dfrac{\frac{5}{6}+\frac{7}{8}\text{ of }\frac{4}{5}\div\frac{3}{4}\text{ of }\frac{9}{10}}{8\frac{1}{3}-\left(\dfrac{4}{1-\frac{7}{8}}\text{ of }2\frac{1}{4}\right)\div\frac{7}{9}\text{ of }12}\text{ of }6\frac{1}{2}+3\frac{1}{9}.$

33. $\dfrac{\left\{\dfrac{\frac{11}{99}+\frac{6}{13}}{1-\frac{11}{99}\times\frac{6}{13}}-\frac{11}{99}\right\}}{1-\frac{11}{99}\left\{\dfrac{\frac{6}{13}+\frac{11}{99}}{1-\frac{11}{99}\times\frac{6}{13}}\right\}}.$

34. $2\frac{1}{2}\div\dfrac{1-\frac{5}{6}}{\frac{1}{3}-\frac{1}{4}}+(\frac{1}{2}+\frac{1}{4})\div\frac{1}{3}+\frac{1}{5}.$

35. $3-\dfrac{1}{2+\dfrac{1}{1-\dfrac{1}{\frac{4}{5}\div\frac{2}{3}}}}\times 2+\dfrac{1}{1+\dfrac{1}{2\times\dfrac{1}{1-\frac{2}{5}}}}\times 2\div\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{1+\frac{4}{5}}}}.$

XXIV. FRACTIONAL MEASURES.

146. *Example* 1. Find the value of $\frac{3}{4}$ of R7. 8*a*. 3*p*.

First Method:

To multiply the compound quantity by $\frac{3}{4}$, we divide it by 4 and multiply the quotient by 3, thus :

$$\begin{array}{rrrr} & \text{R.} & a. & p. \\ 4) & 7 & .\ 8 & .\ 3 \\ \hline & 1 & .\ 14 & .\ 0\frac{3}{4} \\ & & & 3 \\ \hline & \text{R}5 & .\ 10 & .\ 2\frac{1}{4} \end{array} \quad \textit{Ans.}$$

N. B. If we have to multiply by $5\frac{3}{4}$, we multiply first by $\frac{3}{4}$ (as in the above example) and then under the result set down the product by 5, and add the two results.

Note. To divide a compound quantity by $\frac{3}{4}$, we divide it by 3 and multiply the quotient by 4.

Second Method: Now, $\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$.

$$\begin{array}{rrrrl} & \text{R.} & a. & p. & \\ 2) & 7 & .\ 8 & .\ 3 & \\ \hline 2) & 3 & .\ 12 & .\ 1\frac{1}{2} & =\frac{1}{2} \text{ of R}7.\ 8a.\ 3p. \\ \hline & 1 & .\ 14 & .\ \frac{3}{4} & =\frac{1}{4} \text{ of R}7.\ 8a.\ 3p. \\ \hline & \text{R}5 & .\ 10 & .\ 2\frac{1}{4} & =\frac{3}{4} \text{ of R}7.\ 8a.\ 3p. \end{array}$$

This method should be used when the multiplying fraction can be split up into two or fractions having *unity* for their numerators ; as for example, when the multiplier is $\frac{7}{12}(=\frac{1}{2}+\frac{1}{12})$. In this particular case the method of *Example* 3 is the most convenient.

Example 2. Find the value of $1\frac{1}{3}$ of $1\frac{1}{4}$ of R1.

$$1\frac{1}{3} \text{ of } 1\frac{1}{4} \text{ of R}1=\frac{4}{3} \text{ of } \frac{5}{4} \text{ of R}1=\text{R}\frac{5}{3}=\frac{\text{R}5}{3}.$$

$$\begin{array}{rrrr} & \text{R.} & a. & p. \\ 3) & 5 & .\ 0 & .\ 0 \\ \hline & \text{R}1 & .\ 10 & .\ 8 \end{array} \quad \textit{Ans.}$$

Example 3. Find the value of $\frac{2}{3}$ of R5. 2*a*. 3*p*.

Now, $\frac{2}{3}=1-\frac{1}{3}$.

$$\begin{array}{rrrr} & \text{R.} & a. & p. \\ 3) & 5 & .\ 2 & .\ 3 \\ \hline & 1 & .\ 11 & .\ 5 \\ \hline \text{The value reqd.}= & \text{R}3 & .\ 6 & .\ 10 \end{array} \text{ (by subtraction.)}$$

Example 4. Find the value of $\frac{5}{12}$ of £17. 7*s*. 6*d*. + $\frac{2}{3}$ of £5.

First Method:

$$\begin{array}{lllr} & & & £.\ \ s.\ \ d. \\ \frac{5}{12} \text{ of } £17.\ 7s.\ 6d. & =\frac{£17.\ 7s.\ 6d.}{12}\times 5 & =£1.\ 8s.\ 11\frac{1}{2}d.\times 5 & =7\ .\ 4\ .\ 9\frac{1}{2} \\ \frac{2}{3} \text{ of } £5 & =\frac{£10}{3} & =£\frac{10}{3} & =3\ .\ 6\ .\ 8 \\ \hline & & \therefore \text{ the value required} & =£10\ .\ 11\ .\ 5\frac{1}{2} \end{array}$$

Second Method: Now, $\frac{5}{12}=\frac{1}{3}+\frac{1}{12}$.

	£.	s.	d.		£.	s.	a.
3)	17 .	7 .	6				
4)	5 .	15 .	10	....	5 .	15 .	10
	1 .	8 .	$11\frac{1}{2}$	....	1 .	8 .	$11\frac{1}{2}$

∴ $\frac{5}{12}$ of £17. 7*s*. 6*d*. = 7 . 4 . $9\frac{1}{2}$
And $\frac{2}{3}$ of £5 = £$\frac{10}{3}$ = 3 . 6 . 8
∴ the value required = £10 . 11 . $5\frac{1}{2}$

Note 2. When we have to multiply or divide a compound quantity by a fraction, the terms of which are large numbers, it is generally better to adopt the following method.

Example 5. Find the value of $\frac{311}{550}$ of R10. 2*a*. 6*p*.

Process : $\frac{311}{550}$ of R10 . 2 . 6 = $\frac{311}{550}$ of 1950*p*.
$=\frac{311 \times 1950}{550}p. = \frac{311 \times 39}{11}p. = \frac{12129}{11}p.$
$= 1102\frac{7}{11}p.$ = 91*a*. $10\frac{7}{11}p.$ = R5. 11*a*. $10\frac{7}{11}p.$ *Ans.*

EXAMPLES. 87.

Find the value of

1. $\frac{2}{3}$ of R5. 7*a*. 6*p*.
2. $\frac{5}{6}$ of R2.
3. $\frac{3}{5}$ of R3. 2*a*.
4. $\frac{4}{9}$ of R19. 3*a*. 6*p*.
5. $\frac{3}{8}$ of R3. 4*a*.
6. $\frac{5}{8}$ of 12*a*.
7. $\frac{4}{11}$ of £92. 19*s*. 11*d*.
8. $\frac{5}{6}$ of £70. 4*s*.
9. $\frac{3}{10}$ of £99.
10. $5\frac{3}{5}$ of R12. 9*a*. 8*p*.
11. R$\frac{7}{8}$ + R$\frac{11}{12}$.
12. R$2\frac{1}{2}$ − R$1\frac{1}{3}$.
13. $4\frac{3}{8}$ of £2. 11*s*. $7\frac{1}{4}d$.
14. $4\frac{4}{15}$ of £9.
15. $\frac{35}{112}$ of £1.
16. R13. 12*a*. 9*p*. × $3\frac{4}{5}$.
17. R13. 13*a*. 6*p*. × $1\frac{5}{12}$.
18. £1. 7*s*. 6*d*. × $\frac{27}{38}$.
19. £10. 10*s*. $10\frac{1}{2}d$. × $\frac{278}{189}$.
20. R25. 12*a*. 9*p*. ÷ $\frac{66}{727}$.
21. £100. 3*s*. $4\frac{3}{4}d$. ÷ $2\frac{9}{20}$ of $\frac{9}{5}$.
22. $3\frac{3}{4}$ of 1 cwt. 1 qr. 1 lb.
23. $2\frac{2}{3}$ of 128 yd. 2 ft. 7 in.
24. $\frac{5}{12}$ of 1 hr. 1 min. 1 sec.
25. $\frac{4}{21}$ of 3 bus. 2 pk. 1 gall.
26. $3\frac{1}{2}$ of $3\frac{1}{3}$ of R12. 9*a*. 3*p*.
27. $\frac{1}{4}$ of $\frac{3}{7}$ of $1\frac{1}{3}$ of R7. 3*a*.
28. $2\frac{1}{3}$ of $6\frac{2}{5}$ of R7. 9*a*. 3*p*. + $7\frac{1}{2}$ of R1. 3*a*. 4*p*.
29. $\frac{3}{4}$ of $4\frac{1}{5}$ of £2. 12*s*. 6*d*. − $\frac{5}{24}$ of £1. 6*s*. 6*d*.
30. £$7\frac{21}{40}$ + $\frac{8}{9}$ of 15*s*. + 7*s*. ÷ $\frac{3}{4}$ + $4\frac{2}{7}$ of £3. 3*s*.
31. R$13\frac{1}{6}$ − $3\frac{7}{8}$ of 7*a*. − R2. 4*a*. ÷ $\frac{90}{80}$ + $7\frac{1}{9}$ of R3.
32. $\frac{3}{4}$ of $\frac{5}{6}$ of £1 + $\frac{2}{3}$ of $\frac{5}{9}$ of 2*s*. 6*d*. + $\frac{3}{4}$ of $10\frac{1}{2}d$.
33. $\frac{5}{6}$ of $\frac{7}{9}$ of R1 + $\frac{3}{4}$ of $\frac{5}{7}$ of 3*a*. 9*p*. + $\frac{2}{3}$ of $7\frac{1}{2}p$.
34. $\frac{11}{12}$ of £1 + $\frac{3}{4}$ of 2 guineas − $\frac{5}{9}$ of 3*s*. 9*d*. + $\frac{7}{8}$ of 1*s*.
35. $\frac{3}{4}$ of a guinea + $\frac{5}{6}$ of a crown − $\frac{7}{8}$ of 3*s*. 6*d*.

36. $\frac{7}{8}$ of R7. 8*a*. 6*p*. $-\frac{4}{7}$ of 7*a*. 7*p*. $+\frac{2}{3\frac{1}{2}}$ of $\frac{4}{\frac{6}{7}-\frac{2}{9}}$ of R$\frac{5}{9}$.

37. $\frac{2\frac{3}{4}}{7-\frac{1}{8}}$ of R8. 9*a*. $+\frac{3\frac{7}{11}}{4\frac{3}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of R9. 0*a*. 7*p*.

38. $(3\frac{1}{4}\div 3\frac{1}{3})$ of £3. 9*s*. $0\frac{3}{4}$*d*. $+(\frac{2}{3})^2$ of 27*s*. $-\frac{7\frac{3}{4}-3\frac{11}{12}}{18\frac{1}{2}\div\frac{6}{7}}$ of 5*s*.

39. Arrange $\frac{3}{4}$ of R7, $\frac{11}{12}$ of R6. 11*a*. and R$\frac{2}{3}$ in order of magnitude.

40. $\frac{3}{7}$ of $1\frac{1}{6}$ of a sum of money is £7. 7*s*. 7*d*. ; find the sum.

41. What is the sum, $\frac{5}{12}$ of which is R3. 9*a*. 3*p*. ?

42. From $\frac{3}{5}$ of a certain sum of money when $\frac{4}{5}$ of R3. 7*a*. is subtracted the remainder is R1. 1*a*. 1*p*. ; find the sum.

43. Find the value of $\frac{1\frac{1}{3}\div 1\frac{1}{5}}{\frac{2}{3}\text{ of }\frac{5}{9}\div 10\frac{1}{3}}$ of $\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{6\frac{5}{16}\times 5\frac{1}{3}}$ of R50.

44. Simplify $\frac{3\frac{1\frac{8}{14}}{4}}{20}$ of £$1+1\frac{4}{19}$ of $\frac{1}{1+\frac{1}{9+\frac{3}{4}}}$ of 15*s*. $+\frac{5-\frac{1}{4}}{12}$*s*.

147. To express one quantity as the *fraction* of another. First express both the quantities in terms of the same unit and then divide the measure of the first by the measure of the second.

Example 1. Express 13*a*. 4*p*. as the fraction of R1.

The fraction $=\frac{13a.\ 4p.}{\text{R}1}=\frac{13\frac{1}{3}}{16}=\frac{40}{48}=\frac{5}{6}$.

Note 1. R7. 13*a*. 4*p*. $=\text{R}7\frac{13a.\ 4p.}{\text{R}1}=\text{R}7\frac{13\frac{1}{3}}{16}=\text{R}7\frac{5}{6}$.

Example 2. Express R2. 1*a*. 10*p*. as the fraction of R3. 2*a*. 9*p*.

The fraction $=\frac{\text{R}2.\ 1a.\ 10p.}{\text{R}3.\ 2a.\ 9p.}=\frac{\text{R}2.\ 22p.}{\text{R}3.\ 33p.}=\frac{\text{R}1.\ 11p.\times 2}{\text{R}1.\ 11p.\times 3}=\frac{2}{3}$.

Example 3. Express $\frac{2}{3}$ of R2. 3*a*. as the fraction of $\frac{3}{4}$ of R8. 9*a*.

The fraction $=\frac{\frac{2}{3}\text{ of R}2.\ 3a.}{\frac{3}{4}\text{ of R}8.\ 9a.}=\frac{\frac{2}{3}\times 35}{\frac{3}{4}\times 137}=\frac{2\times 35\times 4}{3\times 137\times 3}=\frac{280}{1233}$.

Note 2. The above questions may be put in any of the following forms :

(1) Express R2 as the fraction of R5.

(2) Reduce R2 to the fraction of R5.

(3) What part is R2 of R5 ?

(4) What fraction is R2 of R5 ?

(5) How many times is R5 contained in R2 ?

(6) What is the measure of R2 when the unit is R5 ?

(7) Express R2 in terms of R5 as unit.

Example 4. Reduce $\frac{2}{3}$ of R5 + $\frac{3}{4}$ of R2. 3*a*. to the fraction of R11. 15*a*.

$$\text{The fraction} = \frac{\frac{2}{3}\text{ of R5} + \frac{3}{4}\text{ of R2. 3}a.}{\text{R11. 15}a.} = \frac{\frac{2}{3}\times 80 + \frac{3}{4}\times 35}{191}$$

$$= \frac{2\times 80\times 4 + 3\times 35\times 3}{191\times 12} = \frac{955}{191\times 12} = \frac{5}{12}.$$

EXAMPLES. 88.

1. Express R3. 4*a*. as the fraction of R1.
2. Express 9*a*. 9*p*. as the fraction of 1*a*.
3. Express R5. 5*a*. as the fraction of its *highest* denomination.
4. Express 7*s*. 6*d*. as the fraction of its *highest* denomination.
5. Express £7. 10*s*. 6*d*. *in pounds*.
6. Express 7*s*. 4½*d*. *in shillings*.
7. Reduce R7. 5*a*. 4*p*. as the fraction of R1.
8. Reduce £3. 6*s*. 8*d*. as the fraction of £1.
9. Reduce 8*a*. 9*p*. as the fraction of R3. 10*a*. 8*p*.
10. Reduce 12*s*. 5½*d*. to the fraction of £1. 3*s*. 4*d*.
11. What part is R9. 3*a*. 4*p*. of R10. 6*a*. 4*p*. ?
12. What part is 27 lb. 12 oz. 15 dr. of 3 cwt. 3 qr. 21 lb. ?
13. What part of 1 md. 38 seers is 7 seers 5 ch. ?
14. What part of 6 mi. is 2 mi. 441 yd. 1 ft. ?
15. What fraction is 12*s*. 10½*d*. of £10 ?
16. What fraction is 5 gall. 2 qt. 1 pt. of 10 gall. 2 qt. 1 pt. ?
17. What fraction of a guinea is 7*s*. 6¾*d*. ?
18. What fraction of a ton is 12 lb. 12 oz. ?
19. How many times is R7. 8*a*. 4½*p*. contained in R6. 8*a*. ?
20. How many times is 3 da. 7 hr. 8 min. contained in 8 da. 7 hr. 3 min. ?
21. What fraction is 13*s*. 10½¾*d*. of £2. 9*s*. 7*d*. ?
22. What fraction is 5½ guineas of £10½ ?
23. What fraction of 2½ yd. is 2½ ft. ?

24. How many times does 8 lb. 10 oz. 19 dwt. 9 gr. contain 1 lb. Troy ?

25. Express R20. 7*a*. 9*p*. as the fraction of 7*a*. 9*p*.

26. Express £20. 7*s*. 9*d*. as the fraction of 7*s*. 9*d*.

27. Express $\frac{3}{4}$ of R2. 7*a*. 3*p*. as the fraction of R7.

28. Express $1\frac{3}{4}$ of R8 as the fraction of R10. 10*a*. 10*p*.

29. Express $\frac{7}{8}$ of £3. 6*s*. 2*d*. as the fraction of £9. 7*s*. 6*d*.

30. Reduce $\frac{3}{5}$ of 1*s*. $1\frac{1}{2}$*d*. to the fraction of a crown.

31. Reduce $\frac{11}{12}$ of 8*s*. 9*d*. to the fraction of £3.

32. Reduce $\frac{7}{12}$ of R7. 9*a*. to the fraction of R9. 7*a*. 8*p*.

33. Express $\frac{2}{7}$ of R2. 3*a*. as the fraction of $1\frac{1}{2}$ of R5.

34. Express $3\frac{1}{2}$ of R1. 9*a*. as the fraction of $\frac{15}{16}$ of R7. 8*a*.

35. Reduce $\frac{3}{4}$ of $1\frac{1}{2}$ of 1*s*. 7*d*. to the fraction of $\frac{1}{3}$ of a guinea.

36. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of R10. 10*a*. 10*p*. to the fraction of $1\frac{1}{2}$ of R3.

37. What part of $\frac{1}{5}$ of 3 md. 19 seers 8 ch. is 18 seers 7 ch. ?

38. What part of $\frac{3}{8}$ of 7 cwt. 7 lb. is $\frac{2}{7}$ of a stone ?

39. What fraction of $2\frac{1}{2}$ of $\frac{1}{7}$ of 2 tons is $\frac{3}{8}$ of 3 cwt. 2 lb. ?

40. What fraction of a furlong is $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards ?

41. How many times is $\frac{2}{3}$ of 7 lb. 7 oz. 7 dr. contained in $\frac{5}{8}$ of a quarter ?

42. What fraction of $\frac{6}{11}$ of a foot is a pole ?

43. What fraction is $\frac{3}{4}$ of a gallon of $\frac{2}{3}$ of a pint ?

44. Express $\frac{3}{7}$ of 1 hr. 15 min. as the fraction of 1 day.

45. Express 5 fathoms as the fraction of $\frac{3}{14}$ of $3\frac{1}{2}$ of a pole.

46. What fraction of $\dfrac{7\frac{4}{5}}{4\frac{2}{3}}$ of £30. 13*s*. $2\frac{3}{4}$*d*. is $(8\frac{4}{7}-3\frac{3}{8})$ of £5. 9*s*. $11\frac{1}{2}$*d*. ?

47. Express R$7\frac{3}{4}-\frac{3}{4}$ of R5 as the fraction of R10. 9*a*.

48. Reduce $\frac{7}{12}$*s*. $-\frac{7}{12}$*d*. to the fraction of 12*s*. 10*d*.

49. Reduce R$7\frac{4}{5}-\frac{4}{5}$ of R7 to the fraction of R5.

50. Express $\frac{7}{9}$ of £1 $-\frac{2}{3}$ of 21*s*. as the fraction of 10*s*. 6*d*.

51. Express $\frac{2}{3}$ of 12*s*. 6*d*. $+\frac{3}{4}$ of 16*s*. 6*d*. as the fraction of £1.

52. Express $\frac{4}{15}$ of £1. 10*s*. $+\frac{5}{8}$ of 5*s*. 4*d*. $-8\frac{1}{2}$ of $\dfrac{1}{4\frac{1}{4}}$ of 5*s*. $3\frac{3}{4}$*d*. as the fraction of 2*s*. $1\frac{1}{2}$*d*.

53. What fraction of $\frac{2}{3}$ of 27*s*. is $\dfrac{3\frac{1}{2}}{4\frac{1}{2}}$ of $\{\frac{3}{4}$ of £1 $-\frac{2}{7}$ of 5*s*.$\}$?

MISCELLANEOUS EXAMPLES. 89.

(These examples are intended to give the student a thorough revision, both in the theory and practice, of all previous work.)

1. Distinguish between the *intrinsic* and *local* value of figures. Explain *local* value by the number 555, showing that the figure 5 has a different value according to its position.

2. Distinguish between *abstract* and *concrete* numbers.

3. What is the utility of o in the decimal system of notation ?

4. Write down the least and greatest possible numbers of four digits.

5. In the following example, add together (i) the vertical columns, (ii) the horizontal rows and (iii) each set of answers. The Grand Total will, of course, be the same in each case.

	(*a*)	(*b*)	(*c*)	(*d*)	Totals
(1)	405	769	999	24	....
(2)	678	42	386	139	....
(3)	309	370	123	975	....
Totals	...	...	...	...	.... Grand Total

6. State in figures the least number of 6 figures and the greatest number of 6 figures. Express their difference in words.

7. Supply the missing lines in the following addition sums :

(i)	(ii)	(iii)
3456	8888	235716
6967	10000	538607

(iv)	(v)	(vi)
306	6724	29768
4731	375	3709
8643	7009	43008
21622	19030	76856

8. Prove that 7 times 5 equal 5 times 7.

9. Multiply 86792 by 96164, the multiplication to be done in three lines.

10. Multiply 726 by 525, the multiplication to be done in two lines.

11. Multiply 123321 by 336567, using only three partial products.

12. Explain why a whole number is multiplied by 100 when you place two zeroes to the right of the figures which express it. State and prove the corresponding rule for division by 100.

13. Is it possible for two numbers to have 100 for their sum and 3000 for their product ?

14. If 109 be multiplied by a certain number it is increased by 2071. Find the multiplier.

15. Find the continued product of 16, 64, 125 and 625 as simply as you can.

16. Supply the missing digits so that,

(i) 21*4 may be exactly divisible by 7.
(ii) 98*2 " " " " " 8.
(iii) 346*6 " " " " " 9.

17. Explain the Italian method of division and perform the following divisions by that method :

(i) $52633 \div 123$. (ii) $77926 \div 506$. (iii) $729188 \div 478$.

State the local value of the first digit in the quotient as soon as you write it down.

18. In the following examples employ Short Division :

(i) $4372 \div 24$. (ii) $7356 \div 42$. (iii) $56703 \div 144$.

19. Multiply as shortly as possible :

(i) 324898 by 999. (ii) 98721 by 998. (iii) 825 by 9997.

20. Without actual division find the value of :

(i) $186453 \div 99$. (ii) $378569 \div 999$. (iii) 1298756 by 9999.

21. Complete the divisions below by writing in the missing numbers in the first and second lines :

(i)
2 | ______
3 | ______ remainder 1.
   39 remainder 2.

(ii)
7 | ______
8 | ______ remainder 3.
   1204 remainder 2.

22. If when a number is divided continuously by 4, 6 and 7 the remainders are 3, 4 and 2 respectively, what would be the remainder if the same number were divided by the continued product of 4, 6 and 7 ?

23. Two numbers when divided by a certain divisor have remainders 3 and 4 respectively ; when the two numbers are added and their sum divided by the same divisor, the remainder is 2. What is the divisor ?

24. Which is the correct statement—

R4 = $(4 \times 16)a.$, or R4 = $(16 \times 4)a.$? Are both correct ? Give reasons.

25. Multiply £871. 19*s*. 9*d*. by 420 and divide £1058. 15*s*. 9*d*. by 142. Also ascertain how often £18. 12*s*. is contained in £1864. 13*s*.

26. Convert 41 oz. Troy into Avoirdupois weight.

27. Reduce 535 guineas to half-crowns.

28. In what two senses do we use the word "divide" when we say "divide R100 by 5" and "divide R100 by R5" ?

29. The 1st January, 1929, was on Tuesday, what day of the week was October 1st, of the same year ?

30. In any year that is not a leap year show that the same day of the month in January and October will fall on the same day of the week.

31. How many pice in a continuous straight line would reach from Calcutta to Dum Dum, a distance of 5 miles, the width of a pice being 1 inch ; and what would be their total value in rupees ?

32. Divide £449. 6*s*. 2*d*. among *A*, *B* and *C*, giving *B* £21. 5*s*. 4*d*. more than *C*, and *A* £7. 6*s*. 3*d*. more than *B*.

33. A sum of money is divided between *A*, *B* and *C*. *C* gets twice as much *A* ; *A* and *B* together get R50 ; *B* and *C* together get R60. Find the sum of money, and how much each person gets ?

34. A bill of R9. 6*a*. was paid with an equal number of rupees, half-rupees, quarter-rupees and two-anna pieces. How many were there of each ?

35. A bill of £43. 6*s*. $1\frac{1}{2}$*d*. was paid with an equal number of pounds, shillings, pence and half-pence. How many were there of each ?

36. Of the men engaged in a factory a certain number receives 12 annas a day, twice as many receive 10 annas a day, and thrice as many receive 7*a*. 6*p*. a day. The daily wages amount to R272. 8*a*. How many workmen are employed ?

37. Can two numbers be prime to each other without being themselves prime numbers ?

38. If from any number, say, 756200 any other number with the same digits in different order, say, 620057, be subtracted, the remainder will be exactly divisible by 9. Explain this.

39. Prove without actual subtraction and division that the difference of 987654321 and 123456789 = a multiple of 9.

40. If to any whole number another number 1000 times its value is added, show that the result is always divisible by 11×13.

41. In finding the G. C. M. of two numbers, the last divisor is 49 and the quotients 17, 3, 2. Find the numbers.

42. Write down all the numbers between 100 and 1000 that have 137 for their H. C. F.

43. In a long division sum the dividend is 529565 and the successive remainders from the first to the last are 246, 222, 542. Find the divisor and quotient.

44. In one factory the wages amount to ₨31. 7*a.* 3*p.*, and in another ₨33. 8*a.* 3*p.* If every man receives the same wages, how many men are employed in each factory and how much does each man receive?

45. Find the least number that can be divided exactly by all numbers up to 12 inclusive.

46. Find the least sum of money that can be paid in either florins, half-crowns or guineas.

47. Define fraction, numerator, denominator. Explain as clearly as you can why the value of a fraction is increased if its denominator is diminished.

48. Reduce to their lowest terms:

(i) $\frac{256}{1024}$; (ii) $\frac{6006}{8008}$; (iii) $\frac{8398}{29393}$.

49. Reduce by cancelling to their lowest terms:

(i) $\frac{16\times45}{24\times75}$; (ii) $\frac{76\times87\times65}{114\times116\times143}$.

50. Express as whole or mixed numbers: $\frac{2516}{13}$, $\frac{427}{11}$, $\frac{840}{15}$.

51. Reduce to equivalent fractions having the least common denominator, and arrange in order of magnitude: $\frac{11}{12}$, $\frac{29}{30}$, $\frac{7}{18}$, $\frac{17}{15}$, $\frac{47}{48}$.

52. Find the value of $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}-\frac{1}{6}-\frac{1}{20}$.

53. Subtract $18\frac{4}{7}$ from $19\frac{1}{21}$ without reducing the mixed numbers to improper fractions.

54. If the numerator and denominator of a fraction be *multiplied* or *divided* by the same number, the value of the fraction is not altered; thus $\frac{1}{3}=\frac{4}{12}$. Prove this by a diagram, taking a straight line as the unit of length.

55. Explain the enlarged meanings of the symbols × and ÷ in $\frac{2}{3}\times\frac{9}{7}$ and $\frac{2}{3}\div\frac{9}{7}$.

56. Show that division by $\frac{2}{3}$ is equivalent to multiplication by $\frac{3}{2}$ and division by $\frac{3}{2}$ is equivalent to multiplication by $\frac{2}{3}$.

57. Divide the product of $4\frac{1}{11}$ and $2\frac{13}{15}$ by their difference.

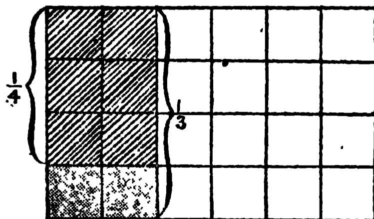
58. Simplify $\frac{1}{2+}\ \frac{1}{3+}\ \frac{1}{4+}\ \frac{1}{5}$.

59. Simplify $\frac{\{(5\frac{1}{3}-2\frac{5}{6})\text{ of }2\frac{3}{5}\}\times\frac{3}{4}}{1\frac{3}{4}+2\frac{7}{12}}$.

60. In an election to the Legislative Assembly $\frac{1}{23}$ of the constituency refused to vote and of the two candidates the one who was supported by $\frac{13}{23}$ of the whole constituency was returned by a majority of 500. How many actually voted ?

61. The sum of three fractions is 2 ; the first exceeds the second by $\frac{1}{2}$; and the second exceeds the third by $\frac{1}{4}$. Find them.

62. Use the diagram given below to show that $\frac{1}{2}(\frac{1}{3}-\frac{1}{4})=\frac{1}{24}$.



63. Make a diagram to show that $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}=\frac{7}{12}$.

64. Show that

$$1-\frac{26\times 366}{27\times 365}=\frac{339}{27\times 365}.$$

65. A man with a fixed yearly income and a fixed daily expenditure saves in ordinary years $\frac{1}{27}$ of his income. In leap years he saves £4. 4*s*. 9*d*. What is his income ?

66. If the day is $1\frac{1}{2}$ hours longer than the night, what is the length of each ?

67. Express the difference between the greatest and least of the fractions, $\frac{7}{24}$, $\frac{3}{14}$ and $\frac{15}{56}$ as the fraction of the other.

68. A clerk commenced work at a salary of R50 a month, which was each month increased by $\frac{1}{8}$ of that of the preceding month ; what was his third month's salary ?

69. *A* gives away $\frac{2}{3}$ of R50. He gives $\frac{1}{4}$ of this to *B*, $\frac{3}{8}$ of it to *C*, and the remainder to *D*. How much does each get ?

70. A sum of money is divided among 3 men. If the first has $\frac{1}{3}$ of it, the second $\frac{7}{12}$, and the third the remainder which is £2. 7*s*. $4\frac{1}{2}$*d*., what is the entire sum divided ?

71. *A* has R14. 7*a*. $4\frac{1}{2}$*p*. and has $3\frac{3}{4}$ times as much as *B* ; what has *B* ?

72. A person owes a guinea to each of 3 creditors ; to one he pays $\frac{1}{2}$ of his debt, to another $\frac{3}{4}$ and to the third $\frac{4}{5}$; what sum will he be still owing altogether ?

73. After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{2}{3}$ of the remainder were found to be 13*s.* $5\frac{1}{2}$*d.*; what sum did the purse contain at first?

74. A post is divided into 3 parts; the first part is $\frac{1}{3}$ of the whole length, the second $\frac{3}{8}$ of the first, and the third is 3 ft. 6 in.: find the length of the post.

75. Five brothers join in paying a sum of money; the eldest pays $\frac{1}{3}$ of it, and the others pay the remainder in equal shares, and thereby each of them pays R20. 7*a.* $7\frac{1}{2}$*p.* less than the eldest brother; what is the sum of money?

76. Find the sum of money that shall be the same part of £3. 10*s.* that 2 lb. 3 oz. Avoir. is of 3 lb. 2 oz.

77. What is the sum of money which is the same fraction of R2. 1*a.* that 7 yd. 1 ft. is of 11 yd.?

78. What fraction of R1. 13*a.* 7*p.* must be added to $\frac{3\frac{1}{2}}{2\frac{1}{3}}$ of $(\frac{3}{4}+\frac{4}{3})$ of 1*a.* 4*p.* to make the sum equal to R1?

79. If the American dollar be equal to £$\frac{5}{24}$, what fraction is $\frac{3}{8}$ of a dollar of $\frac{5}{7}$ of a guinea?

80. Reduce the difference between 1 lb. Avoir. and 1 lb. Troy to the fraction of $\frac{3}{5}$ of 1 lb. Avoir.

81. Reduce the sum of $\frac{3}{4}$ of £1, $\frac{3}{4}$ of 1*s.* and $\frac{3}{4}$ of 1*d.* to the fraction of $\frac{3}{4}$ of a guinea.

82. A cask contains 35 gall. 2 qt. 1 pt. of water; what part of it must be taken out to fill 5 quart bottles?

83. Find the greatest sum of money which is contained in each of $\frac{2}{3}$ of R3. 5*a.* 4*p.*, $\frac{3}{4}$ of R7. 9*a.* 8*p.* and $\frac{2}{5}$ of 8*a.* 9*p.* a whole number of times.

84. Find the least sum of money that contains each of $\frac{7}{8}$ of R1. 3*a.* 3*p.*, $\frac{4}{5}$ of R2. 8*a.* and $\frac{3}{4}$ of R7. 9*a.* 6*p.* an integral number of times.

85. A sum of money increased by its fifth part amounts to R3. 15*a.*; what is the sum?

86. What part of 5 units is $\frac{4}{5}$ of a unit?

87. Standard silver is coined at the rate of R2. 6*a.* $10\frac{3}{8}$*p.* per ounce; find the least integral number of ounces that can be coined into an exact number of rupees.

88. Find the least integral number of pounds Avoir., that contains an exact number of ounces Avoir. and of ounces Troy.

89. From a rope 30 ft. long, as many pieces as possible are cut off, each $3\frac{1}{4}$ ft. long; what fraction of the whole will be left?

XXV. DECIMALS.

148. In the ordinary system of notation the value of a digit in a number depends not only upon the digit itself, but also on the position it occupies in the given number. Thus in the number 5555, each of the four digits has the same *intrinsic* value but they have different *local* values. The digit 5 in the *fourth* place (counting from the right) stands for 5 *thousands*, the digit 5 in the *third* place stands for 5 *hundreds*, the digit 5 in the *second* place stands for 5 *tens*, and the digit 5 in the *first* place stands for 5 *units*.

Thousands.	Hundreds.	Tens.	Units.
5	5	5	5

From the above diagram it is clear that 5 in the tens' column has ten times the value it would have if it were in the units' column, and one-tenth part of the value it would have if it were in the hundreds' column. Hence if 5 be moved from the tens' column to the hundreds' column it would represent 5 *hundreds* instead of 5 *tens*, *i.e.*, *its present value would become ten times its former value.* Similarly, if 5 be moved from the tens' column to the units' column it would represent 5 *units* instead of 5 *tens*, *i.e.*, *its present value would become one-tenth of its former value.* Hence we see that *a movement of a digit from its present place to the place immediately to the left multiplies its value by* 10, *whilst a movement to the place immediately to the right divides its value by* 10.

If by a natural extension of this system of notation, we move 5 to additional new columns to the right of the units' column then by the previous reasoning 5 in the column next to the

etc.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	etc.
	5	5	5	5	5	5	5	

units' column on the right will have its value divided by 10, *i.e.*, its value will be "five-tenths". If it is moved again one place to

the right, its value will again be divided by 10, and it will be "five hundredths".

If this process be carried on further we shall have a complete system of notation in which digits placed to the left of the units' position will represent tens, hundreds, thousands, etc., whilst those placed to the right will represent tenths, hundredths, thousandths, etc. Thus the number in the above diagram represents "five thousand five hundred and fifty-five and five tenths, five hundredths, five thousandths".

But in such a system of notation it is necessary to indicate clearly the position of the units' figure ; and it has been agreed that the figure to whose right a point (·), called the **Decimal point**, is placed shall be the units' figure ; and to distinguish this point from the one used as the sign of multiplication, it is placed towards the top of the figure.

Thus 74·256 represents 74 units, 2 tenths, 5 hundredths, and 6 thousandths ; and is read "*seventy-four* **decimal**, *two, five, six*".

74·056 represents 74 units, no tenths, 5 hundredths and 6 thousandths ; and is read "seventy-four *decimal*, zero, five, six".

0·205 or ·205 represents no units, 2 tenths, no hundredths, and 5 thousandths ; and is read "*decimal*, two, zero, five".

149. A number expressed in the above notation is called a **decimal** or a **decimal fraction**. The part to the left of the point is called the **integral part**, and the part to the right is called the **decimal part** of the given number.

Note. Such numbers are called decimal fractions because each figure to the right of the decimal point represents a fraction which has some power of 10 as its denominator : thus $2·34 = 2 + \frac{3}{10} + \frac{4}{100}$.

150. If the number five thousand is to be written in figures, we put down 5 and then three ciphers to the right of 5 to indicate that there are no hundreds, tens, or units ; similarly, if 5 thousandths are to be written in figure, we put a cipher in the units' position to mark that position, then the decimal point and then place two ciphers in the next two places to indicate that there are no tenths and no hundredths and then put 5 to indicate five thousandths. Thus the number is written 0·005, but in practice the cipher in the units' place is often omitted and the number is simply written ·005.

151. The value of a decimal is not altered by annexing ciphers to the right of the last figure ; thus, 2·35 = 2·350 = 2·3500 ; for, these ciphers do not alter the position of any of the other figures relatively to the decimal point.

Note. An integer may be expressed as a decimal by writing ciphers in the decimal part ; thus $12 = 12{\cdot}00$.

But the value of the *decimal part* of a number decreases ten-fold, a hundred-fold,........., as we place one, two,........., zeroes immediately to the right of the decimal point.

Thus	·1	is one-tenth ;
	·01	is one-hundredth ;
	·001	is one-thousandth ;
		and so on.

152. It will be observed that a decimal is multiplied by 10, 100, 1000,........., by removing the decimal point 1, 2, 3,............, places to the right ; and conversely, a decimal is divided by 10, 100, 1000,........., by removing the point 1, 2, 3,......, places to the left.

Thus $20{\cdot}31 = 2{\cdot}031 \times 10$
$= 203{\cdot}1 \div 10.$

EXAMPLES. 90.

(*Most of the questions to be done orally.*)

Read off as decimals :

1. Three tenths.
2. Two and one hundredth.
3. Seven hundredths.
4. One tenth and four thousandths.
5. Eight ten-thousandths.
6. Nine millionths.
7. Twelve and four hundredths and six hundred-thousandths.
8. One hundredth and three thousandths and five millionths.
9. One ten-thousandth and one hundred-millionth.
10. One hundred and five tenths and two thousandths.
11. $5 + \frac{5}{10} + \frac{5}{100} + \frac{5}{1000}$.
12. $3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$.
13. $7 + \frac{8}{100} + \frac{5}{100000}$.
14. $\frac{1}{100} + \frac{2}{1000} + \frac{3}{100000}$.

Point out the actual values of the digits 3, 5 and 8 where they occur in the following numbers :

15. 203·8.
16. 305·518.
17. ·0835.
18. 105·768.
19. ·000378.
20. 123·456.
21. 80503·2.
22. 33·5588.

Multiply and divide each of the following numbers by 10, and by 1000 :

23. 7.
24. 29.
25. ·2.
26. ·02.
27. 3·4.
28. 7·03.
29. 1·003.
30. ·007.
31. 39·2
32. 23·45.
33. 3000.
34. 123·2.

35. Write down the number which is ten thousand times ·00001.

36. Write down the number which is a millionth part of 10000.

37. How many tenths of an inch are there in 3·5, 7·05 and 4 inches respectively?

38. How many tens-of-inches are there in 2·5, ·6 and 3 inches respectively?

Multiply by 100 and read off the product as a decimal.

39. $1+\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}$.

40. $3+\frac{5}{10}+\frac{7}{1000}+\frac{9}{100000}$.

Divide by 100 and read off the quotient as a decimal.

41. $100+\frac{2}{10}+\frac{4}{100}+\frac{6}{1000}$.

42. $\frac{1}{10}+\frac{4}{100}+\frac{7}{1000}+\frac{5}{10000}$.

153. To convert a decimal into the equivalent vulgar fraction.

Example. Express ·71 and 2·017 as vulgar fractions.

By the preceding Art., we have,

(i) $\cdot 71=71\div 100=\frac{71}{100}$;

(ii) $2\cdot 017=2017\div 1000=\frac{2017}{1000}$;

or, $2\cdot 017=2+\cdot 017=2+17\div 1000=2\frac{17}{1000}=\frac{2017}{1000}$.

Hence the rule: Write down the given number suppressing the decimal point for the numerator, and for the denominator write 1 followed by as many zeroes as there are figures in the decimal part.

154. To convert a vulgar fraction having some power of 10 as its denominator, into the equivalent decimal.

Example. Express $\frac{12}{10}$, $\frac{12}{100}$ and $\frac{12}{1000}$ as decimals.

(i) $\frac{12}{10}=12\div 10=1\cdot 2$.

(ii) $\frac{12}{100}=12\div 100=\cdot 12$.

(iii) $\frac{12}{1000}=12\div 1000=\cdot 012$.

Hence the rule: Take the numerator and in it place the decimal point after as many figures (counting from the right) as there are zeroes in the denominator. If the number of figures in the numerator be less than the number of zeroes in the denominator prefix in the numerator the requisite number of zeroes.

EXAMPLES. 91.

Express as vulgar fractions in their lowest terms :

1.	·4.	2.	·83.	3.	·04.	4.	1·5.
5.	·074.	6.	·0125.	7.	·0025.	8.	·075.
9.	2·88.	10.	·725.	11.	4·00256.	12.	7·225.
13.	·625.	14.	·0625.	15.	1·11.	16.	·0006875.
17.	81·0005.	18.	6·4375.	19.	5·0096875.	20.	70·00004.

Express as mixed numbers with the fractional parts in their lowest terms :

21.	2·5.	22.	7·25.	23.	8·125.	24.	1·75.
25.	2·025.	26.	3·05.	27.	9·0125.	28.	6·0075.
29.	3·0005.	30.	7·0675.	31.	12·225.	32.	11·1.
33.	2·0001.	34.	1·2221875.	35.	1·0007225.	36.	12·08056640625.

Express the following vulgar fractions as decimals :

37.	$\frac{7}{10}$.	38.	$\frac{90}{100}$.	39.	$\frac{120}{10}$.	40.	$\frac{240}{100}$.
41.	$\frac{3}{1000}$.	42.	$\frac{7230}{1000000}$.	43.	$\frac{329}{10000}$.	44.	$\frac{900}{10000}$.
45.	$\frac{12345}{100000}$.	46.	$\frac{20}{100000}$.	47.	$\frac{200030}{1000}$.	48.	$\frac{1000}{1000000}$.
49.	125 ten thousandths.			50.	790 millionths.		

135. The operations of addition, subtraction, multiplication and division of decimals are performed exactly in the same way as in the case of whole numbers. Hence it is an advantage to use decimals in preference to vulgar fractions.

136. Addition of Decimals.

In addition the decimals are to be placed so that units digits are under units, tenths under tenths, hundredths under hundredths, and so on, and consequently the decimal points will be in one vertical line one under another.

Example. Add together 72·305, 7·06 and ·7896.

We set down the decimals one under another, point under point ; thus

```
72·305
 7·06
  ·7896
-------
80·1546  Ans.
```

We first put down 6 under ten-thousandths and then add 5 thousandths and 9 thousandths. The result is 14 thousandths, viz., 1 hundredth + 4 thousandths : therefore we write down 4

under thousandths and carry 1 to the hundredths. Adding the hundredths we have 15 hundredths, *i.e.*, 1 tenth + 5 hundredths: therefore we write down 5 under hundredths and carry 1 to the tenths. Adding the tenths we have 11 tenths, *i.e.*, 1 unit + 1 tenth: therefore we write down 1 under tenths and carry 1 to the units. Adding the units we have 10 units, *i.e.*, 1 ten + 0 unit: therefore we write down 0 under units and carry 1 to the tens. Adding the tens we have 8 tens and this we write down below the tens. Thus it is evident that the numerical work is exactly the same as in the case of adding whole numbers.

EXAMPLES. 92.

Add together

1. 3·12, 12·023, ·32, 4·7
2. ·01, 30, 7·469.
3. 39·007, ·0008, 3, 1·3022.
4. 1·3, ·025, 79, ·005.
5. 1·23, 2·345, 6·7891, ·00001.
6. ·04, ·004, ·93, ·026.
7. 4·07, ·089, 2·7012, 3·1398.
8. ·0009, 900, 9·909.
9. 3·3, 10·70902, ·004, ·4, ·12.
10. 7, ·892, ·01, ·098.
11. 700 + 32·7269 + ·00903 + 3·4 + 263·36407.
12. ·1 + ·00095 + 84·0563 + 7·3 + 325·65432.
13. 6·3 + 617·241 + ·0078 + 37·045 + 8·6943 + ·01.
14. ·74259 + 346·274 + 300 + 10·00001 + ·207.
15. ·0705 + 705 + 7·05 + 20·00007 + ·01 + ·00043.
16. R40·004 + R7·2007 + R·00008 + R300·03.
17. £7·54212 + £39·407 + £·07078 + £700.
18. 30 min. + ·0045 min. + 7·7089 min. + 3·7685 min.
19. 329 ft. + ·01 ft. + 3·1 ft. + ·057 ft. + ·308 ft.
20. 2·2 in. + 30·03 in. + ·369 in. + ·7072 in. + 8·0008 in.

157. Subtraction of Decimals.

Example. Subtract 3·587 from 16·29.

We arrange the numbers as in the case of addition; thus

$$\begin{array}{r} 16·29 \\ -\ 3·587 \\ \hline 12·703 \end{array}\ \textit{Ans.}$$

We then subtract as in the case of whole numbers, supposing a zero (or more where necessary) annexed to the right of the minuend, and taking care to place the decimal point in the remainder under the column of points.

EXAMPLES. 103.

Subtract

1. 37·039 from 44·123.	2. 7·0389 from 9·01.
3. ·00078 from 1·1.	4. 100·389 from 300·09234.
5. 37·35 from 100.	6. 102 from 306·103.
7. ·000725 from ·001.	8. ·0001234 from ·012.
9. ·12345 from 7·6789123.	10. 3·1705 from 345·9875.
11. 7·325 from 8·025.	12. ·9375 from 3·0005.
13. R1·9999 from R9.	14. £32·00051 from £33.

Find the value of

15. 3·789 + 7·002 − ·0079 + ·1 − 1·00001.

16. 700 − ·007 − ·7078 − 3·12345 + ·00025.

17. 100 − ·0072 − 3·9345 − 12 − ·1.

18. 2000 − (·079 + 3·67002 − 3·0012).

19. 1·345 − ·072 − (3·123 − 30·321) + 100.

20. Is 3·1415926535 more accurately represented by 3·14159 or by 3·1416?

21. Is 2·718281828 more accurately represented by 2·7182 or by 2·7183?

158. Multiplication of Decimals.

If we take *any* two decimals, convert them into vulgar fractions and multiply these latter together, we find that the numerator of the product is the product of the two given decimals with their decimal points suppressed, and that the denominator is 1 followed by as many ciphers as there are decimal places in the two given numbers; and if now the product be reduced back to the equivalent decimal, it will contain as many decimal places as there are ciphers in the denominator. Hence we have the following rule for the Multiplication of Decimals:

Multiply the given numbers as if they were integers, and mark off in the product a number of decimal places equal to the sum of the numbers of decimal places in the two factors. If the number of figures in the product be less than the number of decimal places in the two factors, prefix the requisite number of ciphers.

Example 1. Multiply 13·325 by 3·2 and ·00046 by 36.

```
(i)    13·325              (ii)   ·00046
          3·2                         36
       ------                     ------
        26650                        276
       39975                        138
       -----                      ------
       42·6400 = 42·64  Ans.      ·01656  Ans.
```

Second Method.—Write down the multiplier so that the digit in the units' place may fall just below the last figure (on the right) of the multiplicand and then proceed, remembering that in all cases, the first figure on the right of each partial product must be placed in the same vertical column with the figure by which the product is obtained. (See Art. 37.)

Example 2. Multiply (i) 13·326 by 3·2 and (ii) 26·394 by 34·7.

```
(i)  13·326                 (ii)  26·394
        3·2                         34·7
      2·6652                       18·4758
     39·978                       105·576
     42·6432                      791·82
                                  915·8718
```

EXAMPLES. 94.

(*Oral.*)

Obtain the following products :

1. ·4×2. **2.** ·5×3. **3.** ·3×·1.
4. ·2×·2. **5.** ·1×·01. **6.** ·02×·6.
7. ·05×·05. **8.** 5·2×·4. **9.** 7·5×·04.
10. 12·5×·8. **11.** ·0025×·04. **12.** 40×·015.

EXAMPLES. 95.

Multiply

1. 32·4 by 2·3. **2.** 7·24 by 5. **3.** 67·23 by ·002.
4. 30·03 by 200. **5.** ·032 by ·032. **6.** ·045 by ·0072.
7. 800·008 by ·035. **8.** 34·12345 by 72. **9.** ·0202 by 2020.
10. 4030·4 by ·0075. **11.** 4·379 by ·37. **12.** ·00125 by ·25.
13. 10·607 by 402000. **14.** ·000625 by 12800. **15.** 725 by ·0008.
16. 6400 by ·00125. **17.** 5·12 by 42·25. **18.** 46·025 by 12·8.
19. ·0064 by ·0125. **20.** ·00846 by ·005. **21.** ·007853 by ·00476
22. 56·875 by ·0144. **23.** ·015625 by ·0064. **24.** ·0204 by 40·2.
25. 700 by ·005. **26.** 79·235 by 39·02. **27.** 40·25 by 30·04.
28. 12·8 by ·0075. **29.** 1·12005 by ·12005. **30.** 9·006 by 5·40005
31. 2·5×2·5×2·5. **32.** ·25×·25×·25. **33.** ·05×·08×·02.
34. 1·2×15×·12. **35.** 11×1·1×·11. **36.** 20×·2×·25.
37. ·0005×·005×·05. **38.** 7×·7×·07×7000.
39. ·3×·03×·003×30. **40.** 2000×·0055×2·5.

Find the value of

41. $(6{\cdot}25)^2-({\cdot}5)^2$. **42.** $(74{\cdot}5-{\cdot}007)\times{\cdot}035$.

43. $7{\cdot}6-3{\cdot}7\times{\cdot}009$. **44.** $({\cdot}05)^2+4{\cdot}5\times 20$.

45. $7{\cdot}5\times{\cdot}75-75\times{\cdot}075+(7{\cdot}5)^2-(7{\cdot}5-{\cdot}75)\times{\cdot}075$.

159. Division of Decimals.

I. *When the Divisor is an Integer.*

Example 1. Divide 808·9 by 25.

Process :	25) 808·900 (32·356 *Ans.*
tens	75
units	58
”	50
tenths	89
”	75
hundredths	140
”	125
thousandths	150
”	150

Italian Method.

```
        32·356  Ans.
25 ) 808·900
      58
       89
       140
        150
```

Here we divide as in the case of whole numbers, taking care to place the decimal point in the quotient as soon as the division of the integral part is finished.

If there is a remainder (as in the above case) after division, we affix a zero to the remainder, and divide. We treat all successive remainders in the same manner, and continue the division until the required number of decimal places in the quotient is obtained, or until there is no remainder.

Note. The method of short division may be employed with advantage when the divisor does not exceed 20, or when the divisor can be expressed as the product of factors each less than 20.

Example 2. Obtain the quotient to five places of decimals in the division of ·025 by 7.

```
Process :    7 ) ·02500
                 ·00357...  Ans.
```

II. *When the Divisor is a Decimal* :

Remove the decimal point in both the Divisor and Dividend as many places to the right as will make the *divisor* a whole number ; and then divide as in the preceding case.

Note. Observe that removing the decimal point in the divisor and dividend an equal number of places to the right is

equivalent to multiplying the divisor and dividend by the same number; and that if the divisor and dividend be both multiplied by the same number the quotient is not altered.

Example 3. Divide 12·96 by 10·8.

Here we divide 129·6 by 108:

```
          1·2  Ans.
108 ) 129·6
      108
      ---
       216
       216
       ---
```

Example 4. Divide 34·6 by ·08.

Here we divide 3460· by 8:

```
8 ) 3460·0
     432·5  Ans.
```

160. A vulgar fraction may be expressed as a decimal by dividing the numerator by the denominator.

Example. Express $\frac{5}{8}$ as a decimal.

Process: 8) 5·000
·625 *Ans.*

Note. The following results are useful and should be remembered.

$\frac{1}{2}$ = ·5; $\frac{1}{4}$ = ·25; $\frac{3}{4}$ = ·75; $\frac{1}{8}$ = ·125;
$\frac{3}{8}$ = ·375; $\frac{5}{8}$ = ·625; $\frac{7}{8}$ = ·875.

EXAMPLES. 96.

(Oral.)

Divide

1. 6·3 by 3. **2.** 6·4 by 4 **3.** 8·1 by 9. **4.** 3·96 by 3.
5. 29·4 by 7. **6.** ·82 by 2. **7.** ·4 by 8. **8.** ·25 by 5.
9. 2·4 by ·2. **10.** 4·9 by 7. **11.** 4·8 by ·02. **12.** 3·9 by ·03.
13. 4·5 by ·009. **14.** 6·3 by ·09. **15.** 2·7 by ·27.
16. 3·8 by ·038. **17.** ·69 by 69. **18.** 2·5 by ·025.
19. 150 by ·15. **20.** ·0026 by ·0013.

EXAMPLES. 97.

Divide

1. 29·21 by 23. **2.** 34·3 by 25. **3.** 129·6 by 108.
4. ·03096 by 72. **5.** 457·7 by 230. **6.** ·06227 by 1300.
7. ·04009 by 1520. **8.** 3708 by 360. **9.** ·00281 by 1405.
10. 8357 by 488. **11.** ·001007 by 47500. **12.** 431·376 by 8170.

Divide, finding the quotient as far as the fifth decimal place:

13. 42·5 by 23. 14. ·0269 by 281. 15. 197 by 79.
16. ·041326 by 101. 17. ·0079 by 372. 18. 312 by 84.
19. 356·5 by 273. 20. 6·5 by 342. 21. ·0042 by 121.

Find the quotient, by Short Division, to not more than 6 places of decimals, in the division of

22. 4·125 by 2. 23. 3·73 by 8. 24. ·034 by 7.
25. 21·24 by 90. 26. 134 by 11. 27. 36·7 by 16.
28. ·04321 by 80. 29. 8·567 by 13. 30. ·01 by 6.

Divide

31. ·3125 by ·01. 32. 8·454 by ·024. 33. ·5568 by 2·32.
34. 6·33 by ·0025. 35. 17·28 by ·0144. 36. 4 by ·00625.
37. ·00281 by 1·405. 38. 1·77089 by 4·735.
39. ·00005 by ·0000025. 40. 816 by ·0004.
41. 84·375 by ·00375. 42. 2874·465 by ·0495.
43. ·830676 by ·000231. 44. 33·363 by ·00275.
45. 7 by ·0004. 46. ·0007 by ·0005.
47. 5·625 by ·0000075. 48. ·0003738028 by ·0476.

Find the quotient to five places of decimals:

49. 3·461 ÷ ·027. 50. ·3125 ÷ ·06.
51. ·2 ÷ ·006. 52. ·000753 ÷ ·009.
53. ·000001 ÷ ·0000431. 54. ·5 ÷ 70·91342.
55. 4000 ÷ ·000121. 56. ·666666 ÷ ·008.
57. ·007 ÷ ·00073. 58. [illegible] ÷ 329·265.

Employ Short Division in finding the quotient to not more than 6 places of decimals:

59. 28 ÷ ·08. 60. 3·76 ÷ ·005. 61. ·0076 ÷ ·003.
62. ·0101 ÷ ·0016. 63. ·000012 ÷ ·13. 64. 229 ÷ ·007.
65. 39·4 ÷ ·007. 66. 4·767 ÷ ·004. 67. 13·75 ÷ ·012.
68. ·02 ÷ 1·1. 69. ·03 ÷ 1·4. 70. 3·4 ÷ ·009.

Convert into decimals:

71. $\frac{1}{2}$. 72. $\frac{1}{4}$. 73. $\frac{3}{4}$. 74. $\frac{1}{8}$. 75. $\frac{3}{8}$.
76. $1\frac{7}{16}$. 77. $3\frac{3}{32}$. 78. $9\frac{21}{64}$. 79. $3\frac{7}{25}$. 80. $\frac{201}{75}$.

Express as decimals as far as the fifth decimal place:

81. $\frac{1}{3}$. 82. $\frac{1}{6}$. 83. $\frac{2}{7}$. 84. $\frac{8}{11}$. 85. $\frac{9}{13}$.
86. $1\frac{4}{9}$. 87. $7\frac{2}{11}$. 88. $8\frac{7}{21}$. 89. $10\frac{10}{29}$. 90. $\frac{701}{12}$.

Arrange in order of magnitude, by reducing to decimals as far as the fourth decimal place :

91 $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. 92. $\frac{3}{11}, \frac{5}{12}, \frac{7}{14}$. 93. $\frac{11}{20}, \frac{16}{30}, \frac{21}{40}$.

94. $\frac{5}{16}, \frac{7}{32}, \frac{3}{8}$. 95. $\frac{7}{20}, \frac{11}{25}, \frac{13}{30}$. 96. $\frac{3}{5}, \frac{5}{7}, \frac{7}{9}$.

Reduce to decimals :

97. $\frac{4}{5}$ of ·027. 98. ·025 of $4\frac{1}{2}$.

99. $\frac{1}{2}$ of $\frac{3}{4} \times 8{\cdot}36$. 100. $\frac{1}{4}$ of $\frac{1}{15} \div {\cdot}05$ of $2\frac{1}{2}$.

161. H. C. F. and L. C. M. of Decimals.

To find the H. C. F. or the L. C. M. of Decimals, affix ciphers (where necessary) so that all the given numbers may have the same number of decimal places ; then find the H. C. F. or the L. C. M. of them as if they were integers, and mark off in the result as many decimal places as there are in each of the numbers.

Example. Find the H. C. F. and L. C. M. of 3, 1·2 and ·06.

The given numbers are equivalent to 3·00, 1·20 and ·06.

The H. C. F. of 300, 120 and 6 = 6 ; their L. C. M. = 600.

∴ The H. C. F. required = ·06 ;

and the L. C. M. required = 6·00 = 6.

EXAMPLES. 98.

Find the H. C. F. and L. C. M. of

1. 3·75, 7·25. 2. 72·12, ·03. 3. ·02, ·4, ·008.
4. 1·2, ·24, 6. 5. 1·6, ·04, ·005. 6. 2·4, ·36, 7·2.
7. ·08, ·002, ·0001. 8. 3·9, 6·6, 8·22. 9. ·6, ·09, 1·8.
10. ·18, 2·4, 60. 11. 20, 2·8, ·25. 12. 1·5, ·25, ·075.

162. Complex Decimal Fractions.—The methods of simplifying complex decimal fractions are the same as those of complex vulgar fractions. As a general rule when decimals are connected by the signs '+', '−' or '×' they should be added, subtracted or multiplied as decimals and should not be converted into vulgar fractions, but when they are connected by the sign '÷' we may shift the decimal places in both the divisors and dividends equally. For example $\frac{\cdot 002}{\cdot 579} = \frac{2}{579}$. Thus the points may be dropped altogether. The answer should also be presented as a decimal.

Example 1. Simplify

$$\frac{8\cdot5}{\cdot0171} \times \frac{1\cdot33}{\cdot8} \div \frac{11\cdot9}{\cdot0072}.$$

$$\text{Fraction} = \frac{8\cdot5}{\cdot0171} \times \frac{1\cdot33}{\cdot8} \times \frac{\cdot0072}{11\cdot9}$$

$$= \frac{85000}{171} \times \frac{133}{80} \times \frac{72}{119000}$$ Shifting the decimal places in both the divisors and dividends equally.

$$= \tfrac{1}{2} = \cdot5. \quad \textit{Ans.}$$

EXAMPLES. 99.

Simplify without reducing to vulgar fractions :

1. $\frac{\cdot0075 \times 2\cdot1}{\cdot0175}$.
2. $\frac{1\cdot18}{\cdot152} \times \frac{3\cdot04}{2\cdot95}$.
3. $\frac{\cdot081 \times 5\cdot7}{1\cdot71}$.
4. $\frac{\cdot00281 \times \cdot0625}{1\cdot405}$.
5. $\frac{4\cdot255 + \cdot0064}{\cdot00032}$.
6. $\frac{6\cdot501 - 3\cdot07 + 2\cdot124}{5\cdot5}$.
7. $\frac{12\cdot32 - 7\cdot56}{20\cdot35 + 3\cdot45}$.
8. $\frac{2\cdot553 \times \cdot0064}{\cdot00032}$.
9. $\frac{5\cdot9 \times \cdot00152}{3\cdot04 \times 1\cdot18}$.
10. $\frac{5\cdot5}{\cdot66} \times \frac{\cdot081}{4\cdot2} \times \frac{4\cdot9}{\cdot35}$.
11. $\frac{\cdot0075 \times 2\cdot1}{\cdot0175} + \frac{4\cdot255 \times \cdot064}{\cdot00032}$.

XXVI. RECURRING DECIMALS.

163. In the process of reduction of vulgar fractions to decimals, it will be found, in some cases, that the division does not terminate ; so that the quotient can be continued without limit.

Example. Reduce $\frac{19}{55}$ to a decimal.

$$55\ )\ \underline{19\cdot0000000}$$
$$\cdot3454545\cdots$$

164. We can tell beforehand whether, in any particular case, the division will terminate or not.

Let the fraction be in its lowest terms ; then if the prime factors of the dinominator are each of them either 2 or 5, the division will terminate ; and not otherwise.

Thus

(i) $\frac{7}{20}$ $\left(=\frac{7}{2\times2\times5}\right)$ will produce a **terminating** decimal.

(ii) $\frac{7}{12}$ $\left(=\frac{7}{2\times2\times3}\right)$ will produce a **non-terminating** decimal.

EXAMPLES. 100.

State, in each case, whether the equivalent decimal is terminating or non-terminating :

1. $\frac{1}{3}$.	2. $\frac{3}{4}$.	3. $\frac{7}{9}$.	4. $\frac{21}{24}$.	5. $\frac{78}{20}$.
6. $2\frac{49}{63}$.	7. $\frac{33}{121}$.	8. $\frac{12}{170}$.	9. $\frac{34}{510}$.	10. $\frac{101}{155}$.
11. $3\frac{18}{72}$.	12. $\frac{9}{25}$.	13. $7\frac{1}{2}$.	14. $\frac{72}{96}$.	15. $11\frac{10}{12}$.

16. Write down those numbers between 1 and 20, which being denominators of fractions in their lowest terms, will produce non-terminating decimals.

165. In non-terminating decimals, certain digits must recur over and over again.

Consider the fraction $\frac{5}{6}$. In the process of division the only remainders possible are, 1, 2, 3, 4, 5 ; consequently, after five steps at most, we must come to a remainder which has occurred before, and therefore from that point we must have a recurrence of the remainders, and therefore of the digits in the quotient.

Example 1. $\frac{2}{3}$ = ·6666666...

Example 2. $\frac{19}{55}$ = ·3454545...

Note. It may be noticed here that division by 3 or 9 gives a period (See Art. 166) of *one* digit ; division by 11, a period of *two* digits ; division by 7 or 13, a period of *six* digits.

166. Decimals in which certain digits recur are called **recurring decimals.**

Note. A recurring decimal is also called a **periodic, repeating** or **circulating** decimal.

The whole body of digits which recur is called the **period.** Thus, in ·6666... the period is 6 ; in ·3454545... the period is 45.

167. In writing a recurring decimal we usually stop at the end of the first period and place dots over its first and last digits.

Thus ·666666......... is written $\cdot\dot{6}$;

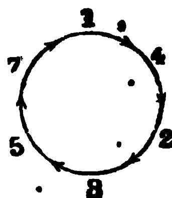
·373737......... .... $\cdot\dot{3}\dot{7}$;

·3454545......... .... $\cdot3\dot{4}\dot{5}$;

·34576576......... .... $\cdot34\dot{5}7\dot{6}$.

A **pure** recurring decimal is one in which the period commences immediately after the decimal point ; as, $\cdot\dot{6}$, $\cdot\dot{3}\dot{7}$.

A **mixed** recurring decimal is one in which one or more figures precede the period ; as, $\cdot3\dot{4}\dot{5}$, $\cdot34\dot{5}7\dot{6}$.



Note. It may be noticed that decimals equivalent to fractions with denominator 7 are all *pure* recurring decimals, all of which contain the same digits 142857. If these digits be arranged in a circle, as in the annexed diagram, we may obtain the decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, by beginning in turn with 1, 2, 4, 5, 7, 8, and reading off the remaining digits in order in the direction of the arrow-heads.

Thus $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$; and so on.

EXAMPLES. 101.

Express each of the following as a recurring decimal:

1. $\frac{1}{3}$. 2. $\frac{2}{9}$. 3. $\frac{5}{7}$. 4. $\frac{7}{9}$. 5. $\frac{13}{11}$.

6. $\frac{20}{13}$. 7. $\frac{7}{15}$. 8. $1\frac{1}{111}$. 9. $\frac{5}{18}$. 10. $3\frac{8}{13}$.

11. $\frac{250}{21}$. 12. $\frac{1}{22}$. 13. $\frac{20000}{5291}$. 14. $\frac{5}{24}$. 15. $\frac{101}{30}$.

16. $\frac{202}{27}$. 17. $5\frac{2}{7}$. 18. $10\frac{1}{13}$. 19. $7\frac{2}{15}$. 20. $9\frac{9}{14}$.

21. $\frac{506}{303}$. 22. $\frac{725}{52}$. 23. $4\frac{45}{56}$. 24. $\frac{3542}{1025}$. 25. $5\frac{11}{30}$.

26. $2 \div 3$. 27. $46 \div 7$. 28. $39 \div 22$. 29. $8 \div 63$. 30. $44 \div 9$.

31. $\frac{1}{6}$. 32. $\frac{1}{66}$. 33. $\frac{1}{666}$. 34. $\frac{1}{6666}$. 35. $\frac{1}{66666}$.

36. $8\frac{8}{75}$. 37. $3\frac{24}{175}$. 38. $\frac{1}{17}$. 39. $\frac{49}{19}$. 40. $\frac{2}{23}$.

41. $1{\cdot}2 \div {\cdot}11$. 42. $1 \div 10{\cdot}01$. 43. ${\cdot}3 \div {\cdot}13$. 44. $\frac{{\cdot}2}{{\cdot}07}$.

45. $\frac{{\cdot}03}{{\cdot}0011}$. 46. $2 + \frac{{\cdot}3}{1{\cdot}1}$. $7 + \frac{2}{2{\cdot}3}$. 48. $1 + \frac{1{\cdot}1}{{\cdot}07}$.

49. $3 + \frac{4}{1{\cdot}3}$. 50. $\frac{4\frac{1}{3}}{{\cdot}007}$. 51. $\frac{3{\cdot}7}{4\frac{1}{2}}$. 52. $\frac{{\cdot}004}{5\frac{1}{2}}$.

168. In a given recurring decimal, the period may be supposed to begin at any point after the first repeating figure.

Thus $\cdot 3272727\ldots = \cdot 3\dot{2}\dot{7} = \cdot 32\dot{7}\dot{2} = \cdot 327\dot{2}\dot{7} = $ etc.

Again, the number of figures in the period of a recurring decimal may be *doubled*, *trebled*,... without altering the value of the decimal.

Thus $\cdot 3\dot{2}\dot{7} = \cdot 3\dot{2}72\dot{7} = \cdot 3\dot{2}7272\dot{7} = $ etc.

169. Recurring decimals are said to be **similar** when they have the same number of non-recurring figures, and also the

same number of recurring figures. Thus $\cdot\dot{3}$ and $\cdot\dot{6}$ are similar recurring decimals ; $\cdot 3\dot{2}\dot{7}$ and $2\cdot 4\dot{5}\dot{6}$ are similar.

170. Two or more given recurring decimals can always be made similar.

Take the recurring decimals $2\cdot\dot{3}$, $\cdot 2\dot{4}\dot{5}$ and $\cdot 25\dot{7}6\dot{8}$.

Now the highest number of non-recurring decimal places in any of these numbers is 2 ; and the numbers of figures in the periods respectively are 1, 2, 3, the L. C. M. of which is 6. Therefore the given recurring decimals may be made similar by extending each of them to eight places of decimals, the first two places being non-recurring and the last six places being recurring.

$$\text{Thus} \qquad 2\cdot\dot{3} = 2\cdot 33\dot{3}3333\dot{3} \;;$$
$$\cdot 2\dot{4}\dot{5} = \cdot 24\dot{5}4545\dot{4} \;;$$
$$\cdot 25\dot{7}6\dot{8} = \cdot 25\dot{7}6876\dot{8} \;;$$

EXAMPLES. 102.

In each of the following recurring decimals begin the period at the fourth decimal place :

1. $\cdot 2\dot{3}4\dot{5}$. 2. $\cdot 34\dot{7}\dot{6}$. 3. $\cdot\dot{6}\dot{7}$. 4. $\cdot 23\dot{4}\dot{5}$.

5. $\cdot 00\dot{1}2\dot{3}$. 6. $\cdot 1\dot{2}34\dot{5}$. 7. $\cdot\dot{1}23\dot{4}$. 8. $\cdot 1\dot{2}345\dot{6}$.

9. Extend $\cdot 3\dot{4}$, $\cdot\dot{2}\dot{4}$ and $\cdot 2\dot{6}7\dot{8}$ so that they may have the same number of figures in the period.

10. Extend $\cdot 1\dot{0}\dot{2}$, $\cdot 1\dot{2}3\dot{4}$ and $\cdot\dot{3}76\dot{5}$ so that they may have the same number of recurring figures.

Make the following sets of recurring decimals similar :

11. $\cdot 2\dot{3}$, $\cdot\dot{7}\dot{8}$. 12. $\cdot 3\dot{4}\dot{5}$, $\cdot\dot{7}\dot{6}$, $\cdot 7\dot{2}$. 13. $\cdot 30\dot{7}$, $\cdot\dot{7}6$.

14. $\cdot 0\dot{7}\dot{6}$, $\cdot\dot{7}$, $\cdot 00\dot{0}12\dot{3}$. 15. $\cdot 23\dot{8}$, $\cdot\dot{1}23\dot{4}$, $\cdot 0\dot{2}\dot{3}$. 16. $\cdot\dot{3}$, $\cdot\dot{7}\dot{6}$, $\cdot\dot{7}23\dot{0}$.

17. $\cdot\dot{7}$, $\cdot 1\dot{2}\dot{4}$, $\cdot 24\dot{7}2\dot{3}$. 18. $3\cdot\dot{4}$, $\cdot 2\dot{6}\dot{8}$, $\cdot\dot{1}2\dot{3}$.

19. $3\cdot 40\dot{2}$, $\cdot 78\dot{2}\dot{3}$, $\cdot 3\dot{1}$. 20. $\cdot 4\dot{2}\dot{3}$, $\cdot\dot{7}\dot{2}$, $\cdot 1\dot{2}0\dot{3}$.

171. To express a recurring decimal as a vulgar fraction.

Example 1. $\cdot\dot{5} = \cdot 55555\ldots$

Now, 10 times $\cdot\dot{5} = 5\cdot 5555\ldots$

and $\cdot\dot{5} = \cdot 5555\ldots$

Subtracting, 9 times $\cdot\dot{5} = 5$;

$\therefore \quad \cdot\dot{5} = \frac{5}{9}$.

Example 2. $\cdot 23\dot{4}\dot{5} = \cdot 23454545\ldots$

Now, 10000 times $\cdot 23\dot{4}\dot{5} = 2345\cdot 4545\ldots$

and 100 times $\cdot 23\dot{4}\dot{5} = 23\cdot 4545\ldots$

Subtracting, 9900 times $\cdot 23\dot{4}\dot{5} = 2345 - 23$;

$\therefore \quad \cdot 23\dot{4}\dot{5} = \frac{2345-23}{9900}$.

Example 3. $3\cdot 6\dot{2} = 3\cdot 622222\ldots$

Now, 100 times $3\cdot 6\dot{2} = 362\cdot 2222\ldots$

and 10 times $3\cdot 6\dot{2} = 36\cdot 2222\ldots$

Subtracting, 90 times $3\cdot 6\dot{2} = 362 - 36$;

$3\cdot 6\dot{2} = \frac{362-36}{90}$.

172. Hence we deduce the following rule for reducing a recurring decimal to a vulgar fraction :

For the *numerator* take the integral number formed by all the figures up to the end of the first period, subtracting the integral number formed by the figures (if any) that precede the first period ; for the *denominator* take the number formed by as many nines as there are figures in the period, followed by as many ciphers as there are figures between the decimal point and the first period.

Example 1. Find the vulgar fraction equivalent to $\cdot\dot{3}$.

Process : $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$. *Ans.*

Example 2. Reduce $\cdot 4\dot{5}$ to a vulgar fraction.

Process : $\cdot 4\dot{5} = \frac{45-4}{90} = \frac{41}{90}$. *Ans.*

Example 3. Express $\cdot 04\dot{7}\dot{6}$ as a vulgar fraction.

Process : $\cdot 04\dot{7}\dot{6} = \frac{476-4}{9900} = \frac{472}{9900} = \frac{118}{2475}$. *Ans.*

Example 4. Express $1\cdot 00\dot{2}7\dot{1}$ as a vulgar fraction.

Process : $\cdot 00\dot{2}7\dot{1} = \frac{271}{99900}$. *Ans.*

Example 5. Express $2\cdot 3\dot{7}$ as an improper fraction.

Process : $2\cdot 3\dot{7} = \frac{237-23}{90} = \frac{214}{90} = \frac{107}{45}$. *Ans.*

Example 6. Express $2\cdot 3\dot{7}$ as a mixed number.

Process : $2\cdot 3\dot{7} = 2 + \cdot 3\dot{7} = 2 + \frac{37-3}{90} = 2 + \frac{17}{45} = 2\frac{17}{45}$. *Ans.*

Note. It follows from the rule that $\cdot\dot{9} = \frac{9}{9} = 1$; similarly $\cdot 0\dot{9} = \cdot 1$ and $\cdot 00\dot{9} = \cdot 01$; and therefore $2\cdot\dot{9} = 3$, $2\cdot 3\dot{9} = 2\cdot 4$, $2\cdot 345\dot{9} = 2\cdot 346$; etc. Also $\cdot\dot{9}\dot{9} = 1$, $\cdot\dot{9}9\dot{9} = 1$, $\cdot 2\dot{9}\dot{9} = \cdot 3$; etc.

Therefore when the recurring part contains the figure 9 *only*, the recurring part should be omitted and the preceding figure increased by unity.

EXAMPLES. 103.

Express as vulgar fractions in their lowest terms :

1. $\cdot\dot{6}$.	**2.** $\cdot\dot{1}\dot{8}$.	**3.** $\cdot\dot{1}4285\dot{7}$.	**4.** $\cdot\dot{7}6923\dot{0}$
5. $\cdot2\dot{7}$.	**6.** $\cdot27\dot{2}$.	**7.** $\cdot3\dot{7}\dot{8}$.	**8.** $\cdot03\dot{2}$.
9. $\cdot00\dot{7}8\dot{5}$.	**10.** $\cdot008\dot{2}\dot{3}$.	**11.** $\cdot00106\dot{4}$.	**12.** $\cdot0\dot{8}\dot{1}$.
13. $3\cdot\dot{0}1\dot{3}$.	**14.** $3\cdot4\dot{3}\dot{2}$.	**15.** $7\cdot0\dot{2}\dot{8}$.	**16.** $31\cdot00\dot{7}$.
17. $\cdot5\dot{9}2\dot{5}$.	**18.** $\cdot0\dot{5}$.	**19.** $2\cdot\dot{6}1904\dot{7}$.	**20.** $10\cdot25\dot{6}\dot{7}$.
21. $\cdot0012\dot{3}$.	**22.** $\cdot011\dot{3}\dot{6}$.	**23.** $\cdot00\dot{7}2\dot{9}$.	**24.** $\cdot38\dot{1}4\dot{8}$.
25. $\cdot00\dot{6}7\dot{5}$.	**26.** $\cdot0\dot{2}\dot{4}$.	**27.** $\cdot\dot{0}37\dot{8}$.	**28.** $\cdot2\dot{2}7\dot{3}$.
29. $\cdot00\dot{0}2\dot{5}$.	**30.** $\cdot100\dot{0}\dot{1}$.	**31.** $3\cdot0\dot{0}0\dot{7}$.	**32.** $\cdot02\dot{1}7\dot{7}$.

Reduce to improper fractions in their lowest terms :

33. $3\cdot\dot{6}$.	**34.** $7\cdot\dot{1}\dot{8}$.	**35.** $1\cdot3\dot{4}$.	**36.** $2\cdot7\dot{6}$.
37. $1\cdot07\dot{2}$.	**38.** $3\cdot03\dot{6}$.	**39.** $10\cdot2\dot{7}\dot{5}$.	**40.** $4\cdot0\dot{0}8\dot{6}$.
41. $7\cdot12\dot{3}\dot{0}$.	**42.** $7\cdot6\dot{5}3\dot{1}$.	**43.** $20\cdot45\dot{9}0\dot{0}$.	**44.** $14\cdot0\dot{1}3\dot{1}$.
45. $10\cdot02\dot{2}\dot{7}$.	**46.** $13\cdot94\dot{2}30769\dot{9}$.	**47.** $11\cdot00\dot{1}20\dot{0}$.	**48.** $100\cdot00\dot{1}0\dot{0}$.

49. Prove that $\frac{1}{9}=\frac{\cdot\dot{1}}{1}=\frac{\cdot\dot{2}}{2}=\frac{\cdot\dot{3}}{3}=\frac{\cdot\dot{4}}{4}=\frac{\cdot\dot{5}}{5}=\frac{\cdot\dot{6}}{6}=\frac{\cdot\dot{7}}{7}=\frac{\cdot\dot{8}}{8}$.

50. Prove that $\frac{1}{11}=\frac{\cdot\dot{0}\dot{9}}{1}=\frac{\cdot\dot{1}\dot{8}}{2}=\frac{\cdot\dot{2}\dot{7}}{3}=\frac{\cdot\dot{3}\dot{6}}{4}=\frac{\cdot\dot{4}\dot{5}}{5}=\frac{\cdot\dot{5}\dot{4}}{6}$.

51. Prove that $\frac{1}{13}=\frac{\cdot\dot{0}7692\dot{3}}{1}=\frac{\cdot\dot{1}5384\dot{6}}{2}=\frac{\cdot\dot{2}3076\dot{9}}{3}=\frac{\cdot\dot{3}0769\dot{2}}{4}$.

52. Prove that $\frac{\cdot\dot{1}0\dot{1}}{1}=\frac{\cdot\dot{2}0\dot{2}}{2}=\frac{\cdot\dot{3}0\dot{3}}{3}=\frac{\cdot\dot{4}0\dot{4}}{4}=\frac{\cdot\dot{5}0\dot{5}}{5}$.

Express as non-recurring decimals :

53. $\cdot0\dot{9}$.	**54.** $\cdot367\dot{9}$	**55.** $1\cdot6\dot{9}$.	**56.** $\cdot000\dot{9}$.
57. $\cdot2\dot{9}\dot{9}$.	**58.** $3\cdot\dot{9}\dot{9}$.	**59.** $3\cdot9\dot{9}\dot{9}$.	**60.** $9\cdot\dot{9}9\dot{9}$.

173. Addition and Subtraction of Recurring Decimals.

Rule for Addition : Make the decimals *similar* : add in the usual way and *increase* the last figure in the result by the figure (if any) carried from the first column (to the left) of the period ; then the sum will be a recurring decimal similar to the summands.

Subtraction is effected in exactly the same way, the only difference being that the last figure in the result in this case is diminished (and not increased) by the figure carried.

Example 1. Add together $2{\cdot}3\dot{7}\dot{5}$, $\cdot8\dot{1}7\dot{3}$ and $4{\cdot}31$.

(i)

Process

$$\begin{array}{lll} 2{\cdot}3\dot{7}\dot{5} & = 2{\cdot}37 & \dot{5}7575\dot{7} \\ \cdot8\dot{1}7\dot{3} & = \cdot81 & \dot{7}3173\dot{1} \\ 4{\cdot}31 & = 4{\cdot}31 & \\ \hline & 7{\cdot}50 & 307488 \\ & & \quad\quad\quad 1 \end{array}$$

Ans. $7{\cdot}50\,\dot{3}0748\dot{9}$

(ii)

$$\begin{array}{llll|l} 2{\cdot}3\dot{7}\dot{5} & = 2{\cdot}37 & 575757 & & 57 \\ \cdot8\dot{1}7\dot{3} & = \cdot81 & 731731 & & 73 \\ 4{\cdot}31 & = 4{\cdot}31 & & & \\ \textit{Ans.} & 7{\cdot}50 & \dot{3}0748\dot{9} & & \end{array}$$

Make the decimals similar so that they may recur from the 3rd place and have a period of 6 places (L. C. M. of 2 and 3). It is now clear that there is a recurrence of the same group of decimals after every six places and that the figure 1 has to be carried from each group to the next on the left (since 1, *carried*, 5 and 7 in the first column, to the left, of the period make 13). Hence the sum is $7{\cdot}50\dot{3}0748\dot{9}$.

Example 2. Add together $7{\cdot}63\dot{4}$ and $\cdot8\dot{5}\dot{2}$.

Process :

$$\begin{array}{lll} 7{\cdot}63\dot{4} & = 7{\cdot}63 & \dot{4}\dot{4} \\ \cdot8\dot{5}\dot{2} & = \cdot85 & \dot{2}\dot{5} \\ \hline & 8{\cdot}48 & \dot{6}\dot{9} \quad \textit{Ans.} \end{array}$$

Example 3. Add together $\cdot76\dot{8}$, $\cdot0\dot{7}$ and $1{\cdot}0\dot{3}$.

Process :

$$\begin{array}{lll} \cdot76\dot{8} & = \cdot76 & \dot{8} \\ \cdot0\dot{7} & = \cdot07 & \dot{7} \\ 1{\cdot}0\dot{3} & = 1{\cdot}03 & \dot{3} \\ \hline & 1{\cdot}87 & 8 \\ & & 1 \\ \hline & 1{\cdot}87 & \dot{9} = 1{\cdot}88 \quad \textit{Ans.} \end{array}$$

Example 4. Subtract $\cdot78\dot{3}7\dot{2}$ from $4{\cdot}0\dot{7}\dot{1}$.

Process :

$$\begin{array}{lll} 4{\cdot}0\dot{7}\dot{1} & = 4{\cdot}07 & \dot{1}7171\dot{7} \\ \cdot78\dot{3}7\dot{2} & = \cdot78 & \dot{3}7237\dot{2} \\ \hline & 3{\cdot}28 & 799345 \\ & & \quad\quad\quad 1 \\ \hline & 3{\cdot}28 & \dot{7}9934\dot{4} \quad \textit{Ans.} \end{array}$$

As in addition, first make the decimals similar. It is now clear that there is a recurrence of the same group of decimals after every six places and that the figure 1 has to be carried from each group to the next on the left.

Example 5. Subtract $\cdot8\dot{6}\dot{2}$ from $6{\cdot}74\dot{5}$.

Process :

$$\begin{array}{lll} 6{\cdot}74\dot{5} & = 6{\cdot}74 & \dot{5}\dot{5} \\ \cdot8\dot{6}\dot{2} & = \cdot86 & \dot{2}\dot{6} \\ \hline & 5{\cdot}88 & \dot{2}\dot{9} \quad \textit{Ans.} \end{array}$$

EXAMPLES. 104.

Perform the operations indicated below :

1. $3.7\dot{6}+.0\dot{2}$.
2. $.7\dot{8}\dot{9}+.00\dot{3}$.
3. $1.0\dot{4}+2.0\dot{3}+8.01\dot{7}$.
4. $3.07\dot{2}+3.4+.01\dot{2}\dot{3}$.
5. $5.45+.\dot{6}+.7\dot{1}\dot{2}$.
6. $.0312+.02\dot{3}\dot{1}+.\dot{9}7\dot{6}$.
7. $2.8\dot{2}+.0\dot{3}\dot{4}+.001\dot{4}$.
8. $8.3\dot{1}+.\dot{6}+.00\dot{2}$.
9. $10.0\dot{1}+.000\dot{5}+.\dot{3}$.
10. $7.392+.3\dot{7}+.23\dot{2}$.
11. $.00\dot{7}+.08\dot{2}+.0\dot{1}2\dot{3}$.
12. $1.12\dot{3}+3.\dot{7}\dot{6}+.\dot{4}57\dot{6}$.
13. $1.30103+9.\dot{7}+8.0\dot{9}3\dot{4}$.
14. $.00\dot{3}+.0\dot{0}\dot{3}+.\dot{0}0\dot{3}$.
15. $1.\dot{3}+.0\dot{2}\dot{3}+.\dot{1}23\dot{4}+9.7$.
16. $.004+.\dot{3}\dot{7}+.\dot{2}3\dot{4}+1.\dot{1}$.
17. $7.31\dot{2}347\dot{6}+1.68\dot{7}652\dot{3}$.
18. $.\dot{7}\dot{4}+3.00\dot{1}+2.123\dot{4}$.
19. $72+3.0\dot{1}2\dot{3}+.00123\dot{4}$.
20. $1.3\dot{4}56\dot{3}+2.\dot{6}54\dot{3}$.
21. $3.\dot{1}34\dot{7}+7.0\dot{3}\dot{2}+.0\dot{7}+1.345+.00\dot{7}\dot{9}$.
22. $1.3\dot{7}\dot{6}+.23\dot{7}0\dot{2}+.0001+.\dot{6}+.\dot{3}\dot{7}$.
23. $4.0\dot{3}4\dot{5}+7.234+81+.0\dot{4}56\dot{7}+.0\dot{3}+.\dot{1}\dot{2}$
24. $3.7\dot{6}-.00\dot{7}\dot{2}$.
25. $4.1\dot{3}0\dot{2}-1.0\dot{5}\dot{2}$.
26. $.\dot{4}32\dot{5}-.03\dot{7}6\dot{4}$.
27. $3-.\dot{7}\dot{6}-.\dot{3}2\dot{1}$.
28. $3.46-.072\dot{3}\dot{4}$.
29. $3.4\dot{7}6\dot{8}-1.004$.
30. $7-.23\dot{4}7\dot{6}$.
31. $.9-.00\dot{8}\dot{9}$.
32. $9.46\dot{8}-3.123$.
33. $2.\dot{4}67\dot{9}-.00\dot{3}4\dot{5}$.
34. $1-.\dot{1}0\dot{2}-.4\dot{6}$.
35. $3.\dot{8}97\dot{2}-.00\dot{3}\dot{4}$.
36. $.\dot{7}28\dot{4}-.0\dot{1}2\dot{3}$.
37. $3.\dot{7}\dot{6}-.\dot{1}234\dot{5}$.
38. $.1\dot{2}34\dot{5}-.000\dot{3}\dot{7}$.
39. $789.07\dot{3}\dot{8}-18.0003\dot{2}5\dot{6}$.
40. $30-.376\dot{9}803\dot{4}$.

174. Multiplication and Division of Recurring Decimals.

RULE. Reduce the decimals to vulgar fractions ; find the product or quotient as a vulgar fraction and reduce it back to the equivalent decimal. *But in the case of division, if the dividend and divisor are both recurring decimals, it will be generally convenient to make them similar before reducing to vulgar fractions.*

Example 1. Multiply $.\dot{0}\dot{9}$ by $7.\dot{3}$.

$.\dot{0}\dot{9}\times 7.\dot{3}=\frac{9}{99}\times\frac{73-7}{9}=\frac{1}{11}\times\frac{22}{3}=\frac{2}{3}=\frac{6}{9}=.\dot{6}$. *Ans.*

Example 2. Divide $.\dot{6}$ by $.75$.

$.\dot{6}\div .75=\frac{6}{9}\div\frac{75}{100}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}=.\dot{8}$. *Ans.*

Example 3. Divide $.7\dot{3}\dot{2}$ by $.02\dot{7}$.

$.7\dot{3}\dot{2}\div .02\dot{7}=.7\dot{3}2\dot{3}\div .02\dot{7}\dot{7}=\frac{7250}{9900}\div\frac{275}{9900}$

$=\frac{7250}{275}=\frac{290}{11}=26.\dot{3}\dot{6}$. *Ans.*

EXAMPLES. 105.

Find the value of

1. $.0\dot{3}\times.0\dot{6}$.
2. $4.\dot{8}\times.\dot{2}\dot{4}$.
3. $.\dot{2}\dot{7}\times4.\dot{9}\dot{0}$.
4. $.\dot{1}\dot{2}\times1.\dot{3}$.
5. $2.\dot{4}\times.0\dot{4}$.
6. $7.\dot{6}\times6.\dot{7}$.
7. $.\dot{3}\div.\dot{6}$.
8. $.3\dot{4}\div.003\dot{2}$.
9. $8.0\dot{2}\div.003\dot{4}$.
10. $.3\dot{4}5\dot{6}\div.22\dot{7}\dot{6}$.
11. $3.9\dot{2}\div1.40\dot{3}$.
12. $.\dot{1}4285\dot{7}\div.\dot{1}\dot{8}$.
13. $.0\dot{8}\dot{1}\div.\dot{3}4\dot{6}$.
14. $.0\dot{2}3\dot{4}\div.\dot{2}\dot{8}$.
15. $.3\dot{1}2\dot{3}\div.\dot{0}04\dot{5}$.

175. Complex Fractions involving Recurring Decimals.

Example. Simplify $\dfrac{.\dot{3}\text{ of }\frac{5}{6}}{.5\times.\dot{1}}+\dfrac{.3\dot{5}}{.0\dot{8}}$.

$$\frac{.\dot{3}\text{ of }\frac{5}{6}}{.5\times.\dot{1}}+\frac{.3\dot{5}}{.0\dot{8}}=\frac{\frac{1}{3}\times\frac{5}{6}}{\frac{1}{2}\times\frac{1}{9}}+\frac{\frac{32}{90}}{\frac{8}{90}}=\frac{1}{3}\times\frac{5}{6}\times\frac{2}{1}\times\frac{9}{1}+\frac{32}{90}\times\frac{90}{8}$$

$$=5+4=9. \quad \textit{Ans.}$$

EXAMPLES. 106.

Simplify, giving each answer in decimals,

1. $\dfrac{4.\dot{4}-2.8\dot{3}}{1.\dot{6}+2.\dot{6}2\dot{9}}$.
2. $\dfrac{.00\dot{3}\times.05}{.0022}$.
3. $\dfrac{.125\times(.175\text{ of }.\dot{2}8571\dot{4})}{.00025}$.
4. $\dfrac{6.27\times0.5}{(\frac{1}{2}\text{ of }\frac{3}{4})\times8.36}\div\dfrac{(\frac{1}{4}\text{ of }1\frac{1}{10})\times(.75\text{ of }21.\dot{3})}{(\frac{2}{3}\text{ of }\frac{5}{6})+1.\dot{4}}$.
5. $\dfrac{4.\dot{2}-3.1\dot{4}}{1.3+2.\dot{1}0\dot{2}}$ of $\dfrac{1.3\text{ of }4}{.37\text{ of }8.\dot{8}\dot{1}}$.
6. $\dfrac{1.8\dot{3}+2.0416+.\dot{3}-3\frac{1}{3}}{1.0025+.0625-1\frac{1}{16}}$.
7. $\dfrac{.\dot{1}4285\dot{7}\times.\dot{0}7692\dot{3}}{.\dot{0}1098\dot{9}}+\dfrac{2.7\dot{5}\times11.25}{6.2}$.
8. $\dfrac{3.125}{2.16}$ of $\dfrac{.2\dot{4}}{.125}\div\dfrac{2.\dot{2}}{1.5}$ of $\dfrac{187.5}{3.42}$.
9. $\left\{37+\dfrac{3.7\dot{0}3\dot{7}}{100}\right\}\times.54$.
10. $\dfrac{\frac{5}{8}\text{ of }\frac{18}{25}+\frac{1}{7}\times2.\dot{3}}{3-(\frac{3}{5}+\frac{7}{10})\div2.\dot{3}\dot{6}}$.
11. $\left(\dfrac{.0019}{3.1\dot{6}}\text{ of }\dfrac{4.\dot{4}}{.005}\right)\div\left(\dfrac{8.\dot{8}}{7}\text{ of }\dfrac{4}{5.625}\right)$.
12. $\dfrac{.044\times2.1}{.000035}\div\dfrac{3.\dot{0}7692\dot{3}}{2.\dot{3}\times5.\dot{6}}$.

13. $\dfrac{2\cdot8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot\dot{3}\dot{6}} + \left\{\dfrac{4\cdot\dot{4} - 3\cdot8\dot{3}}{1\cdot\dot{3} + 2\cdot6\dot{2}\dot{9}} \text{ of } 8\cdot2\right\}.$

14. $\dfrac{\cdot175 - \cdot116 \text{ of } \dfrac{1\frac{1}{4}}{3\frac{1}{3}}}{\cdot08\dot{3} \text{ of } \dfrac{1\frac{7}{8}}{2\frac{6}{7}} + \cdot55}.$

15. $\dfrac{\cdot\dot{0}7692\dot{3}}{\cdot\dot{0}3\dot{7}} \times \dfrac{999}{\cdot\dot{0}2\dot{7}} \times \dfrac{\cdot001}{111} \times \dfrac{13}{\cdot\dot{0}0\dot{9}}.$

16. $\dfrac{9\cdot\dot{6}}{3\cdot\dot{3}} \times \dfrac{14\cdot02\dot{3}}{\cdot9} \times 1\dfrac{1}{29} \times \cdot\dot{3} \times 1\cdot\dot{7}4\dot{1} \div \cdot\dot{0}0\dot{6} \times \dfrac{30}{4207}.$

XXVII. DECIMAL MEASURES.

176. ***Example* 1.** Reduce R3·4 to pies.

Process :

$$\begin{array}{r} \text{R}3\cdot4 \\ 16 \\ \hline 54\cdot4a. \\ 12 \\ \hline 652\cdot8p. \end{array} \quad \textit{Ans.}$$

Or, observing that R1 = 192*p*. = (200 − 8)*p*., we may proceed thus :

$$\begin{array}{rcl} \text{R}3\cdot4 & & \\ 200 & & \\ \hline 680\cdot0 & = & \text{R}3\cdot4 \times 200. \\ 27\cdot2 & = & \text{R}3\cdot4 \times 8. \end{array}$$

Ans. 652·8 pies. (By subtraction.)

***Example* 2.** Find the value of 4·135 of £1.

Process : £4·135 — The £4 is not reduced to shillings.
20
s. 2·700 — The 2*s*. is not reduced to pence.
12
d. 8·4

∴ 4·135 of £1 = £4. 2*s*. 8·4*d*.

***Example* 3.** How many rupees, annas and pies are there in ·522 of R5 ?

Process :

$$\begin{array}{r} \cdot522 \\ 5 \\ \hline \text{R}2\cdot610 \\ 16 \\ \hline a.\ 9\cdot76 \\ 12 \\ \hline p.\ 9\cdot12 \end{array}$$

∴ ·522 of R5 = R2. 9*a*. 9·12*p*.

Example 4. Find the value of ·25 of £9. 7*s*. 6*d*.

Process :

First Method : £9. 7*s*. 6*d*. = 2250*d*.

```
        ·25
       2250
        125
        50
       50
12 )  562·50d.
20 )  46s. 10·5d.
     £2. 6s. 10·5d.
```

Second Method :

	£.	s.	d.
·25 of £9 = £2·25 =	2 .	5 .	0
·25 of 7*s*. = 1·75*s*. =		1 .	9
·25 of 6*d*. = 1·50*d*. =			1½
·25 of £9. 7*s*. 6*d*. =	2 .	6 .	10½

∴ ·25 of £9. 7*s*. 6*d*. = £2. 6*s*. 10½*d*.

Or, since ·25 = ¼ ; ∴ ·25 of £9. 7*s*. 6*d*. = ¼ of £9. 7*s*. 6*d*.

```
    £.  s.  d.
4 ) 9 . 7 . 6
   £2 . 6 . 10·5d.   Ans.
```

Example 5. Find the value of ·2$\dot{3}$ of R10. 5*a*.

Process : ·2$\dot{3}$ of R10. 5*a*. = $\frac{7}{30}$ of R10. 5*a*. = etc.

Example 6. Find the value of 2 tons 3 cwt. 2 qr. 8 lb. × ·45.

```
   ·45

   ·90          = 2 tons × ·45.
   20
   18  cwt.
   1·35         = 3 cwt. × ·45
  19·35 cwt.
      4
   1·40 qr.
    ·90         = 2 qr. × ·45.
   2·30 qr.

   1·20
      7
   8·40 lb.     = ·30 qr. × 28.
   3·60         =   8 lb. × ·45.
  12 lb.
```

The value required = 19 cwt. 2 qr. 12 lb. *Ans.*

Example 7. Simplify :

$\frac{3}{8}$ of £2. 7*s*. 11*d*. + $\frac{8}{17}$ of ·1$\dot{8}$ of 1*s*. 3*d*. − $\frac{1}{3}$ of 2*s*. 7*d*.

Process :

	£.	*s*.	*d*.
$\frac{3}{8}$ of £2. 7*s*. 11*d*. = 9*s*. 7*d*. × 3 =	1	8	9
$\frac{8}{17}$ of ·1$\dot{8}$ of 1*s*. 3*d*. = $\frac{8}{17} \times \frac{17}{90}$ of 1*s*. 3*d*. =	0	0	1·$\dot{3}$
	1	8	10·$\dot{3}$
$\frac{1}{3}$ of 2*s*. 7*d*. = $\frac{31d.}{3}$ =	0	0	10·$\dot{3}$
Ans.	£1	8	0*d*.

EXAMPLES. 107.

(*Examples* 1—10 *should be taken orally.*)

Find the number of pies in

1. ·5*a*. **2.** ·75*a*. **3.** ·125*a*. **4.** ·$\dot{3}$*a*. **5.** ·$\dot{9}$*a*.

Find the number of shillings in

6. £0·2. **7.** £0·7. **8.** £·05. **9.** £·75. **10.** £2·55.

Reduce

11. R7·15 to pies. **12.** ·0234375 of R1 to pies.

13. £·134375 to pence. **14.** ·00375 of £1 to farthings.

15. ·03125 of R5 to pies. **16.** ·045 of £7 to farthings.

17. R8·2$\dot{3}$ to pies. **18.** ·0$\dot{7}$ of £5 to pence.

19. ·895 cwt. to ounces. **20.** 3·985 poles to inches.

Express as compound quantities :

21. R7·325. **22.** £3·35. **23.** R2·02.

24. 2·575 of 15*a*. **25.** 3·45 of 16*s*. **26.** ·06 of R13·5.

27. 3·725 of R9·2. **28.** ·032 of 12 yd. **29.** ·234 ton.

Find the value of

30. ·625 of R1. 4*a*. 4*p*. **31.** ·725 of R9. 6*a*. **32.** R9. 2*a*. × 1·35.

33. ·6 of R7. 9*a*. 10*p*. **34.** 3·9 of R11. 9*a*. **35.** ·079 of R35·5.

36. ·256 of £3. 4*s*. 9*d*. **37.** ·1875 of 9*s*. 4½*d*. **38.** ·0625 of 3·6*s*.

39. R3. 3*a*. 8*p*. × ·785. **40.** £6 × ·78125. **41.** 3*s*. 6½*d*. × ·45.

42. 3 md. 7 seers 9 ch. × 3·24. **43.** 2 tons 3 cwt. 2 qr. 8 lb. × ·65.

44. 3 po. 2 yd. 1½ in. × ·725. **45.** 1 da. 3 hr. 3 min. 7 sec. × ·825.

46. 3·$\dot{4}$ of R2. 4*a*. **47.** ·6$\dot{3}$ of 3*s*. 6½*d*. **48.** R7. 9*a*. ÷ ·0$\dot{6}$.

49. R3. 4*a*. 9*p*. ÷ ·422. **50.** £7. 8*s*. 2*d*. ÷ ·044.

51. $11{\cdot}1375$ of R6. 8*a*. $-{\cdot}56$ of R7. 8*a*.

52. ${\cdot}8\dot{3}$ of R2. 8*a*. $+{\cdot}6$ of R4. 11*a*. $+2{\cdot}05$ of R5.

53. ${\cdot}375$ of R9 $+{\cdot}8\dot{3}$ of 10*a*. $-{\cdot}\dot{6}$ of 6*p*.

54. ${\cdot}01\dot{6}$ of R260. 2*a*. 6*p*. $+{\cdot}\dot{3}5\dot{1}$ of R13. 14*a*. $+1{\cdot}0\dot{0}03\dot{3}$ of R7. 14*a*. 3*p*.

55. ${\cdot}03125$ of R2 $+{\cdot}\dot{7}2\dot{9}$ of R$3\frac{1}{12}$ $+{\cdot}\dot{7}2\dot{9}$ of R$3\frac{5}{8}$.

56. £$\cdot634375$ $+{\cdot}025$ of 25*s*. $+{\cdot}325$ of 30*s*.

57. $8{\cdot}71875$ of 8*d*. $+1{\cdot}146875$ of 6*s*. 8*d*. $-{\cdot}0625$ of 1 guinea.

58. $6{\cdot}8\dot{3}$ of £$3{\cdot}867708\dot{3}$ $+5{\cdot}8$ of £$2{\cdot}411458\dot{3}$ $-4{\cdot}375$ of £$1{\cdot}3$.

Arrange in order of magnitude :

59. $\frac{11}{12}$ of R3. 9*a*., ${\cdot}025$ of R100. 10*a*., ${\cdot}\dot{3}\dot{2}$ of R5. 8*a*.

60. ${\cdot}003\dot{4}$ of £1, ${\cdot}256$ of 1*s*., $3\frac{1}{8}$ of 1*d*.

61. What is the sum, ${\cdot}75$ of which is R3. 9*a*. 2*p*. ?

62. $\frac{3}{4}$ of ${\cdot}72$ of a sum of money is 3*s*. 6*d*. ; what is ${\cdot}0\dot{3}$ of the sum ?

63. Simplify $\dfrac{{\cdot}625 \text{ of £}143.\ 12s. + {\cdot}625 \text{ of £}71.\ 16s.}{\frac{5}{8} \text{ of } 5175}$.

64. Simplify ${\cdot}42\dot{6}$ of $\dfrac{3{\cdot}\dot{3}}{{\cdot}08}$ of $\dfrac{{\cdot}3}{{\cdot}\dot{7}3\dot{5}}$ of $\dfrac{{\cdot}147\times4{\cdot}\dot{4}}{11{\cdot}1}$ of £1. 17*s*. 6*d*.

65. Multiply ${\cdot}892$ of R16. 5*a*. 4*p*. by $4{\cdot}678$.

66. Find the value of ${\cdot}\dot{8}5714\dot{2}$ of $2{\cdot}0625$ tons $+{\cdot}\dot{5}7142\dot{8}$ of $3{\cdot}375$ cwt. $+{\cdot}\dot{7}1428\dot{5}$ of $1{\cdot}25$ qr. $+{\cdot}\dot{2}8571\dot{4}$ of $10{\cdot}5$ lb.

67. Find the value of ${\cdot}\dot{0}\dot{9}$ of $1{\cdot}5$ md. $+{\cdot}\dot{2}\dot{7}$ of $2{\cdot}25$ md. $+{\cdot}6\dot{3}$ of $7{\cdot}75$ md. $+{\cdot}4\dot{5}$ of ${\cdot}7$ md.

68. Find the greatest sum of money which is contained in each of ${\cdot}25$ of 5*s*. 6*d*. and ${\cdot}0\dot{5}$ of £1 a whole number of times.

177. The following examples illustrate the *converse* operation :

Example 1. Reduce 1000 pies to rupees.

First Method :

$1000p. = \text{R}\dfrac{1000}{12\times16} = \text{R}\dfrac{125}{24}$

$= \text{R}5{\cdot}208\dot{3}$. *Ans.*

Second Method :

12) 1000 pies

16) $83{\cdot}\dot{3}$ annas

$1000p. = 5{\cdot}208\dot{3}$ rupees.

Example 2. Reduce £1. 3*s*. 6*d*. to the decimal of £1.

First Method : £1. 3*s*. 6*d*. = £1. 42*d*. = £$1\dfrac{42}{12\times20}$ = £$1\dfrac{7}{40}$ = £$1{\cdot}175$;

the decimal $= 1{\cdot}175$.

Second Method : $3s.\ 6d. = £\frac{3\cdot5}{20} = £\frac{\cdot35}{2} = £\cdot175$;

$\therefore$ $£1.\ 3s.\ 6d. = £1\cdot175$.

Example 3. Express $\cdot\dot{3}$ of R1. 3*a*. 6*p*. as the decimal of 4*a*. 10*p*.

$$\text{The decimal} = \frac{\cdot\dot{3} \text{ of R}1\ .\ 3\ .\ 6}{4a.\ 10p.} = \frac{\frac{1}{3}\times 234}{58} = \frac{234}{3\times 58} = \frac{39}{29} = 1\cdot3448\ldots$$

EXAMPLES. 108.

Reduce

1. 3333 pies to rupees.
2. 8446*f*. to pounds.
3. 10000 lb. to tons.
4. 90000 in. to miles.
5. 66656 sec. to days.
6. 39 guineas to pounds.

Express each of the following as a decimal of its *highest* denomination :

7. 7*a*. 9*p*.
8. R3. 10*a*. 3*p*.
9. R5. 5*a*. 5*p*.
10. 8*s*. 6*d*.
11. £1. 3*s*. 8*d*.
12. £7. 6*s*. $4\frac{1}{2}$*d*.
13. 1 md. 15 seers.
14. 3 cwt. $3\frac{3}{8}$ qr.
15. 5 po. 4 yd.
16. 7 da. $5\frac{3}{4}$ hr.
17. 1 ac. 20 yd. 3 ft.
18. $7°.\ 2'.\ 20''$.

In the following examples, reduce the first of the two given quantities to the decimal of the second :

19. R3. 4*a*. 9*p*. ; R5.
20. £7. 10*s*. $4\frac{1}{2}$*d*. ; £10.
21. 9*a*. 4*p*. ; 11*a*. 3*p*.
22. R7. 9*a*. 10*p*. ; R12. 4*a*. 4*p*.
23. 7*s*. 6*d*. ; 15*s*. 7*d*.
24. £3. 10*s*. $9\frac{1}{2}$*d*. ; £6. 2*s*. $4\frac{1}{2}$*d*.
25. $\frac{3}{4}$ of £1. 8*s*. 6*d*. ; £1.
26. $\frac{7}{8}$ of R3. 9*a*. 4*p*. ; R3.
27. $\cdot375$ of R10. 10*a*. 10*p*. ; R3. 13*a*. 3*p*.
28. 9*a*. 8*p*. ; $\cdot3\dot{8}$ of R3. 4*a*.
29. $\cdot35$ of £7. 3*s*. $4\frac{1}{2}$*d*. ; $\cdot0\dot{5}$ of £3.
30. $\cdot00\dot{3}$ of £1 ; $\cdot\dot{7}$ of 9*s*. $4\frac{1}{2}$*d*.
31. $\cdot25$ of 3*a*. 4*p*. ; $\cdot0\dot{6}$ of R3.
32. $2\frac{22}{23}$ of £2. 6*s*. $5\frac{3}{4}$*d*. ; £18. 17*s*. $10\frac{3}{4}$*d*.

33. Express $\frac{5}{8}$ of 12*s*. 6*d*. + $\cdot625$ of 7*s*. 6*d*. − $\cdot5\dot{0}\dot{5}$ of 16*s*. 6*d*. as the decimal of £1.

34. Reduce $\frac{3}{8}$ of R$\cdot05 + \frac{7}{10}$ of 4*a*. + $\frac{3}{8}$ of R1 to the decimal of R$\frac{21}{20}$.

35. Express $\cdot\dot{4}2857\dot{1}$ of £$1\cdot05 + \cdot3\dot{8}$ of $1\cdot5$*s*. as the decimal of £43. 2*s*. 6*d*.

36. Reduce $\cdot\dot{2}4\dot{6}$ of 9*s*. 3*d*. + $\cdot25\dot{9}$ of £1. 5*s*. + $\cdot0\dot{2}$ of £3. 7*s*. 6*d*. to the decimal of $\cdot0\dot{3}$ of £90.

37. Reduce $\cdot062435$ of £100 + $7\cdot4375$ of 10*s*. + $1\cdot35\dot{6}$ of 7*s*. 6*d*. + $2\cdot784$ of $2\frac{1}{2}$*d*. to the decimal of £29. 10*s*. $7\frac{1}{2}$*d*.

38. What decimal of ₨3. 9*a*. must be added to $\cdot0\dot{7}\dot{5}$ of 5*a*. 6*p*. to make the sum equal to 1 anna?

39. What decimal of £6. 10*s*. must be taken from $\frac{3}{4}$ of £9 that the remainder may be £6. 10*s*.?

40. Express £874. 13*s*. 4*d*. × 3·75 as the decimal of £10000.

MISCELLANEOUS EXAMPLES. 109.

1. Give the local value of each of the significant digits in ·02073.

2. Express the difference between $2\cdot\dot{7}\dot{6}$ and 2·76, (i) by a circulating decimal, and (ii) by a vulgar fraction.

3. Express $\frac{1}{8}(3\frac{1}{2}+2\frac{2}{3}-4)$ as a decimal, and $\cdot6+\frac{2}{11}$ of $\cdot025+3\cdot0\dot{6}$ as a vulgar fraction.

4. Reduce $\frac{3}{25}$ of 2·35 ÷ 1000 to a decimal.

5. Find the least number which must be subtracted from the sum of 2·36 and 3·002 that the remainder may be an integer.

6. Find the price of 321 yards of cloth at 11·25 annas per yard.

7. Find the total weight of 324 bags, each 13·75 lb.

8. By what decimal do we divide $3\frac{3}{8}$, if the quotient is 7·5?

9. ₨720 is ·08 of what amount?

10. If the divisor be 2·36 and the quotient ·125 of the divisor, what must the dividend be?

11. Divide 64·09 by 49·3, and arrange the divisor, dividend and quotient in order of magnitude.

12. If the diameter of a pice be 1·025 inches, how many must be placed in contact along a straight line to extend from Calcutta to Hughly, a distance of 24·6 miles?

13. How often will a wheel, 2·75 yards in circumference, turn in a distance of 12·5 miles?

14. A vessel holds 3·256 gallons; how many times can it be filled from a cask of 96 gallons? Will there be any remainder?

15. How many times can you subtract 3·01 from 65·23, and what is the remainder?

16. Express as a decimal the continued product of $\frac{3}{8}$, $\frac{2\frac{1}{3}+1\cdot5}{8\cdot75}$ and $\frac{2\frac{1}{12}}{\cdot\dot{9}}$.

17. Express 21·43 crowns + 18·52 shillings in pence.

18. Subtract 4·42 cwt. from 7·28 tons.

19. Express 2·75 oz. + ·075 cwt. in pounds.

20. Find the rent of 32·25 acres at £1·025 per acre.

21. If the product of ·064 and a certain number be divided by ·00008, the quotient is 3404; find the number.

22. A book containing 219 leaves is 1·34 inches thick; allowing ·06 of an inch for the cover, find to 5 decimal places the thickness of the paper.

23. A roller 4·03 ft. in circumference makes 34·04 revolutions in passing from one end of a lawn to another; what is the length of the lawn?

24. From a rod 2 yards long, portions each ·063 of an inch in length are cut off; how many such portions can be cut off, and what will be the length of the remaining piece?

25. Find a decimal which shall differ from $\frac{17}{21}$ by less than $\frac{1}{10000}$

26. Multiply 9·036 by itself in two lines.

27. Multiply 37·056 by 12·10411 in three lines.

28. Find the least number of articles, costing R2·375 each, that can be purchased for an integral number of rupees.

29. Find the smallest number of articles, costing £2. 6*s*. 2·37*d*. each, that you can buy for an exact number of pounds.

30. *A* did ·025 of a piece of work, and *B* ·825; how much was left to be done?

31. A boy, after giving away ·8 of his pocket-money to one companion, and ·06 of the remainder to another, has 7*a*. 10*p*. left: how much had he at first?

32. A man received $\cdot 3\dot{8}$ of $\cdot 0\dot{3}$ of a property, and sold $\cdot\dot{3}$ of his own share for R350; what would be the value of the whole property at the same rate?

33. A gallon contains 277·274 cubic inches; how many cubic yards are there in 200 bushels?

34. A cubic foot of water weighs 62·35 lb. Avoir.; what would be the error in calculating the weight of 30 cubic feet on the approximate supposition that a cubic foot of water weighs 1000 oz.?

35. *A* is ·75 times as old as *B*, and *C* $\cdot 7\dot{5}$ times as old as *B*; *A* is 15 years old: how old is *C*?

36. Four bells toll at intervals of 1·3, 1·4, 1·5 and 1·6 seconds, beginning together; after what interval will they toll together again?

37. Find the largest sum of money which is contained in £3·75 and £2·125 a whole number of times.

38. Divide ₨50 into two parts such that one part may be $\cdot\dot{6}$ of the other.

39. Divide £52 between *A*, *B*, *C* in such a manner that *B* may receive $\cdot\dot{3}$ of *A*, and *C* $\cdot\dot{3}$ of *B*.

40. Express $\dfrac{8\frac{11}{63}}{\frac{7}{8}\text{ of }2\cdot25}$ of $\dfrac{\cdot1625}{\frac{1}{18}\text{ of }5\frac{4}{7}} \div \left(\dfrac{2}{21}+\dfrac{7}{81}\right)$ as a fraction of $\left\{37+\dfrac{3\cdot7\dot{0}3\dot{7}}{100}\right\}$ of $\cdot54$.

DECIMALIZATION OF MONEY.

178. We have already explained the *general* methods of ascending and descending decimal reductions of simple and compound quantities. Now we propose to discuss some simple rules by which the work may be shortened in the case of decimal reductions of money. Before proceeding to do so, we shall explain briefly *approximate* decimals and also a new notation.

179. Approximate Decimals.—It is often inconvenient, and not always possible, to find an *exact* decimal equivalent to a proposed number. In such cases we may proceed to a few places of decimals and indicate by dots (...) that the work has not terminated. Thus $\frac{22}{23}=\cdot95652\ldots$. If however we wish to *approximate* to the result by terminating our work at any specified place, we should increase the last digit retained by 1 if the first digit rejected be 5 or greater than 5. Thus $\frac{22}{23}=\cdot957$ *correct to three places of decimals* or *to the nearest thousandth*; also $\frac{22}{23}=\cdot9565$ *to four places.*

[The subject of Approximate Decimals will be resumed and more fully discussed in the next Section.]

180. A New Notation.—A modern practice has grown up of using such mixed expressions as $\cdot49\frac{3}{4}$. When fully expressed as a decimal $\cdot49\frac{3}{4}=\cdot4975$. Since $\cdot49=\frac{49}{100}$, so $\cdot49\frac{3}{4}$

$$=\frac{49\frac{3}{4}}{100}=\frac{49\cdot75}{100}=\cdot4975.$$

This notation will be found to be very convenient in converting certain types of vulgar fractions into decimals and *vice versa.*

Example 1. $\frac{3}{14}\left[=\cdot2\frac{2}{14}=\cdot2\frac{1}{7}\right]=\cdot2\dot{1}4285\dot{7}$.

Example 2. $\frac{7}{16}\left[=\cdot4\frac{6}{16}=4\frac{3}{8}\right]=\cdot4375$.

Example 3. $\frac{5}{24}\left[=\cdot2\frac{2}{24}=\cdot2\frac{1}{12}=\cdot206\frac{4}{12}=\cdot206\frac{1}{3}\right]=\cdot206\dot{3}$.

Example 4. $\cdot525\left[=\cdot5\frac{1}{4}=\frac{5\frac{1}{4}}{10}\right]=\frac{21}{40}$.

Example 5. $\cdot 09375\left[=\cdot 09\tfrac{3}{8}=\frac{9\frac{3}{8}}{100}=\frac{75}{8\times 100}\right]=\frac{3}{32}$.

Example 6. $\cdot 89375\left[=\cdot 89\tfrac{3}{8}=\frac{89\frac{3}{8}}{100}=\frac{715}{8\times 100}\right]=\frac{143}{160}$.

Example 7. $\cdot 06875\left[=\cdot 06\tfrac{7}{8}=\frac{6\frac{7}{8}}{100}=\frac{55}{8\times 100}\right]=\frac{11}{160}$.

[The intermediate work within brackets may be easily done mentally.]

It is clear from the above examples that by this notation the exact decimal value of a fraction or the exact fractional value of a decimal may be completely represented at any stage.

181. Decimalization of Indian Money.

(i) To *decimalize* (that is, *to express as the decimal of a rupee*) any number of annas not exceeding 15.

Now, $1a. = R\frac{1}{16} = R\cdot 06\frac{1}{4} = R\cdot 0625$;

$\therefore\ 3a. = R\frac{3}{16} = R\cdot 18\frac{3}{4} = R\cdot 1875$;

also, $4a. = R\frac{4}{16} = R\cdot 24\frac{4}{4} = R\cdot 2500$; $[4a. = R\frac{1}{4} = R\cdot 25.]$

also, $7a. = R\frac{7}{16} = R\cdot 42\frac{7}{4} = R\cdot 4375$;

also, $8a. = R\frac{8}{16} = R\cdot 48\frac{8}{4} = R\cdot 5000$; $[8a. = R\frac{1}{2} = R\cdot 5.]$

and, $15a. = R\frac{15}{16} = R\cdot 90\frac{15}{4} = R\cdot 9375$.

Hence to decimalize annas we get the following rule :

To decimalize to **two** places, *multiply the number of annas by* 6 *and add* 1 *if the number of annas is* 4 *or more ; add* 2 *if the number of annas is* 8 *or more ; add* 3 *if the number of annas is* 12 *or more. Prefix the decimal point.*

To obtain the **third** and **fourth** places, *divide the number of annas by* 4 *and take the decimal part of the quotient for the third and fourth places.*

Note. It should be observed that any number of annas can be expressed as an exact decimal of a rupee. The decimal equivalent consists of not more than four figures.

Example 1. Decimalize *at sight*—9*a.* (i) to two places, (ii) completely.

(i) $9a. = R\cdot 56$. Mental Process : $9\times 6 = 54$, add 2, = 56

(ii) $9a. = R\cdot 5625$. [$\because\ \frac{9}{4} = 2\cdot 25$. We place 25 to the right of 56.]

Example 2. Decimalize *at sight*—12*a*. (i) to two places, (ii) completely.

(i) 12*a*. = R·75. Mental Process : 12 × 6 = 72
add 3
75

(ii) 12*a*. = R·7500 = R·75. [∵ $\frac{12}{4}$ = 3·00.]

(2) *To decimalize any number of pies not exceeding* 11.

Now, 1*p*. = $\frac{1}{12}$*a*. = R$\frac{1}{12 \times 16}$ = R$\frac{1}{192}$ = R·005$\frac{5}{24}$ = R·0052... ;

∴ 3*p*. = R·015$\frac{15}{24}$ = R·0156... ;

also, 5*p*. = R·025$\frac{25}{24}$ = R·0260... ;

and 10*p*. = R·050$\frac{50}{24}$ = R·0520....

Hence we get the following rule :

To decimalize pies to **three** places, *multiply the number of pies by* 5 *and add* 1 *if the number of pies is* 5 *or more ; add* 2 *if the number of pies is* 10 *or* 11. *Place the decimal point three places to the left.*

To obtain the **fourth** decimal place, *multiply the number of pies by* 2 *and place the units' digit of the product in the fourth place.*

Note. The student will observe that pies cannot be expressed as exact decimals of R1 unless their number is 3 or a multiple of 3. But we can get as close an approximation as we like by decimalizing the terminal fraction to the required number of figures.

Example 1. Decimalize *at sight*—9*p*. (i) to 3 places, (ii) 4 places.

(i) 9*p*. = R·046. Mental Process : 9 × 5 = 45
add 1
46

(ii) 9*p*. = R·0468. [∵ 9 × 2 = 18.]

Example 2. Decimalize 14*a*. 9*p*. to three places.

14*a*. = R·875. Mental Process : 14 × 6 = 84
add 3
87. $\frac{14}{4}$ = 3·5.

9*p*. = R·046. Mental Process : 9 × 5 = 45
∴ 14*a*. 9*p*. = R·921. add 1
46

Example 3. Decimalize 11*a*. 7*p*. to four places.

Process : 11*a*. = R·6875
7*p*. = R·0364
∴ 11*a*. 7*p*. = R·7239. R·7239

Note 1. When the result is required correct to the 3rd decimal place, the fourth place should be taken into account. *E. g.*, in *Example* 1, 9*p.*=R·047 correct to three places; in *Example* 2, 14*a.* 9*p.*=R·922 correct to three places; in *Example* 3, 11*a.* 7*p.*=R·724 correct to three places.

Note 2. In decimalizing annas and pies it is not really necessary to get the result to more than three places (corrected), for any decimal of a rupee correct to the third decimal place gives a result correct to the *nearest pie.* Hence the error in the resulting value is less than $\frac{1}{2}$*p.* which for ordinary practical purposes may be left out of account, for there is no coin less than a pie and so a fraction of a pie can neither be taken nor given.

EXAMPLES. 110.

(*Examples* 1—10 *should be taken orally.*)

Decimalize *at sight* (i) to two places, (ii) completely :

1. 3*a.* **2.** 5*a.* **3.** 6*a.* **4.** 11*a.* **5.** 15*a.*

Decimalize *at sight* (i) to three places ; (ii) to four places :

6. 4*p.* **7.** 5*p.* **8.** 8*p.* **9.** 10*p.* **10.** 11*p.*

Decimalize to three places :

11. 6*a.* 2*p.* **12.** 8*a.* 4*p.* **13.** 11*a.* 6*p.* **14.** 12*a.* 9*p.*
15. 13*a.* 10*p.* **16.** 15*a.* 11*p.*

Decimalize to four places :

17. 11*a.* 1*p.* **18.** 14*a.* 5*p.* **19.** 13*a.* 9*p.* **20.** 12*a.* 11*p.*
21. R14. 3*a.* 8*p.*

182. If in any case it is required to decimalize pies, or annas and pies, completely, we may get the result by decimalizing the terminal fraction until the decimal terminates or the recurring figure is reached ; or we may proceed thus :

Decimalize to four places as usual. Then we observe that no decimal part of any number of annas can occur after the fourth place (see Art. 181) and we also observe that

1*p.* = ·0052 08$\frac{1}{3}$.

Hence to get the remaining places, *we multiply the number of pies in the given sum by* ·08$\frac{1}{3}$ *and put down the product after the fourth place.*

Example 1. Decimalize 5*p.* completely.

(i) 5*p.* [=R·026$\frac{1}{24}$=R·02604$\frac{1}{6}$]=R·02604$1\dot{6}$.

Or thus : (ii) 5*p.*=R·0260... to four places ;

∴ 5*p.*=R·02604$1\dot{6}$. [Mental Process : 5×·08$\frac{1}{3}$=·40$\frac{2}{3}$=·41$\dot{6}$.]

Example 2. Decimalize 11*a*. 7*p*. completely.

(i) 11*a*. 7*p*. = R·7239$\frac{7}{12}$ = R·72395$8\dot{3}$.

Or thus : (ii) 11*a*. 7*p*. = R·7239... to four places ;

∴ 11*a*. 7*p*. = R·72395$8\dot{3}$. [Mental Process : 7 × ·08$\frac{1}{3}$ = ·56$\frac{2}{3}$ = ·5$8\dot{3}$.]

Note. The student should remember that when any number of pies is decimalized completely, the forms which always occur towards the end are 41$\dot{6}$, 8$\dot{3}$, 91$\dot{6}$, or 5.

EXAMPLES. 111.

(*Examples* 1—8 *should be taken orally.*)

Decimalize completely :

1. 3*p*.	**2.** 4*p*.	**3.** 6*p*.	**4.** 7*p*.
5. 8*p*.	**6.** 9*p*.	**7.** 10*p*.	**8.** 1*a*. 2*p*.
9. 2*a*. 4*p*.	**10.** 5*a*. 11*p*.	**11.** 8*a*. 6*p*.	
12. 10*a*. 7*p*.	**13.** 9*a*. 10*p*.	**14.** 15*a*. 9*p*.	
15. 15*a*. 11*p*.	**16.** R4. 3*a*. 7*p*.	**17.** R12. 14*a*. 9*p*.	

183. Decimalization of English money.

(i) *To decimalize any number of shillings not exceeding* 19.

Now, 1*s*. = £$\frac{1}{20}$ = £·05 ;

∴ 3*s*. = £$\frac{3}{20}$ = £·15 ;

also 19*s*. = £$\frac{19}{20}$ = £·95.

Hence to decimalize shillings we get the following rule :

Multiply the number of shillings by 5, *and prefix the decimal point.*

Example 1. Decimalize *at sight*—(i) 12*s*., (ii) 15*s*.

(i) 12*s*. = £·6. [∵ 12 × 5 = 60.]

(ii) 15*s*. = £·75. [∵ 15 × 5 = 75.]

(ii) *To decimalize any number of farthings not exceeding* 47.

Now, 1*f*. = $\frac{1}{4}$*d*. = $\frac{1}{4\times12}$*s*. = £$\frac{1}{4\times12\times20}$ = £$\frac{1}{960}$ = £·001$\frac{1}{24}$;

∴ 7*f*. = £·007$\frac{7}{24}$ = £·007 correct to three places ;

so 12*f*. = £·012$\frac{12}{24}$ = £·013 correct to three places ;

also, 24*f*. = (6*d*.) = £·024$\frac{24}{24}$ = £·025 (exactly) ;

also, 36*f*. = £·036$\frac{36}{24}$ = £·037$\frac{1}{2}$ = £·0375 (exactly) ;

and 47*f*. = £·047$\frac{47}{24}$ = £·048$\frac{23}{24}$ = £·049 correct to 3 places.

Hence to decimalize farthings we get the following rule :

To decimalize to **three** places, *put down the number of farthings. If the number of farthings is* 24 (6*d.*) *or more, add* 1. *Place the decimal point* 3 *places to the left.*

To decimalize **correct** to the third decimal place, *see if the terminal fraction is* $\frac{1}{2}$ *or more. When it is so, add* 1.

By observing the terminal fractions in the above examples we get the following alternative rule to decimalize farthings **correct** to three places :

Put down the number of farthings. If the number of farthings is 12 *or more, add* 1. *If the number of farthings is* 36 *or more, add* 2. *Place the decimal point* 3 *places to the left.*

Note 1. The student will observe that any number of shillings can always be expressed as an exact decimal of £1. The decimal equivalent consists of one or two places. But farthings can only be expressed as exact decimals of £1 when their number is 3 or a multiple of 3. But in other cases we can get as close an approximation as we like by decimalizing the terminal fraction to the required number of places.

Note 2. In decimalizing shillings, pence and farthings, it is not really necessary to get the result to more than 3 places (corrected), for any decimal of a pound correct to the third decimal place yields a result correct to the nearest farthing. Hence the error is less than $\frac{1}{2}$*f.* which for practical purposes may be left out of account.

(iii) *To decimalize any number of pence not exceeding* 11.

Reduce pence to farthings and proceed as in the case of farthings.

Example 1. Decimalize *at sight* to three places :

(i) 5*f.* (ii) 30*f.* (iii) 38*f.*

(i) 5*f.* = £·005.

(ii) 30*f.* = £·031. (Adding 1 as the number of farthings is greater than 24.)

(iii) 38*f.* = £·039. (Adding 1 as the number of farthings is greater than 24.)

Example 2. In *Example* 1, decimalize *at sight* the farthings *correct* to three places.

(i) 5*f.* = £·005$\frac{5}{24}$ = £·005.
(ii) 30*f.* = £·031$\frac{6}{24}$ = £·031.
} The terminal fractions are less than $\frac{1}{2}$.

(iii) 38*f.* = £·038$\frac{38}{24}$ = £·039$\frac{14}{24}$ = £·040. [The terminal fraction is greater than $\frac{1}{2}$.]

By alternative rule :

(i) 5*f*. = £·005. [Number of farthings is less than 12.]

(ii) 30*f*. = £·031. [Adding 1 as number of farthings is greater than 12.]

(iii) 38*f*. = £·040. [Adding 2 as number of farthings is greater than 36.]

Example 3. Decimalize *at sight* 14*s*. $4\frac{1}{2}$*d*. to three places.

14*s*. $4\frac{1}{2}$*d*. = £·718.

Mental Process : 14*s*. = £·70
$4\frac{1}{2}$*d*. = 18*f*. = £·018
∴ 14*s*. $4\frac{1}{2}$*d*. = £·718.

Example 4. Decimalize *at sight* 11*s*. $7\frac{1}{2}$*d*. to three places *correctly*.

11*s*. $7\frac{1}{2}$*d*. = £·581.

Mental Process : 11*s*. = £·55
$7\frac{1}{2}$*d*. = 30*f*. = £·031 (Adding 1 as the number of farthings is greater than 12.)
∴ 11*s*. $7\frac{1}{2}$*d*. = £·581

184. By combining the rules for decimalizing shillings and farthings and observing the solutions of the examples given above, we get the following rule for decimalizing any number of shillings, pence and farthings taken together to three places :

Reduce pence to farthings and put down the total number of farthings adding 1 *if the number of farthings is* 24 (6*d*.) *or more. Multiply the number of shillings by* 5 *and put down the units' figure of this product under the tens' figure of the number of farthings. Add together and place the decimal point three places to the left.*

When the result is required *correct* to three decimal places, the only modification necessary in the above rule is this : *Reduce pence to farthings and put down the total number of farthings adding* 1 *if their number is* 12 (3*d*.) *or more ; adding* 2 *if their number is* 36 (9*d*.) *or more.* Then proceed as before. (See alternative rule, Art. 183.)

After a little practice the student will find it easy to work according to the above rules.

Example 1. Decimalize *at sight* 5*s*. $3\frac{3}{4}$*d*. to three places.

5*s*. $3\frac{3}{4}$*d*. = £·265.

Mental Process : $3\frac{3}{4} \times 4 = 15$
$5 \times 5 = 25$
265

Example 2. Decimalize *at sight* 9*s*. $7\frac{1}{4}$*d*. to three places.

9*s*. $7\frac{1}{4}$*d*. = £·480.

Mental Process : $7\frac{1}{4} \times 4 + 1 = 30$ (Adding 1 as pence are over 6*d*.)
$9 \times 5 = 45$
480.

Example 3. Decimalize *at sight* 19*s*. $9\frac{3}{4}$*d*., *correct* to the third decimal place.

19*s*. $9\frac{3}{4}$*d*. = £·991.

Mental Process : $9\frac{3}{4} \times 4 + 2 = 41$ (Adding 2 as farthings are over 36.)
$19 \times 5 = 95$
991.

EXAMPLES. 112.

(*Oral*.)

Read off as decimals of £1 :

1. 3*s*.	**2.** 5*s*.	**3.** 6*s*.	**4.** 8*s*.
5. 11*s*.	**6.** 12*s*.	**7.** 13*s*.	**8.** 17*s*.
9. 4*s*. 6*d*.	**10.** 7*s*. 6*d*.	**11.** 9*s*. 6*d*.	**12.** 10*s*. 6*d*.

Read off as decimals of £1 to three places :

13. $\frac{1}{2}$*d*.	**14.** $\frac{3}{4}$*d*.	**15.** $1\frac{1}{4}$*d*.	**16.** $1\frac{1}{2}$*d*.
17. $1\frac{3}{4}$*d*.	**18.** $2\frac{1}{4}$*d*.	**19.** $2\frac{1}{2}$*d*.	**20.** $2\frac{3}{4}$*d*.

Read off as decimals of £1 (i) to three places ; (ii) *correct* to three places :

21. $3\frac{3}{4}$*d*.	**22.** $4\frac{1}{2}$*d*.	**23.** $5\frac{1}{4}$*d*.	**24.** $5\frac{3}{4}$*d*.
25. $6\frac{1}{4}$*d*.	**26.** $7\frac{3}{4}$*d*.	**27.** $9\frac{1}{4}$*d*.	**28.** $10\frac{1}{2}$*d*.

Decimalize *at sight* (i) to three places ; (ii) *correct* to the third decimal place :

29. 4*s*. $1\frac{1}{2}$*d*.	**30.** 6*s*. $3\frac{3}{4}$*d*.	**31.** 9*s*. $4\frac{1}{4}$*d*.
32. 13*s*. 6*d*.	**33.** 15*s*. $7\frac{1}{4}$*d*.	**34.** 19*s*. $9\frac{1}{4}$*d*.
35. 18*s*. $10\frac{3}{4}$*d*.	**36.** 19*s*. $11\frac{1}{2}$*d*.	**37.** £5. 16*s*. $6\frac{1}{2}$*d*.
38. £20. 0*s*. $8\frac{3}{4}$*d*.	**39.** £16. 0*s*. $9\frac{1}{2}$*d*.	**40.** £18. 18*s*. $11\frac{3}{4}$*d*.

185. It has already been pointed out that in decimalizing shillings, pence and farthings it is not necessary for practical purposes to get the result to more than three places (corrected). But if in any case it is required to decimalize completely, we may get the result by decimalizing the terminal fraction until the decimal terminates or the recurring figure is reached. Or we may proceed thus :

Decimalize to three places (not corrected) as usual. Then we observe that no decimal part of a shilling can occur after the second place and we also observe that

$$1f. = \cdot 001\,|04\tfrac{1}{6}.$$

Hence to get the remaining places after the third, *multiply the number of farthings (including pence reduced to farthings) by* $\cdot 04\frac{1}{6}$ *and place the decimal part of the product after the third decimal place.*

Example 1. Decimalize 14*s.* $1\frac{1}{2}$*d.* completely.

(i) 14*s.* $1\frac{1}{2}$*d.* = £$\cdot 706\frac{1}{4}$ = £$\cdot 70625$.

Or thus : (ii) 14*s.* $1\frac{1}{2}$*d.* = £$\cdot 706$... to three places.

∴ 14*s.* $1\frac{1}{2}$*d.* = £$\cdot 70625$.

[Mental Process : $\cdot 04\frac{1}{6} \times 6 = \cdot \mathbf{25}$.]

Example 2. Decimalize 12*s.* $11\frac{1}{4}$*d.* completely.

(i) 12*s.* $11\frac{1}{4}$*d.* = £$\cdot 646\frac{7}{8}$ = £$\cdot 646875$.

Or thus : (ii) 12*s.* $11\frac{1}{4}$*d.* = £$\cdot 646$... to three places.

∴ 12*s.* $11\frac{1}{4}$*d.* = £$\cdot 646875$.

[Mental Process : $\cdot 04\frac{1}{6} \times 45 = 1\cdot \mathbf{875}$.]

Example 3. Decimalize 16*s.* $6\frac{1}{4}$*d.* completely.

(i) 16*s.* $6\frac{1}{4}$*d.* = £$\cdot 826\frac{1}{24}$ = £$\cdot 82604\frac{1}{6}$ = £$\cdot 826041\dot{6}$.

Or thus : (ii) 16*s.* $6\frac{1}{4}$*d.* = £$\cdot 826$... to three places.

∴ 16*s.* $6\frac{1}{4}$*d.* = £$\cdot 826041\dot{6}$.

[Mental Process : $\cdot 04\frac{1}{6} \times 25 = 1\cdot \mathbf{041\dot{6}}$.]

Note. The student should remember that, as in the case of Indian money, the forms which always occur towards the end are $41\dot{6}$, $8\dot{3}$, $91\dot{6}$, or 5.

EXAMPLES. 113.

(Oral.)

Decimalize *at sight* completely :

1. 12*s.* 3*d.*	**2.** 4*s.* $1\frac{1}{2}$*d.*	**3.** 6*s.* $7\frac{1}{2}$*d.*
4. 12*s.* $10\frac{1}{2}$*d.*	**5.** 8*s.* $4\frac{1}{2}$*d.*	**6.** 10*s.* $2\frac{1}{4}$*d.*
7. 16*s.* $5\frac{1}{4}$*d.*	**8.** 18*s.* $3\frac{3}{4}$*d.*	**9.** 3*s.* $4\frac{1}{4}$*d.*
10. 18*s.* $8\frac{1}{2}$*d.*	**11.** £1. 6*s.* $4\frac{3}{4}$*d.*	**12.** £10. 3*s.* $1\frac{1}{4}$*d.*
13. £23. 13*s.* $2\frac{3}{4}$*d.*	**14.** £30. 9*s.* $4\frac{1}{4}$*d.*	**15.** £15. 12*s.* $11\frac{1}{4}$*d.*

Decimalize *at sight*, correct to five decimal places :

16. £1. 5*s.* 1*d.*	**17.** £2. 6*s.* $4\frac{1}{4}$*d.*	**18.** £4. 6*s.* $1\frac{3}{4}$*d.*
19. £5. 3*s.* $7\frac{1}{2}$*d.*	**20.** £8. 8*s.* $8\frac{1}{4}$*d.*	**21.** £10. 7*s.* $10\frac{1}{2}$*d.*
22. £12. 17*s.* $9\frac{3}{4}$*d.*	**23.** £20. 16*s.* $11\frac{1}{2}$*d.*	**24.** £25. 0*s.* $0\frac{3}{4}$*d.*

186. Converse Process.—*To read off a decimal of £1 to three places, in shillings and pence correct to the nearest farthing.*

The following rule known as the **Five-and-Four** Rule is the most convenient. If more than three figures are given, take the decimal *correct* to the 3rd decimal figure. The student will note that this rule is the *inverse* of the process explained before for decimalizing shillings and pence, with correction for farthings.

Divide first two places by 5—*quotient is shillings. Divide by* 4 *the remainder with the 3rd place brought down (diminished by* 1 *if the number so formed is from* 13 *to* 36, *and diminished by* 2 *if the number so formed is from* 37 *to* 48)—*quotient is pence correct to the nearest farthing.*

Example 1. Evaluate *at sight* £·484 *correct* to the nearest farthing.

£·484 = 9*s.* 8¼*d.*, correct to the nearest farthing.

Mental Process : 48 ÷ 5 = **9**, remainder 3.

(34 *less* 1) ÷ 4 = **8¼**.

Example 2. Evaluate *at sight* £3·494 *correct* to the nearest farthing.

£3·494 = £3. 9*s.* 10½*d.*, correct to the nearest farthing.

Mental Process : 49 ÷ 5 = **9**, remainder 4.

(44 *less* 2) ÷ 4 = **10½**.

Example 3. Evaluate *at sight* £11·221875 *correct* to the nearest farthing.

£11·221875 = 11·222 (correct to 3 places) = £11. 4*s.* 5¼*d.*

Mental Process : 22 ÷ 5 = **4**, remainder 2.

(22 *less* 1) ÷ 4 = **5¼**.

EXAMPLES. 114.

(Oral.)

Read off in £. *s.* *d.* *correct* to the nearest farthing :

1. £0·361.	**2.** £0·561.	**3.** £0·129.	**4.** £0·460.
5. £4·081.	**6.** £6·194.	**7.** £10·219.	**8.** £11·866.
9. £1·943.	**10.** £14·647.	**11.** £20·826.	**12.** £2·73̇.
13. £3·45208.	**14.** £5·16458.	**15.** £10·37188.	**16.** £8·51771.

187. An interesting application of decimalization of money gives a brief method of reducing English money to farthings or pence.

(1) To reduce £. s. d. f. to farthings.

Rule. *Decimalize to three places as usual and omit the decimal point. Multiply by 4, beginning from the 3rd figure from the right, using the 1st and 2nd figures for carrying purposes only. Subtract this result from the former. The resulting answer will give the number of farthings required.*

Example 1. Reduce £165. 15*s*. 4½*d*. to farthings.

Process :	£165. 15*s*. 4½*d*. = 165·768.	165768
		6630
	= 159138 *f*. *Ans.*	159138

Example 2. Reduce £3150. 14*s*. 2¼*d*. to farthings.

Process :	£3150. 14*s*. 2¼*d*. = £3150·711	3150711
		126268
	= 3030443 *f*. *Ans.*	3030443

Multiplication and subtraction can be combined and the result may be put down *at once*. (See Art. 42.)

Example 3. Reduce £4125. 18*s*. 10¾*d*. to farthings.

Process :	£4125. 18*s*. 10¾*d*. = £4125·942	4125942
	= 3960805 *f*. *Ans.*	3960805

Explanation of the Rule.—What we have really done is that we have multiplied the decimalized money by 1000 and subtracted from the product 40 times the same money, *i.e.*, we have multiplied it by (1000 − 40) or 960. £1 = 960*f*.

The explanation of the accuracy of these approximations is that the decimal parts are the same. The verification of this is left to the student as an exercise.

(2) To reduce £. s. d. to pence.

Rule. *Decimalize to three places as usual. Omit the decimal point. Divide by 4, and reject the remainder. Again divide by 100 and reject the remainder. Subtract the second result from the first. The answer will give the number of pence required.*

Example. Reduce £125. 16*s*. 9*d*. to pence.

£125. 16*s*. 9*d*. = £125·837 4) 125837

31459—rem. 1 rejected.

125837 ÷ 100 = 1258—rem. 37 rejected.

= 30201*d*. *Ans.* 30201

The explanation of the rule is left to the student as an exercise.

EXAMPLES. 115.

Reduce to farthings :

1. £312. 10*s*. $4\frac{1}{2}$*d*. 2. £475. 8*s*. $6\frac{1}{4}$*d*. 3. £3125. 12*s*. $5\frac{3}{4}$*d*.
4. £4524. 15*s*. $9\frac{1}{2}$*d*. 5. £975. 18*s*. $10\frac{3}{4}$*d*. 6. £999. 19*s*. $11\frac{3}{4}$*d*.
7. £2135. 17*s*. $10\frac{1}{2}$*d*. 8. £7651. 19*s*. $5\frac{1}{2}$*d*. 9. £7796. 17*s*. $11\frac{1}{4}$*d*.

Reduce to pence :

10. £302. 12*s*. 6*d*. 11. £409. 0*s*. 10*d*. 12. £567. 16*s*. 11*d*.

XXVIII. APPROXIMATION.

188. No measurements can be made with perfect accuracy. When we try to make two pieces of sticks equally long, all that we can ensure is that they shall be of the same length so far as the eye can judge. But if examined by a microscope they may be found to differ by some hundredths or thousandths of an inch. A shopkeeper weighing out a seer of *ghee* only gives approximately a seer ; probably he gives a quantity just a little over or just a little below a seer. Similarly, if a student has found by measurement the length of a line to be 5·368 in., it is probable that the last figure 8, is only a guess-work.

In calculations involving lengths, weights, etc., we have to be content with approximate results, more or less accurate. Measurements made very carefully may be taken to be correct to not more than the nearest thousandth, *i.e.*, correct to not more than the third decimal place. Beyond that all measurements may be suspected.

189. It is often inconvenient, and not always possible, to find an *exact* decimal equivalent to a proposed number. In such cases we may proceed to a few places of decimals and indicate by dots (...) that the work has not terminated. Thus $\frac{22}{23}$ = ·95652... If however we wish to *approximate* to the result by terminating our work at any specified place, we should increase the last digit retained by 1 if the first digit rejected be 5 or greater than 5. Thus $\frac{22}{23}$ = ·957 *correct to three places of decimals* or *to the nearest thousandth* ; also $\frac{22}{23}$ = ·9565 *to four places.*

Note. It will be easily seen that the difference of ·957 and ·95652... is less than the difference of ·95652... and ·956 ; hence ·957 represents ·95652... more accurately than ·956. It may be noticed that the approximate result is less than the actual result when the first figure rejected is less than 5, but greater when not less.

190. Significant Figures.—*Significant figures* are those which in any approximate result express the *number* of units, correct to such unit. The following illustrations will make the meaning clear :

(1) Suppose 15,27,18,463 passengers travelled on Indian Railways in 1929. We may say *roughly* that 15,00,00,000 passengers travelled in that year, meaning thereby 15 **crores** *correct to the nearest* **crore**. Here the figures **15** which express the *number* of units, namely, crores, are *significant*, while the zeroes towards the right which indicate the *magnitude* of the unit are *non-significant*. We also say that the number of passengers was 15,00,00,000 correct to two significant figures. It was 15,27,00,000 correct to four significant figures ; and so on.

(2) Suppose the distance from Calcutta to London is given *approximately* as 7300 miles.

(i) If it is meant to convey that the distance is correct to the nearest *hundred*, then the unit taken is *one hundred* miles and the distance is **73 hundreds** of miles, correct to the nearest **hundred**. On this supposition the figures **73** are significant, while the two ciphers to the right are non-significant.

(ii) If it is meant to convey that the distance is correct to the nearest *ten*, then the unit taken is *ten* miles and the distance is **730 tens** of miles, correct to the nearest **ten**. In this case the figures **730** are significant and the last cipher is non-significant.

(iii) Lastly, if the distance has been measured much more accurately and is correct to the nearest **mile**, then *one mile* is the assumed unit of measurement. Then the distance is 7300 miles correct to the nearest mile, and all the figures **7300** are significant.

(3) Suppose it is given that the length of a line is ·09 of an inch, correct to the second decimal place. This means that the length is 9 *hundredths* of an inch, correct to the nearest *hundredth*. Here the assumed unit is *one-hundredth* of an inch. Therefore the significant digit is **9** ; and the cipher to the left is non-significant.

(4) Again, suppose the length of a line is given to be 3·0 metres, correct to the nearest decimetre. Here the unit taken is one decimetre and the length is 30 decimetres, correct to the nearest decimetre. The figures **30** are both significant.

Note. From the above illustrations we conclude that :

(i) ciphers at the *beginning* of a decimal are always non-significant ;

(ii) ciphers at the *end* of a whole number or decimal may or may not be significant according to the degree of accuracy implied ;

(iii) a cipher preceded and followed by a significant figure is significant ;

(iv) the figures 1, 2,...9, are always significant.

191. Limits of Error.—Suppose the real length of an object is 5·247 in. If the length is taken as 5·24 in., the error is ·007 in., while if it is taken as 5·25, the error is only ·003 in., so that the error in the first case is much greater than in the second case.

Hence it would be better if 5·249, 5·248, 5·247 and 5·246 be taken as 5·25 correct to 2 decimal places; while 5·241, 5·242, 5·243 and 5·244 be taken as 5·24 correct to 2 decimal places. The number 5·245 correct to 2 decimal places may be taken both as 5·24 and 5·25; but it is generally taken as 5·25. Now, if 5·25 correct to 2 decimal places is given, it is clear that the number cannot be less than 5·245 (for 5·244 would be 5·24 correct to 2 decimal places) and is less than 5·255 (for 5·255 would be 5·26 correct to 2 decimal places), that is, the number may have any value lying between 5·255 and 5·245. The number may be as much as ·005 greater than, or as much as ·005 less than, 5·250, *i.e.*, the possible error in the value of the number is ·005 *in excess or in defect*. Therefore the error in taking 5·25 for the decimal lies between $+$·005 and $-$·005, that is, the error is not greater than $+$·005 and not less than $-$·005. This may be expressed thus:

5·25 correct to 2 decimal places $=$ 5·25 $\pm$ ·005.

Hence the **Limits of Error** of a decimal correct to *two* places are $\pm$·005. Similarly, the **Limits of Error** of a decimal correct to *three* places are $\pm$·0005; and so on. The actual error in any particular case may, of course, have any value between these limits.

Note. 5·25 in. "correct to 2 decimal places" is also expressed as "correct to ·01 in.". Similarly, 5·247 in. "correct to 3 decimal places" is also expressed as "correct to ·001 in.". and 5·2 in. "correct to 1 decimal place" as "correct to ·1 in.". And a decimal "correct to the 2nd place" is also expressed as "correct to the nearest hundredth." Similarly, a decimal "correct to the 3rd place" is also expressed as "correct to the nearest thousandth."

Example. To what place of decimals must a decimal of **R**1 be correct so that the resulting value may be correct to the nearest pie?

The error in the resulting value must be less than $\frac{1}{2}p$.

Now, $1p. = \text{R}\frac{1}{12 \times 16} = \text{R}\frac{1}{192} = \text{R}0{\cdot}0052\ldots$

$\therefore \quad \frac{1}{2}p. = \text{R}\frac{0{\cdot}0052}{2} = \text{R}{\cdot}0026\ldots$

Hence an error less than R0·005 may be greater than $\frac{1}{2}p.$, but an error less than R0·0005 will always be less than $\frac{1}{2}p.$

Therefore a decimal of R1, correct to the third place, will always give a result correct to the nearest pie.

Similarly it may be shewn that a decimal of £1, correct to the third place, will give a result correct to the nearest farthing.

Note. The student should observe that the phrases "correct *within* a pie" and "correct *to the nearest* pie" do not mean the same thing, the latter being the closer approximation. In the first case the error is less than ± 1 pie: in the second, less than $\pm\frac{1}{2}$ pie. So also the phrases "correct *within* one thousandth" and "correct *to the nearest* thousandth" are not identical. In the first case the error is less than $\pm$·001; in the second, less than $\pm$·0005.

192. Absolute Error, Relative Error, Percentage Error.

The **absolute error** of an approximation is the actual difference between the approximate value and the true value. The **relative error** of an approximation is the fraction whose numerator and denominator are respectively the absolute error and the true value.

Suppose, in judging a distance of 50 yards we are wrong by 2 yards. Then:

(1) the absolute or actual error 2 yards.

(2) the relative error $= \dfrac{\text{absolute error}}{\text{true value}} = \dfrac{2 \text{ yards}}{50 \text{ yards}} = \dfrac{1}{25}$.

(3) the percentage error $= \dfrac{\text{absolute error}}{\text{true value}} \times 100$

$= \text{relative error} \times 100 = 4.$

193. Addition and Subtraction.—If the decimals whose sum or difference is required are only approximate, that is, correct to a certain number of decimal places, it is evident that their sum or difference will also be approximate. In such cases it may be required that the limits of error of the sum or difference should be indicated.

Example. The numbers 4·36, 5·82, 5·04, and 6·39 are correct to two decimal places. Find their sum and indicate the limits of error.

We know that the limits of error of a decimal correct to two places of decimals are $\pm$·005. The operation of addition may now be performed in the ordinary way. The sums of positive and negative decimals on the right will give respectively the higher and lower limits of error.

$$\begin{array}{r} 4{\cdot}36 \pm {\cdot}005 \\ 5{\cdot}82 \pm {\cdot}005 \\ 5{\cdot}04 \pm {\cdot}005 \\ 6{\cdot}39 \pm {\cdot}005 \\ \hline 21{\cdot}61 \pm {\cdot}020 \end{array}$$

Thus the sum is 21·61 with a possible error in excess or defect not exceeding ·02.

In the process of subtraction of two approximate decimals, we can get the maximum difference by subtracting the *least* possible value of the smaller quantity from the *greatest* possible value of the greater quantity, and the minimum difference by subtracting the *greatest* possible value of the smaller quantity from the *least* possible value of the greater quantity.

194. Multiplication.—When we speak of a decimal as being correct to, say, 2 places of decimals, we mean that it is either greater or smaller than the actual value of the decimal. It is, therefore, evident that when an approximate decimal is multiplied by a number greater than unity the product is, relatively speaking, much greater or much smaller than the product of the real value of the decimal by the same number. In other words, the error is *increased* in the product.

Example 1. If ·123 is known correct to 3 places, find the limits of error in ·123 × 67.

As ·123 is correct to 3 places, the limits of error are $\pm$·0005. Therefore the limits of error in the product ·123 × 67 are ($\pm$·0005 × 67), or $\pm$·0335. The product is, therefore, reliable only to the first place and there is a possible error of $\pm$3 in the second place, while there is absolutely no certainty about the third place.

Again, consider the following cases :

If	5·926 ft. is correct to 3 decimal places,	
then	5·926 ft. × 10 = 59·26 ft.	is correct to 2 decimal places,
	5·926 ft. × 100 = 592·6 ft.	„ „ „ 1 „ place,
and	5·926 ft. × 1000 = 5926 ft.	„ „ „ the nearest foot.

Hence, it is clear that if an approximate decimal known correct to a given number of places is multiplied by a definite number lying between 1 and 10, 10 and 100, 100 and 1000, etc., the number of places to which the product may be considered reliable is diminished by one, two, three, etc. But this proposition will not necessarily hold good if the multiplier is also an approximate decimal correct to a certain number of places, as will be clear from the following example.

Example 2. The decimals 15·64 and 8·17 are correct to two decimal places only. Find the limits of error in their product.

By actual multiplication we find that the product is 127·78 corerct to two decimal places.

Now, the limits of error of a decimal correct to two places are ±·005. Therefore 15·64 lies between 15·635 and 15·645. Similarly, 8·17 lies between 8·165 and 8·175. Hence the true product lies between

15·645 × 8·175, or, nearly 127·90,

and 15·635 × 8·165, or, nearly 127·66.

Hence the result 127·78 is liable to an error of about ±·12.

193. Division of Approximate Decimals.—As division is the *inverse* of multiplication the following results can be easily deduced The actual reasoning is left to the student as an exercise.

(i) If an approximate decimal is divided by any number greater than unity, the error is *diminished* in the quotient.

(ii) If an approximate decimal known correct to a given number of places is divided by a definite number lying between 1 and 10, 10 and 100, etc., the number of places to which the quotient may be considered reliable is increased by one, two, etc. But, as in the case of the corresponding proposition in multiplication, this is not necessarily true if the divisor is also approximate, as will be clear from the following example.

Example. Divide 524·6 by 7·15, the dividend being correct to one and the divisor to two decimal places. Between what limits will the true quotient lie?

The upper limit of the quotient will be obtained by giving to the dividend the *greatest* possible value and to the divisor the *least* possible value, while the lower limit will be obtained by giving to the dividend the *least* possible value and to the divisor the *greatest* possible value.

Now, the limits of error of the dividend are ±·05 and of the divisor, ±·005.

∴ The upper limit of the quotient $=\frac{524·65}{7·145}=73·43$ correct to 2 decimal places and the lower limit of the quotient $=\frac{524·55}{7·155}$ $=73·31$ correct to 2 decimal places.

The true quotient, therefore, lies between 73·43 and 73·31.

EXAMPLES. 116.

(*Examples* 1—43 *should be taken orally.*)

Read off the following numbers correct to the nearest *thousands*; and name the significant figures in your answers:

1. 75829. **2.** 30076. **3.** 509708.

Read off the following correct to four significant figures:

4. 378361. **5.** 735982. **6.** ·520681. **7.** 7·38512.
8. 2·00972. **9.** 2·00023. **10.** ·034071. **11.** ·0090628.

Read off the following numbers correct to 2 decimal places:

12. 5·8762. **13.** 19·3075. **14.** ·0291.
15. ·00353. **16.** 73·0729. **17.** 6·3$\dot{8}$.

How many significant figures are there in each of the following numbers;

18. 79·087. **19.** ·00753. **20.** 3·8710.

Read off the following decimals correct to the nearest thousandth:

21. 1·86427. **22.** ·023748. **23.** 5·40072.
24. 6·31724. **25.** 9·38$\dot{2}$. **26.** ·$\dot{8}$0$\dot{9}$.

Read off the following lengths to the nearest ·01 of a foot:

27. 3·14159 ft. **28.** ·00789 ft. **29.** ·0076 ft.

30. Read off 3456792 correct to the nearest *hundred*, and 80057123 correct to the nearest *thousand*.

31. Find the approximate value of 3·9281 (1) correct to the nearest *unit*, (2) correct to the nearest *tenth*, (3) correct to the nearest *hundredth*.

Name the significant digits in the following:

32. Calcutta to Delhi, 890 miles, to the nearest 10 miles.

33. Cawnpore to Allahabad, 120 miles, to the nearest mile.

34. Length of Equator, 24900 miles, to the nearest 100 miles.

35. If 1·01 were written for 1·001, what error is made ?

36. What error is made in writing 1 for ·998 ? What is the error per cent. ?

37. What error is made in writing 10 for 9·997 ? What is the error per cent. ?

The following quantities are correct to the *second decimal places* : between what limits must the actual quantities lie ?

38. R5·63. **39.** £6·27. **40.** 3·47 in.

41. 50·08 cm. **42.** 24·80 lb. **43.** 36·63 tons.

In expressing the following quantities correct to *three significant digits* find (i) the absolute error, and (ii) the relative error.

44. ·0090015 in. **45.** 849·83 miles. **46.** R207·74.

47. Express £15. 3*s*. 9¼*d*. as pounds and decimals of a pound correct to four significant figures.

48. Express 6 yd. 0 ft. 10½ in. as yards and decimals of a yard correct to three significant figures.

49. Find a decimal that is within ·001 of $\frac{1}{7}$.

50. Find a decimal that is within $\frac{1}{100000}$ of $\frac{355}{113}$.

51. Add together 53·078 in., 37·132 in., and 7·364 in., each length being correct to 3 decimal places. What is the greatest possible error in excess or defect ?

52. Add together 3·192, 0·803, 1·014 and 5·000. If each decimal is correct to 3 decimal places, to how many decimal places can the sum be trusted ?

53. Subtract 6·05 from 8·62. If each decimal is correct to 2 decimal places, between what limits will the remainder lie ?

54. The numbers 3·47, 2·90 and 3·58 are each correct to two decimal places. Between what limits will their sum lie ?

55. Find approximately the limits of error in the product of 80·42 and 5·16, each factor being true to the second decimal place.

56. A boy writes 4 yd. 2 ft. for 4 yd. 1 ft. 10 in. ; what decimal place will be affected if these lengths are stated as yards and decimals of a yard ? Will the results be affected if they are expressed correct to the third significant figure ?

XXIX. CONTRACTED METHODS.

196. We have seen in the previous Section that it is unnecessary to give a result correct to more than a specified number of decimal places. Hence, in order to get the result correct to the required number of decimal places, we reject unnecessary figures and employ contracted methods.

197. Addition and Subtraction.—To find an approximate answer, correct to a certain decimal place, two additional places are ordinarily required in addition and subtraction for the following reasons :—

(1) the carrying figure in the first additional place may alter the required place ;

(2) the carrying figure in the second additional place may alter the figure in the first additional place from below 5 to above 5, or from above 5 to below 5 and thus increase or diminish the required place by 1.

Example 1. Find the sum of ·23674, ·31782, 1·62, and 7·05156 correct to two decimal places.

(1) Long Method.

```
 ·23674
 ·31782
1·62
7·05156
-------
9·22612
```

The required answer is 9·23.

(ii) Omitting all additional places.

```
 ·23
 ·31
1·62
7·05
----
9·21
```

The error in the second decimal place is 2. It is due to the omission of the carrying figure from the third decimal place.

(iii) Taking one additional place.

```
 ·236
 ·317
1·62
7·051
-----
9·224
```

The answer is 9·22. The error in the second decimal place is 1. It is due to the omission of the carrying figure from the fourth decimal place to the third.

(iv) Taking two additional places.

```
 ·2367
 ·3178
1·62
7·0515
------
9·2260
```

The answer is 9·23. This is correct.

Example 2. Find the difference between $\cdot 6\dot{3}2\dot{1}$ and $\cdot 00\dot{8}$ correct to five places of decimals.

Taking two additional decimal places,

·63213	21
·00888	88
·62324	3... = ·62324. *Ans.*

Example 3. Find the sum of $72\cdot\dot{6}\dot{5}$, $8\cdot79\dot{0}\dot{8}$ and 4·02, true to 6 significant figures.

72·6565	65
8·7908	90
4·02	
85·4734	09... = 85·4735. *Ans.*

Example 4. Find the value of $1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \text{.........}$ correct to 3 places of decimals.

$$1 = 1\cdot000|000$$

$$\therefore \quad \frac{1}{1.2} = \frac{1}{2} = \cdot500|000$$

$$\therefore \quad \frac{1}{1.2.3} = \frac{\cdot 5}{3} = \cdot166$$

$$\therefore \quad \frac{1}{1.2.3.4} = \frac{\cdot166666}{4} = \cdot041|666$$

$$\therefore \quad \frac{1}{1.2.3.4.5} = \frac{\cdot041666}{5} = \cdot008|333$$

$$\therefore \quad \frac{1}{1.2.3.4.5.6} = \frac{\cdot008333}{6} = \cdot001|388$$

$$\frac{1}{1.2.3.4.5.6.7} = \frac{\cdot001388}{7} \qquad \cdot000|198$$

$$\frac{1}{1.2.3.4.5.6.7.8} = \frac{\cdot000198}{8} \qquad \cdot000|024$$

$$\frac{1}{1.2.3.4.5.6.7.8.9} = \frac{\cdot000024}{9}$$

and ∴ the expression $= 1\cdot718|2...$

$= 1\cdot718$, to 3 places.

Here we stop at $\frac{1}{1.2.3.4.5.6.7.8.9}$, as in the decimals equivalent to the succeeding fractions, the first six figures will be zeroes.

Example 5. Find the value of

$1-\frac{1}{1.3}+\frac{1}{1.3.5}-\frac{1}{1.3.5.7}+\ldots\ldots$ correct to 3 places of decimals.

[Add up the positive and the negative terms separately to 5 places of decimals and then subtract one sum from the other.]

		Positive terms	Negative terms.
$1=1$		$=1\cdot000\,00$	
$\therefore \frac{1}{1.3}$	$=\frac{1\cdot00000}{3}$		$=\cdot333\,33$
$\therefore \frac{1}{1.3.5}$	$=\frac{\cdot33333}{5}$	$=\cdot066\,66$	
$\therefore \frac{1}{1.3.5.7}$	$=\frac{\cdot06666}{7}$		$=\cdot009\,52$
$\therefore \frac{1}{1.3.5.7.9}$	$=\frac{\cdot00952}{9}$	$=\cdot001\,05$	
$\therefore \frac{1}{1.3.5.7.9.11}$	$=\frac{\cdot00105}{11}$		$=\cdot000\,09$
		$1\cdot067\,71$	$\cdot342\,94$
		$\cdot342\,94$	
		$\cdot724\,77$	
		1	

Ans. $\cdot725$, correct to 3 places.

EXAMPLES. 117.

1. Find the value of $\cdot0312+\cdot02\dot{3}\dot{1}+\cdot\dot{9}7\dot{6}$ correct to four places of decimals.

2. Find the sum of 72, $3\cdot0\dot{1}2\dot{3}$ and $\cdot00\dot{1}23\dot{4}$ correct to three places of decimals.

3. Find the difference between $\cdot432\dot{5}$ and $\cdot0\dot{3}76\dot{4}$ correct to four places of decimals.

Find the value, correct to 2 places of decimals, of

4. $1+\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots$

5. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$

6. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

7. $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$

Find the value, correct to 3 places of decimals, of

8. $1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots$

9. $1+\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\ldots$

Find the value, correct to 5 places of decimals, of

10. $\cdot 25+(\cdot 25)^3+(\cdot 25)^5+\dots$ **11.** $1+\frac{1}{1.3}+\frac{1}{1.3.5}+\frac{1}{1.3.5.7}+\dots$

12. $\frac{1}{1}\cdot\frac{1}{2^2}+\frac{1}{2}\cdot\frac{1}{2^4}+\frac{1}{3}\cdot\frac{1}{2^6}+\frac{1}{4}\cdot\frac{1}{2^8}+\dots$

[First express as decimals $\frac{1}{2^2}, \frac{1}{2^4}, \frac{1}{2^6}, \dots$, then divide the results respectively by 1, 2, 3,..., and add.]

13. $\frac{1}{1}\cdot\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}+\frac{1}{7}\cdot\frac{1}{5^7}+\frac{1}{9}\cdot\frac{1}{5^9}+\dots$

Find the value, correct to 3 places of decimals, of

14. $1-\frac{1}{10}+\frac{1}{10^2}-\frac{1}{10^3}+\dots$ **15.** $1-\frac{1}{1.2}+\frac{1}{1.2.3}-\frac{1}{1.2.3.4}+\dots$

16. $\frac{1}{1}\cdot\frac{1}{5}-\frac{1}{2}\cdot\frac{1}{5^2}+\frac{1}{3}\cdot\frac{1}{5^3}-\frac{1}{4}\cdot\frac{1}{5^4}+\dots$

Note. The following algebraical method may be used with great advantage when each term of the series is the product of the preceding term by a constant proper fraction, positive or negative.

Example 1. Find the value, correct to 4 decimal places, of

$$1+\frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\dots$$

Let S denote the sum of the series. Then

$$S=1+\frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\dots;$$

multiplying both sides by $\frac{1}{25}$ (the constant multiplier), we have

$$\frac{1}{25}S=\frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\dots$$

Hence, by subtraction, $S-\frac{1}{25}S=1$,

or $\frac{24}{25}S=1$;

$\therefore$ $S=\frac{25}{24}=1\cdot 0417$. *Ans.*

Example 2. Find the value, correct to 3 decimal places, of

$$1-\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\dots$$

Let S denote the sum of series. Then

$$S=1-\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\dots;$$

multiplying both sides by $-\frac{1}{2}$ (the constant multiplier), we have

$$-\tfrac{1}{2}S = -\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\dots$$

Hence, by subtraction, $S+\frac{1}{2}S=1$,

or $\frac{3}{2}S=1$;

$\therefore$ $S=\frac{2}{3}=\cdot 667$. *Ans.*

EXAMPLES. 118.

Find the value, correct to 5 places of decimals, of

1. $1+\frac{1}{8}+\frac{1}{8^2}+\frac{1}{8^3}+\dots$

2. $1+\frac{1}{50}+\frac{1}{50^2}+\frac{1}{50^3}+\dots$

3. $1-\frac{1}{6}+\frac{1}{6^2}-\frac{1}{6^3}+\dots$

4. $1-\frac{1}{20}+\frac{1}{20^2}-\frac{1}{20^3}+\dots$

CONTRACTED MULTIPLICATION.

198. The following rule will shorten the process of multiplication when the product is required only to a certain number of decimal places.

First express the multiplier (if necessary) so that it may have only one figure before the decimal point, and make the corresponding change in the multiplicand. This can be done by moving the decimal point in the multiplier to the right or left and moving the decimal point in the multiplicand also an equal number of places to the left or right. Begin the multiplication in the *reverse* order, that is, from the left hand figure of the multiplier using the last figure rejected in the multiplicand for carrying. Do not set down from this product with the last figure rejected but carry its *nearest ten** to the next. Go on rejecting from the multiplicand the last figure from the right after each successive multiplication, using them, however, for carrying purposes.

Since in multiplication we have to add the various partial products, to find the complete product, it is necessary to work with two additional places so that the result may be sufficiently accurate. Other unnecessary figures may be crossed out. The method will be fully illustrated by the following examples.

---

*That is, carry 1 if the product is a number from 5 to 14; carry 2 if it is from 15 to 24; carry 3 if it is from 25 to 34; etc.; if the product is 4 or less, we ignore it.

Example 1. Multiply 123·052348 by ·7153286 correct to 2 decimal places.

In order to get one figure before the decimal point in the multiplier, we multiply it by 10, and to keep the product unaltered we divide the multiplicand by 10.

Hence 123·052348 × ·7153286 = 12·3052348 × 7·153286.

```
12·3052|348 × 7·153286
86·1366|
 1·2305
  ·6153
    369
     25
     10
      1
-------
88·0229|
```

Work to 2+2, *i.e.*, 4 decimal places.

Draw a line below the multiplicand and a barrier line after 4 decimal places, figures beyond which, *i.e.*, 348 are unnecessary and are, therefore, crossed out, but the figure 3 is multiplied in order to see what figure should be carried. 7 × 3 = 21, 1 is not written but 2 is carried. 7 × 2 = 14, 14 + 2 = 16; the figure 6 is written down and 1 is carried. 7 × 5 = 35, 35 + 1 = 36; the figure 6 is written down, and so on. The first line goes immediately below the multiplicand.

Before multiplying by 1, the second figure of the multiplier, the last figure 2 of the multiplicand is rejected. There is nothing to carry as the product of 1 and 2 (rejected) is 2, which is less than 5. The product of 1 and the multiplicand is then placed just below the first line.

Again from the multiplicand the last figure 5 is rejected. But this cannot be neglected for 5 × 5 = 25; therefore 3 is carried. 5 × 0 = 0; 0 + 3 = 3; the figure 3 is written down. The product is then placed in the next line.

[Proceeding thus, it is clear that each succeeding product comes one place further to the right, *i.e.*, one place further beyond the barrier line: therefore each time one figure can be rejected from the multiplicand and considered only to find carrying figure.]

Add the partial products. Thus the product = 88·02 correct to two decimal places.

Note 1. The method explains itself as the work in full given below will show. The figures in italics are those that have been omitted in the contracted work. The work in full will also enable the student to visualize the amount of labour saved by the contracted method.

Full work.

12·3052	348 × 7·153286	
86·1366	*436*	product by 7,
1·2305	*2348*	.... ·1,
·6152	*61740*	·05,
·0369	*157044*	·003,
·0024	*6104696*	·0002,
·0009	*84418784*	·00008,
·0000	*738314088*	·000006,
88·0228	*638315528*.	7·153286.

Note 2. As in the case of ordinary multiplication, it is convenient to take as the multiplier the number which has the *fewer effective* digits. The form of the multiplier when it has only one integral figure is called the **standard form.** It should be observed that when the multiplier is brought to the standard form and the compensating change is made in the multiplicand, the decimal points move in *opposite* directions.

Note 3. As the most common mistakes in working with decimals arise from the misplacing of the decimal point, the student would do well, before proceeding to find the product, to make a *rough estimate* of the answer, so that any large error due to misplacement of the decimal point may be readily detected.

Note 4. In many cases it is sufficient to work to only one place more than the required number of decimal places.

***Example* 2.** Multiply 29·82 by ·00727 correct to 4 decimal places.

[*Rough Estimate.* The product should be less than ·29.]

Here 29·82 × ·00727 = ·029820 × 7·27.

```
·0298 2 | 0 × 7·27
·208 7 4
   5 9 6
   2 0 9
·216 7 9
```

∴ The product correct to 4 decimal places is ·2168.

199. After a little practice it should be easy for the student to perform the multiplication by taking the digits of the multiplier from left to right and without placing it under the multiplicand. But if it is still found inconvenient the following process may be used.

Reverse the multiplier itself. Strike out the decimal point and place it under the multiplicand so that the units' figure in its altered form may fall just to the left of the barrier line under the last retained figure of the multiplicand. Now multiply from the right. The remaining work is the same as explained before.

It should be clearly understood that this process is only a variation of the first and no new principle is involved.

Example 1. Multiply ·13005 by 25·6957 correct to 3 places of decimals.

[*Rough Estimate.* The product should be less than 26 × ·13 or 3·38.]

We take ·13005 as the multiplier as it contains fewer effective digits.

Here 25·6957 × ·13005 = 2·56957 × 1·3005.

(i) Taking digits of multiplier in reverse order.	(ii) Reversing digits of multiplier itself.
	2·56957
2·56957 × 1·3005	50031
2·5696	2·5696
·7709	·7709
13	13
3·3418	3·3418

∴ The product correct to 3 decimal places is 3·349.

Example 2. Find the value of 80·6251 × 0·6317, correct to three significant digits.

[*Rough Estimate.* The product should be a little more than 80 × ·6 or 48·0. It is clear, therefore, that in the integral part of the product there will be two figures, and as the product is required correct to three significant figures, it is to be correct to one decimal place.]

Now, 80·6251 × 0·6317 = 8·06251 × 6·317.

(i)	(ii)
	8·06|251
8·06|251 × 6·317	7136
48·37	48·37
2·42	2·42
8	
6	6
50·93	50·93

The product correct to 3 significant digits = 50·9. *Ans.*

Note. The method of finding approximate products when recurring decimals are involved is the same, for we may extend the recurring decimals to the required number of places.

For example, if we have to multiply $0{\cdot}5\dot{7}\dot{3}$ by $30{\cdot}52$, we write the product in the form $5{\cdot}737373\ldots \times 3{\cdot}052$, and then proceed as before.

CONTRACTED DIVISION.

200. The following rule will shorten the process in division when the quotient is required to be correct only to a certain number of decimal places.

First express the divisor (if necessary) so that it may have only one figure before the decimal point, and make the corresponding change in the dividend, by moving the decimal point both in the divisor and the dividend an equal number of places to the right or left as the case may be; and determine by inspection (or by taking one step in the ordinary way), how many figures there will be in the integral part of the quotient. In the divisor retain (from the left) one more than the number of figures there are to be in the whole quotient—integral part as well as decimal; and in the dividend keep as many figures as are needed to take the first step of division. Proceed one step with this new divisor, but to the product of its first figure by the quotient-figure, carry the *nearest ten* from the preceding figure. Instead of bringing down a figure to the remainder, strike off another figure from the right of the divisor, and proceed as before, until no figure is left in the divisor, taking care, however, when multiplying, to use the figure last rejected for the purpose of carrying number, if any.

If the number of figures in the divisor be less than the number of quotient figures to be obtained, proceed in the ordinary way until the number of quotient-figures still to be obtained is one less than the number of figures in the divisor. As soon as this happens, instead of bringing down a figure to the remainder, strike off a figure from the end of the divisor, and then proceed as in the preceding case. The difficulty may also be met by affixing to the divisor as many ciphers as necessary to be rejected successively as already stated.

When, by inspection, it is found that there is no integral part of the quotient and there are ciphers just after the decimal point in the quotient, subtract the number of ciphers from the total number of decimal places required, and take the remainder as the number of decimal places required in the quotient. Then proceed as above.

Since addition does not enter into the operation, two additional places need not be considered, only *one* additional place beyond the required figure would be sufficient.

Note. The student should remember that when the divisor is brought to the standard form, and the compensating change is made in the dividend, the decimal points move in the *same* direction.

Example 1. Divide 67·31429 by ·414323 correct to the first decimal place.

In order to get one figure before the decimal point in the divisor, we multiply it by 10, and to keep the quotient unaltered we multiply the dividend also by 10. Hence the quotient

$$=\frac{67{\cdot}31429}{{\cdot}414323}=\frac{673{\cdot}1429}{4{\cdot}14323}.$$

By inspection we see that the quotient will contain *three* integral figures. These together with one decimal figure, and one additional figure, make *five* figures to be obtained by division. Retain only 5 figures in the divisor from the left and cross out 3.

```
              162·47|
4·14323 ) 673·14|29
          414 32|
          258 82|
          248 59|
           10 23|
            8 29|
            1 94|
            1 65|
              29|
              29|
```

Since no figures are required in the dividend after hundredths, draw a barrier line after hundredths and cross out 29.

(6 hundreds ÷ 4) gives 1 hundred. There is nothing to carry as the product of 1 and 3 (crossed out) is 3, which is less than 5. The product of 1 and the divisor is then placed below the dividend.

Again (25 tens ÷ 4) gives 6 tens. If the divisor 4·1432 were kept, work would pass beyond the barrier line. Therefore reject 2 and have 4·143 as divisor. 6 × 2 = 12, carry 1. 6 × 3 = 18, and 1, 19. Write down 9, carry 1, and so on.

(10 units ÷ 4) gives 2 units. To keep the work to the left of the barrier line, reduce the divisor to 4·14.

(19 tenths ÷ 4) gives 4 tenths. Divisor is now 4·1.

(29 hundredths ÷ 4) gives 7 hundredths. Divisor is now 4.

Therefore the quotient correct to 1 decimal place is 162·5.

Example 2. Divide 2·9431542 by 325·3 correct to six decimal places.

$$\text{The quotient} = \frac{2·9431542}{325·3} = \frac{·029431542}{3·253}.$$

By inspection we see that in the quotient two ciphers will follow the decimal point. Hence (4+1) or 5 figures must be got by division. This requires in the divisor five figures. Since divisor has four figures, one cipher may be added to the right. Now divide as before.

```
              ·0090475
3·2530 )      ·0294315|42
               292770
                 1545
                 1301
                  244
                  228
                   16
                   16
```

The quotient correct to six decimal places is ·009048.

EXAMPLES. 119.

(Answers should be given *correct* to the required number of places.)

Multiply

1.	21·1324 by ·345721	to 3 decimal places.
2.	·32504 by 13·0254	to 3 ... ...
3.	·453 by ·01694	to 4 ... ...
4.	375·76843 by 3·14159	to 1 ... ... place.
5.	71·032751 by 2·6719238	to 2 ... ... places.
6.	65·00763 by ·9876	to 2 ... ...
7.	·03281674 by 234·781	to 3 ... ...
8.	·0008127 by 483·2716	to 3 ... ...
9.	4·562 by ·07408	to 5

10.	6·2438 by $3{\cdot}83\dot{0}6$	to 5 decimal places.
11.	4·683 by $14{\cdot}\dot{2}9\dot{3}$	to 1 ... ... place.
12.	$1{\cdot}8\dot{2}35\dot{7}$ by ${\cdot}0\dot{7}8\dot{5}$	to 4 ... ... places.
13.	·01385 by 61·37	retaining 4 ... ...
14.	·346875 by ·119808	retaining 4 ... ...
15.	32·34 by ·32056	correct to 3 ... ...
16.	·342 by $3{\cdot}2\dot{5}\dot{3}$	correct to 3 ... ...
17.	·00926347 by 280·435	correct to 4 ... ...

18. 421·619 by ·547 correct to the nearest integer.

19. 70870096 by 404 correct to the nearest million.

Divide

20.	76·2307 by 47·12345	to 3 decimal places.
21.	3·3706 by 9·7846	to 3 ... ...
22.	32·791 by 26·67	to 3 ... ...
23.	378·325 by 30·732	to 2 ... ...
24.	36·7802 by 312·32	to 3 ... ...
25.	728·389 by 3·76	to 4 ... ...
26.	3892·762 by 7·343	to 5 ... ...
27.	23·78934 by ·00289	to 2 ... ...
28.	13·2346891 by ·01234031	to 2 ... ...
29.	132·405678 by ·000122134	to 2 ... ...
30.	·5 by 76·91342	to 4 ... ...
31.	·0003738028 by ·0476	to 5 ... ...
32.	$3{\cdot}7\dot{2}\dot{5}$ by $13{\cdot}\dot{2}3\dot{4}$	to 3 ... ...
33.	$1{\cdot}\dot{8}235\dot{7}$ by ${\cdot}0\dot{7}8\dot{5}$	to 3 ... ...
34.	·32165 by ·35216	to 4 ... ...
35.	1·59587 by 4·3062	to 3 ... ...

201. In order to find the continued product of three quantities, first we should find the product of the first two quantities and then the product of this product and the third quantity.

***Example* 1.** Find the continued product of 12·7053, ·003725 and 4·532 correct to three decimal places.

[*Rough Estimate.* The product should be a little less than $13 \times \frac{4}{1000} \times 5$ or ·26.]

We take for the multiplicand here 12·7053 which contains the largest number of significant digits. Reducing first the multipliers to the standard form and making the compensating change in the multiplicand we have

$$12·7053 \times ·003725 \times 4·532 = ·0127053 \times 3·725 \times 4·532.$$

Here the product of the multipliers is roughly 4×5 or 20.

Hence the multiplicand will in the final result be multiplied by a number lying between 10 and 100. We therefore retain in the multiplicand *two* decimal figures beyond the number required in the answer, that is, $(3+2)$ or 5 decimal places.

We first find the product $12·7053 \times ·003725$, that is, $·0127053 \times 3·725$ to $(3+2)$ or 5 decimal places.

(i)

```
·0 1 2 7 0 |53 × 3·725
·0 3 8 1 2 |
     8 8 9 |
       2 5 |
         6 |
-----------
·0 4 7 3 2 |
```

(ii)

```
·01270|53
  5273|
------
·03812|
·00889|
·00025|
·00006|
------
·04732|
```

Next we find the product of ·04732 and 4·532 to 5 decimal places.

(i)

```
0 4 7 3 2 | × 4·532
·1 8 9 2 8|
  2 3 6 6 |
    1 4 2 |
        9 |
----------
·2 1 4 4 5|
```

(ii)

```
·04732
  2354
------
·18928
·02366
·00142
·00009
------
·21445
```

∴ the product correct to three decimal places is ·214.

***Example* 2.** Find the value of $\dfrac{0·34567 \times 0·73456}{0·67345}$ correct to 4 decimal places. [C. U. 1918.]

[*Rough Estimate.* The answer should be about $\dfrac{·3 \times 7}{7}$ or ·3.]

Here $0{\cdot}34567 \times 0{\cdot}73456 = {\cdot}034567 \times 7{\cdot}3456$. We first find this product to $4+2$, *i.e.*, 6 decimal places.

$$
\begin{array}{l}
{\cdot}034567 \times 7{\cdot}3456 \\
{\cdot}241969 \\
\quad 10370 \\
\quad\; 1382 \\
\quad\;\; 173 \\
\quad\;\;\; 20 \\
\hline
{\cdot}253914
\end{array}
$$

Now the given expr. $= \dfrac{{\cdot}253914}{0{\cdot}07345} = \dfrac{2{\cdot}53914}{6{\cdot}7345}$.

By inspection we see that there is no digit in the integral part of the quotient. Hence the number of digits required is $4+1=5$.

$$
\begin{array}{r|l}
 & {\cdot}37703 \\
6{\cdot}7345) & 2{\cdot}53914 \\
 & 2{\cdot}02035 \\
 & 51879 \\
 & 47142 \\
 & 4737 \\
 & 4714 \\
 & 23 \\
 & 20
\end{array}
$$

the answer correct to 4 decimal places is ${\cdot}3770$.

EXAMPLES. 120.

Find the value, correct to 3 places of decimals, of

1. ${\cdot}023045 \times 2{\cdot}03 \times 1{\cdot}32$.
2. ${\cdot}15304 \times 10{\cdot}25 \times 1{\cdot}206$.
3. $\dfrac{32{\cdot}302 \times 23{\cdot}54}{36{\cdot}403}$.
4. $\dfrac{{\cdot}12345 \times {\cdot}51234}{{\cdot}45123}$.
5. $\dfrac{{\cdot}348662}{{\cdot}28501 \times {\cdot}608175}$.

[*Hint* :—Change ${\cdot}348662$ and ${\cdot}28501$ to $3{\cdot}48662$ and $2{\cdot}8501$ respectively. Multiply ${\cdot}608175$ by $2{\cdot}8501$ correct to 5 decimal places. Then divide $3{\cdot}48662$ by the product correct to 3 decimal places.]

$\dfrac{{\cdot}12345}{{\cdot}23451 \times {\cdot}34512}$.

XXX. PRACTICE.

202. An **aliquot part** of a quantity is a quantity which can be expressed as a fraction of that quantity, having *unity* for its numerator.

Thus 4*a*., being $\frac{1}{4}$ of R1, is an aliquot part of R1; 2*s*. 6*d*., which is $\frac{1}{8}$ of £1, is an aliquot part of £1.

Table of Aliquot Parts.

Of a Rupee.

8*a*.	= $\frac{1}{2}$ R.
5*a*. 4*p*.	= $\frac{1}{3}$ R.
4*a*.	= $\frac{1}{4}$ R.
2*a*. 8*p*.	= $\frac{1}{6}$ R.
1*a*. 4*p*.	= $\frac{1}{12}$ R.
1*a*.	= $\frac{1}{16}$ R.

Of an Anna.

6*p*.	= $\frac{1}{2}$*a*.
4*p*.	= $\frac{1}{3}$*a*.
3*p*.	= $\frac{1}{4}$*a*.
2*p*.	= $\frac{1}{6}$*a*.
1*p*.	= $\frac{1}{12}$*a*.

Of a Cwt.

2 qr.	= $\frac{1}{2}$ cwt.
1 qr.	= $\frac{1}{4}$ cwt.
16 lb.	= $\frac{1}{7}$ cwt.
14 lb.	= $\frac{1}{8}$ cwt.

Of a Quarter.

14 lb.	= $\frac{1}{2}$ qr.
7 lb.	= $\frac{1}{4}$ qr.
4 lb.	= $\frac{1}{7}$ qr.
3 lb. 8 oz.	= $\frac{1}{8}$ qr.
2 lb.	= $\frac{1}{14}$ qr.
1 lb. 12 oz.	= $\frac{1}{16}$ qr.
1 lb.	= $\frac{1}{28}$ qr.

Of a Pound.

10*s*.	= $\frac{1}{2}$ £.
6*s*. 8*d*.	= $\frac{1}{3}$ £.
5*s*.	= $\frac{1}{4}$ £.
4*s*.	= $\frac{1}{5}$ £.
3*s*. 4*d*.	= $\frac{1}{6}$ £.
2*s*. 6*d*.	= $\frac{1}{8}$ £.
2*s*.	= $\frac{1}{10}$ £.
1*s*. 8*d*.	= $\frac{1}{12}$ £.
1*s*. 4*d*.	= $\frac{1}{15}$ £.
1*s*. 3*d*.	= $\frac{1}{16}$ £.
1*s*.	= $\frac{1}{20}$ £.

Of a Maund.

20 sr.	= $\frac{1}{2}$ md.
10 sr.	= $\frac{1}{4}$ md.
8 sr.	= $\frac{1}{5}$ md.
5 sr.	= $\frac{1}{8}$ md.
4 sr.	= $\frac{1}{10}$ md.
2 sr. 8 ch.	= $\frac{1}{16}$ md.
2 sr.	= $\frac{1}{20}$ md.
1 sr. 4 ch.	= $\frac{1}{32}$ md.
1 sr.	= $\frac{1}{40}$ md.

Of a Mile.

4 fur.	= $\frac{1}{2}$ mi.
2 fur.	= $\frac{1}{4}$ mi.
1 fur.	= $\frac{1}{8}$ mi.

Of a Shilling.

6*d*.	= $\frac{1}{2}$*s*.
4*d*.	= $\frac{1}{3}$*s*.
3*d*.	= $\frac{1}{4}$*s*.
2*d*.	= $\frac{1}{6}$*s*.
1½*d*.	= $\frac{1}{8}$*s*.
1*d*.	= $\frac{1}{12}$*s*.

Of a Ton.

10 cwt.	= $\frac{1}{2}$ ton.
5 cwt.	= $\frac{1}{4}$ ton.
4 cwt.	= $\frac{1}{5}$ ton.
2 cwt. 2 qr.	= $\frac{1}{8}$ ton.
2 cwt.	= $\frac{1}{10}$ ton.
1 cwt. 1 qr.	= $\frac{1}{16}$ ton.
1 cwt.	= $\frac{1}{20}$ ton.

Of a Seer.

8 ch.	= $\frac{1}{2}$ sr.
4 ch.	= $\frac{1}{4}$ sr.
2 ch.	= $\frac{1}{8}$ sr.
1 ch.	= $\frac{1}{16}$ sr.

Of a Foot.

6 in.	= $\frac{1}{2}$ ft.
4 in.	= $\frac{1}{3}$ ft.
3 in.	= $\frac{1}{4}$ ft.
2 in.	= $\frac{1}{6}$ ft.
1½ in.	= $\frac{1}{8}$ ft.
1 in.	= $\frac{1}{12}$ ft.

Of a Pound.
(Avoir.)

8 oz.	$\frac{1}{2}$ lb.
4 oz.	$\frac{1}{4}$ lb.
2 oz.	$\frac{1}{8}$ lb.
1 oz.	$\frac{1}{16}$ lb.

Of a Furlong.

110 yd.	=	$\frac{1}{2}$ fur.
55 yd.	=	$\frac{1}{4}$ fur.

Of a Yard.

1 ft. 6 in.	=	$\frac{1}{2}$ yd.
1 ft.	=	$\frac{1}{3}$ yd.

Of a Month.

1 wk.	=	$\frac{1}{4}$ mo.
2 wk.	=	$\frac{1}{2}$ mo.
15 da.	=	$\frac{1}{2}$ mo.
10 da.	=	$\frac{1}{3}$ mo.

Of a Week.

$3\frac{1}{2}$ da.	=	$\frac{1}{2}$ wk.
$1\frac{3}{4}$ da.	=	$\frac{1}{4}$ wk.
1 da.	=	$\frac{1}{7}$ wk.

EXAMPLES. 121.

(*Oral.*)

Read off as aliquot parts of R1 :

1. 8*a.* **2.** 2*a.* **3.** 5*a.* 4*p.* **4.** 2*a.* 8*p.*
5. 1*a.* 4*p.* **6.** 6*p.*

Read off as aliquot parts of £1 :

7. 10*s.* **8.** 5*s.* **9.** [illegible]*s.* **10.** 6*s.* 8*d.*
11. 2*s.* 6*d.* **12.** 1*s.* 4*d.*

Read off as aliquot parts of 1 md.

13. 10 sr. **14.** 8 sr. **15.** 4 sr. **16.** 2 sr. 8 ch
17. 8 ch. **18.** 2 ch.

Read off as aliquot parts of 6*a.*

19. 2*a.* **20.** 1*a.* 6*p.* **21.** 9*p.* **22.** $4\frac{1}{2}$*p.*

Read off as aliquot parts of 1*a.* 6*p.*

23. 9*p.* **24.** 3*p.* **25.** $1\frac{1}{2}$*p.* **26.** $\frac{1}{2}$*p.*

Read off as aliquot parts of 2*s.* 6*d.*

27. 1*s.* 3*d.* **28.** 10*d.* **29.** 5*d.* **30.** $1\frac{1}{2}$*d.*

203. Simple Practice is a convenient method of finding, by means of aliquot parts, the cost of a *simple quantity*, when the cost is given of the unit-quantity, in terms of which the simple quantity is expressed. Simple Practice is, therefore, also known as Multiplication by Aliquot Parts.

Example. Find the value of 32 cwt. of wheat at R3. 8*a.* per cwt.

Compound Practice is a convenient method of finding, by means of aliquot parts, the cost of a *compound quantity*, when the cost is given of one of the units, in terms of which the compound quantity is expressed.

Example. Find the value of 7 cwt. 3 qr. of wheat at R3. 8*a.* per cwt.

SIMPLE PRACTICE.

204. The following examples will explain the method of Simple Practice.

No rule can be given for determining what aliquot parts should be taken. A little practice will soon enable the student to select the most convenient.

Example 1. Find the price of 23 md. of rice at R3. 13*a.* 9*p.* per md.

	R.	*a.*	*p.*	
	23	0	0	= price at R1 per md.
			3	
	69	0	0	= price at R3 per md.
8*a.* = $\frac{1}{2}$ of R1	11	8	0	= " " 8*a.* " "
4*a.* = $\frac{1}{2}$ of 8*a.*	5	12	0	= " " 4*a.* " "
1*a.* = $\frac{1}{4}$ of 4*a.*	1	7	0	= " " 1*a.* " "
6*p.* = $\frac{1}{2}$ of 1*a.*		11	6	= " " 6*p.* " "
3*p.* = $\frac{1}{2}$ of 6*p.*		5	9	= " " 3*p.* " "
	R88	12	3	= price at R3. 13*a.* 9*p.* per md.

Note 1. Since R3 . 13 . 9 is the difference between R4 and 2*a.* 3*p.*, a shorter method would be to find the price at 2*a.* 3*p.* per md. and subtract it from the price at R4 per md.

Thus

R	*a.*	*p.*	
23	0	0	
		4	
92	0	0	= price at R4 per md.
3	3	9	= " " 2*a.* 3*p.* " "
R88	12	3	= price at R3. 13*a.* 9*p.* per md.

	R	*a.*	*p.*	
	23	0	0	
2*a.* = $\frac{1}{8}$ of R1.	2	14	0	
3*p.* = $\frac{1}{8}$ of 2*a.*		5	9	
	R3	3	9	= price at 2*a.* 3*p.* per md.

Example 2. Find the cost of 9 articles at £10. 12*s*. 6*d*. each.

First Method :

		£.	s.	d.	
		9	0	0	=cost at £1 each.
				10	
		90	0	0	=cost at £10 each.
10*s*.	½ of £1.	4	10	0	= „ „ 10*s*. „
2*s*.	⅕ of 10*s*.		18	0	= „ „ 2*s*. „
6*d*.	¼ of 2*s*.		4	6	= „ „ 6*d*. „
		£95	12	6	=cost at £10. 12*s*. 6*d*. each.

Note 2. Shorter thus : 10*s*. = ½ of £1 ; 2*s*. 6*d*. = ¼ of 10*s*.

Second Method :

	£	
	9	=cost at £1 each.
	10	
	90·	=cost at £10 each.
10*s*. = ½ of £1.	4·5	= „ „ 10*s*. „
2*s*. 6*d*. = ¼ of 10*s*.	1·125	= „ „ 2*s*. 6*d*. „
	£95·625	=cost at £10. 12*s*. 6*d*. each.
	20	
	s. 12·500	
	12	
	d. 6·0	*Ans*. £95. 12*s*. 6*d*.

Example 3. Find the value of 13½ cwt. at R7. 10*a*. 3*p*. per cwt.

		R	a.	p.	
		13	8	0	=value at R1 per cwt.
				7	
		94	8	0	=value at R7 per cwt.
8*a*.	½ of R1.	6	12	0	= „ „ 8*a*. „ „
2*a*.	¼ of 8*a*.	1	11	0	= „ „ 2*a*. „ „
3*p*.	⅛ of 2*a*.		3	4½	= „ „ 3*p*. „ „
		R103	2	4½	=value at R7. 10*a*. 3*p*. per cwt.

Or thus :

		R.
		13·5
		7
		94·5
8*a*.	½ of R1.	6·75
2*p*.	¼ of 8*a*.	1·6875
3*p*.	⅛ of 2*a*.	·2109375
		R103·1484375

R. ·1484375
16
a. 2·3750000
12
p. 4·500

Ans. R103. 2*a*. 4½*p*.

Example 4. Find the value of 42⅔ things at 16*s*. 2¾*d*. each.

		£.	*s*.	*d*.	
		42	13	4	= value at £1 each.
10*s*.	of £1.	21	6	8	= value at 10*s*. each.
5*s*.	½ of 10*s*.	10	13	4	= „ „ 5*s*. „
1*s*.	of 5*s*.	2	2	8	= „ „ 1*s*. „
2*d*.	„ of 1*s*.		7	1⅓	= „ „ 2*d*. „
½*d*.	¼ of 2*d*.		1	9⅓	= „ „ ½*d*. „
¼*d*.	½ of ½*d*.			10⅔	= „ „ ¼*d*. „
		£34	12	5⅓	= value at 16*s*. 2¾*d*. each.

EXAMPLES. 122.

Find, by Practice, the cost of the following articles :

1. 400 at R3. 4*a*. each.
2. 375 at £2. 5*s*. each.
3. 789 at 1*a*.
4. 728 at 3*d*.
5. 439 at 3*p*.
6. 399 at £4. 4*s*.
7. 874 at 6*a*.
8. 723 at 15*s*.
9. 939 at R2. 11*a*.
10. 275 at 4*d*.
11. 475 at 13*a*. 6*p*.
12. 342 at 2*s*. 6*d*.
13. 500 at 7*a*. 3*p*.
14. 942 at 7*s*. 3*d*.
15. 700 at 10*a*. 4½*p*.
16. 374 at 5½*d*.
17. 321 at R2. 5*a*. 3*p*.
18. 230 at £7. 10*s*. 6*d*.
19. 366 at R7. 11*a*. 9*p*.
20. 767 at £10. 8*s*. 8*d*.
21. 839 at R5. 13*a*. 4*p*.
22. 339 at 14*s*. 10¼*d*.
23. 454 at R15. 7*a*. 10½*p*.
24. 900 at £50. 11*s*. 9¾*d*.
25. 900 at R42. 10*a*. 7½*p*.
26. 5013 at £55. 19*s*. 1½*d*.
27. 768 at R19. 9*a*. 3 *pice*.
28. 1010 at £11. 11*s*. 11¾*d*.
29. 8760 at R21. 14*a*. 2 *pice*.
30. 4596 at 12*s*. 0¾*d*.
31. 555 at R89. 3*a*. 5¼*p*.
32. 3111 at £12. 12*s*. 3¾*d*.
33. 8001 at R80. 8*a*. 8½*p*.
34. 10000 at £7. 17*s*. 11½*d*.
35. 346½ at R8. 10*a*. 8*p*.
36. 27⅜ at £8. 16*s*. 7¾*d*.
37. 703⅜ at R29. 13*a*. 4½*p*.
38. 301¼ at £2. 15*s*. 7¾*d*.
39. 821¾ at R41. 7*a*. 5¾*p*.
40. 442⅜ at £76. 2*s*. 4¾*d*.
41. $600\frac{5}{12}$ at R12. 12*a*. 2*p*.
42. $249\frac{7}{10}$ at £20. 2*s*. 8¼*d*.
43. 39·5 at R1. 13*a*. 4*p*.
44. 84·75 at £2. 15*s*. 9*d*.
45. 101·375 at R10. 9*a*. 6*p*.
46. 10·875 at £2. 17*s*. 10¼*d*.

COMPOUND PRACTICE.

205. The method of Compound Practice is illustrated by the following examples.

Example 1. Find the price of 15 md. 12½ seers at R2. 5*a*. 3*p*. per md.

	R.	*a*.	*p*.	
	2	5	3	= price of 1 md.
			3	
	6	15	9	
			5	
	34	14	9	= price of 15 md.
10 seers | ¼ of 1 md.		9	$3\frac{3}{4}$	= „ „ 10 seers.
2½ seers | ¼ of 10 seers		2	$3\frac{15}{16}$	= „ „ 2½ seers.
	R35	10	$4\frac{11}{16}$	= price of 15 md. 12½ seers.

Example 2. Find the cost of 2 tons 3 cwt. 3 qr. 5 lb. at £15. 17*s*. per cwt.

	£.	*s*.	*d*.	
2 tons 3 cwt. = 43 cwt.	15	17	0	= cost of 1 cwt.
			10	
	158	10	0	
			4	
	634	0	0	= cost of 40 cwt.
	47	11	0	= „ „ 3 cwt.
	681	11	0	= „ „ 43 cwt.
2 qr. ½ of 1 cwt.	7	18	6	= „ „ 2 qr.
1 qr. ½ of 2 qr.	3	19	3	= „ „ 1 qr.
4 lb. 1/7 of 1 qr.		11	$3\frac{6}{7}$	= „ „ 4 lb.
1 lb. ¼ of 4 lb.		2	$9\frac{27}{28}$	= „ „ 1 lb.
	£694	2	$10\frac{23}{28}$	= cost of 2 tons 3cwt. 3qr. 5 lb.

Example 3. Find the value of 25 sacks of flour, each weighing 3 md. 10 seers, at R5. 8*a*. per maund.

	R.	*a*.	*p*.	
	5	8	0	= value of 1 md.
			3	
	16	8	0	= „ „ 3 md.
10 seers = ¼ of 1 md.	1	6	0	= „ „ 10 seers.
	17	14	0	= value of 1 sack.
			5	
	89	6	0	
			5	
	R446	14	0	= value of 25 sacks.

EXAMPLES. 123.

Find, by Practice, the value of

1. 7 md. 15 seers at R3. 7*a*. 8*p*. per md.

2. 9 md. 17½ seers at R4. 10*a*. 8*p*. per md.

3. 27 cwt. 2 qr. 7 lb. at £3. 7*s*. 6*d*. per cwt.

4. 11 tons 14 cwt. at £5. 17*s*. 3*d*. per ton.

5. 17 tons 15 cwt. 2 qr. 21 lb. at £3. 15*s*. 9*d*. per cwt.

6. 6 tons 3 cwt. 2 qr. 24 lb. at 17*s*. 7*d*. per cwt.

7. 2 tons 13 cwt. 3 qr. 7 lb. at £1. 1*s*. 4*d*. per cwt.

8. 3 md. 27 seers 8 ch. at R10. 5*a*. 8*p*. per md.

9. 7 md. 18 seers 9 ch. at R13. 7*a*. 5*p*. per md.

10. 8 md. 3 seers 12 ch. at 3*a*. 4*p*. per seer.

11. 1 md. 17 seers 10 ch. at 7*a*. 6*p*. per seer.

12. 4 cwt. 3 qr. 14 lb. at £1. 13*s*. 4*d*. per ton.

13. 7 cwt. 2 qr. 21 lb. at £6 per ton.

14. 3 tons 17 cwt 3 qr. 13 lb. 12 oz at £1. 18*s*. 9*d*. per cwt.

15. 3 md. 37 seers 12 ch at 7*s*. 6*d*. per seer.

16. 2 tons 7 cwt. 1 qr. 13 lb. 14 oz. at R9. 11*a* per qr.

17. 7 sacks of flour, each 3 md. 15 seers, at R7. 10*a*. per md.

18. 24 bales of cotton, each 5 cwt. 2 qr., at 16*s*. 7¾*d*. per cwt.

19. 35 chests of tea, each 1 md. 17 seers 9 ch., at R80. 12*a*. per md.

20. 321 boxes of coffee, each 1 cwt. 2 qr. 21 lb., at £7. 18*s*. per cwt.

21. Find the total produce of a field of 3 ac. 3 ro. 25 po. at 3 qr. 6 bus. 2 pk. per acre.

22. Find the produce of 2 ac. 2 ro. 88 sq yd. at 7 cwt. 3 qr. 14 lb. per acre.

23. Find the price of 29 yd. 2 ft. 9 in. of silk at 7*s*. 10½*d*. per yd.

24. Find the weight of 231 bales of cloth, each weighing 2 cwt. 2 qr. 14 lb.

25. Find the weight of 329 boxes, each weighing 7 md. 27½ seers.

26. Find the tax on £329. 15*s*. at 1*s*. 7½*d*. in the £.

27. Find the tax on £3090. 8*a*. at 1*a*. 4½*p*. in the R.

28. Find the cost of 5 qr. 3 bus. 2 pk. of oats at £2. 14*s*. 4*d*. per qr.

29. Find the price of 12 gall. 3 qt. $1\frac{1}{2}$ pt. of milk at R3. 8*a.* per gallon.

30. Find the value of 225 cwt. at £21. 5*s.* 7*d.* per ton.

31. Find the value of 257 things, 10 of which cost R3. 9*a.* 4*p.*

32. Find, to the nearest pie, the rent of 275·365 bighas at R3. 7*a.* 9*p.* per bigha.

33. Find the value of 1 ton 11 cwt. 1 qr. 11 lb. at £6·285 per ton.

34. Find the dividend on R5145. 12*a.* at 14*a.* 6*p.* in the R.

35. If a man's debts amount to R37925. 14*a.*, and he can pay only 3*a.* $4\frac{1}{2}$*p.* for each rupee, how much do his creditors get?

XXXI. SQUARE ROOT.

206. A number whose square is equal to a given number is called the **square root** of that number. Thus 2 is the square root of 4; 3 is the square root of 9.

The square root of a number is indicated by the symbol $\sqrt{}$ placed before it. Thus $\sqrt{4}$ indicates the *square root of* 4, that is, 2.

207. A number whose square root can be expressed exactly either by a whole number or by a fraction is called a **perfect square.**

Note. It may be noticed that, no number, integral or decimal, which ends with 2, or 3, or 7, or 8, is a perfect square.

208. When the square root of a whole number which is a perfect square does not exceed 20, we obtain it from the multiplication table. Thus from the table we know that the square root of 81 is 9; of 169 is 13.

When a number, which is a perfect square, can be easily separated into prime factors, its square root may be found by inspection.

Thus $\sqrt{8100} = \sqrt{2^2 \times 5^2 \times 3^2 \times 3^2} = 2 \times 5 \times 3 \times 3 = 90.$

Example 1. Find, by factors, the square root of 17424.

$$
\begin{array}{r|l}
2^2=4 & 17424 \\
2^2=4 & 4356 \\
3^2=9 & 1089 \\
\hline
 & 11^2=121
\end{array}
$$

∴ The square root $= 2 \times 2 \times 3 \times 11 = 132.$

Example 2. What is the smallest whole number by which 1260 must be multiplied in order to become a perfect square ?

Since $1260 = 2^2 \times 3^2 \times 5 \times 7$, $\therefore$ the number required $= 5 \times 7 = 35$.

EXAMPLES. 124.

Find, by factors, the square root of

1. 900. **2.** 1600. **3.** 324. **4.** 576.

5. 1296. **6.** 4096. **7.** 1764. **8.** 7056.

9. 11025. **10.** 53361. **11.** 99225. **12.** 571536.

13. $27 \times 12 \times 14 \times 56$. **14.** $182 \times 77 \times 66 \times 39$.

15. $609 \times 290 \times 165 \times 154$.

16. Find the smallest whole number by which 450 must be multiplied in order to become a perfect square.

17. Find the least number by which 2940 must be multiplied in order to become a perfect square.

18. Find the least number by which 968 must be divided in order to become a perfect square.

19. Find the least square number which is divisible by 10, by 16 and by 24.

20. What must be the least number of soldiers in a regiment, that will allow it to be drawn up 10, 15 or 25 deep, and also to be formed into a solid square ?

209. We now proceed to explain a general rule by which we can find the square root of any number consisting of more than two figures.

210. We observe that the square root of 100 is 10, of 10,000 is 100, of 1,000,000 is 1,000 ; and so on. Hence it follows that the square roots of numbers less than 100 consist of only *one* figure in their integral parts ; of numbers between 100 and 10,000, of two figures in their integral parts ; of numbers between 10,000 and 1,000,000, of three figures in their integral parts ; and so on. If then a point be placed over every second figure in any number beginning with the *units'* the number of points will be the same as the number of figures in the integral part of the square root. Thus the square root of $3\dot{1}3\dot{6}$ consists of two figures in its integral part ; the square root of $\dot{1}5\dot{6}2\dot{5}$ consists of three figures in its integral part.

211. We know from Algebra that

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= a^2 + (2a+b)b.$$

Hence $(50+6)^2=50^2+2\times50\times6+6^2$

$=50^2+(2\times50+6)\times6$;

and $125^2=(120+5)^2$

$=120^2+(2\times120\times5)+5^2$

$=120^2+(2\times120+5)\times5.$

212. Now suppose we have to extract the square root of 3136

We first divide the number into periods of two figures each, by placing dots over every second figure beginning with the units'*. We see that there are two digits in the required root.

Since $50^2=2500$ and $60^2=3600$, it is clear that the root lies between 50 and 60. The tens digit will, therefore, be 5. If 50^2 be subtracted from 3136, the remainder will be 636, and by the previous article, the remainder is equal to $(2\times50+\text{the units' digit})\times(\text{the units' digit})$. Hence by trial we find that the units' digit is 6, for $(2\times50+6)\times6=636$.

$$
\begin{array}{r|l}
 & 3\dot{1}3\dot{6}\ (\ 50+6 \\
 & 2500 \\
2\times50+6=106\ ) & 636 \\
 & 636 \\
\end{array}
$$

In actual practice the work is shortened, as in ordinary division, by omitting zeroes thus :

$$
\begin{array}{r|l}
 & 3\dot{1}3\dot{6}\ (\ 56 \\
 & 25 \\
106\ ) & 636 \\
 & 636 \\
\end{array}
$$

and the process is as follows :

We first find the greatest number (5) whose square is contained in the first period ; this is the first figure of the root ; then subtract its square (25) from the first period and to the remainder (6) bring down the second period, thus getting 636 for the new dividend. Next, we divide this number omitting the last figure, by twice the part of the root already found (*i.e.*, we divide 63 by 10), and annex the quotient (6) to the root and also to the *trial divisor* (10) ; then multiply the divisor as it now stands *i.e.*, (106) by the figure of the root last found, *i.e.*, (6). Now, subtracting this product from 636, we have no remainder ; and we conclude that 56 is the square root of 3136.

If there be more periods to be brought down, the above operation must be repeated, as in the following example.

****N. B.* Each period consists of the figure over which a dot is placed and the figure to its left. Here the first period is 31 and second . The *first* period may consist of only one figure.**

Example. Find the square root of 15625.

Work in full.		Work in short.	
	15625 (100 + 20 + 5		15625 (125
	10000		1
100 × 2 + 20 = 220)	5625	22)	56
	4400		44
120 × 2 + 5 = 245)	1225	245)	1225
	1225		1225

The process of the 'work in short' is as follows :

Here, after two figures in the root have been obtained, the remainder is 12 ; to this we bring down the third period, thus getting 1225 as the last dividend. We divide this number, last figure omitted, by twice the part of the root already found (*i.e.*, we divide 122 by 24), getting 5 as the quotient. We then annex 5 to the root and also to the trial divisor 24 : etc.

213. In obtaining the *second* figure of the root by division we sometimes get a quotient which is too large. In such a case we find the root-figure by trial, as in the two following examples.

(i)

```
      125 ( 15
      1
 25 ) 125
      125
```

Here, dividing 12 by 2, the quotient is 6. Taking 6 as the required figure we find that the product (26 × 6) is greater than 125. We then take 5 which is found to be the required root-figure.

(ii)

```
      361 ( 19
      1
 29 ) 261
      261
```

Here, division gives 13 which is obviously inadmissible. By trial we find 9 to be the required root-figure.

214. When the trial divisor is greater than the number to be divided by it (or when the quotient is 1 but found too large) we set down 0 in the root, annex 0 to the divisor, bring down the next period, and proceed in the usual way. The two following examples are given for illustration.

(i)

```
        41209 ( 203
        4
  403 ) 1209
        1209
```

(ii)

```
         4401604 ( 2098
         4
   409 ) 4016
         3681
  4188 ) 33504
         33504
```

215. In the process of extracting the square root, a remainder is often left, which is greater than the divisor. In the following example the second remainder 35 is greater than the divisor 29.

```
       3'9'01 ( 199
       1
  29 ) 290
       261
 389 )  3501
        3501
```

EXAMPLES. 123.

Find the square root of

1. 441.	**2.** 576.	**3.** 729.	**4.** 961.
5. 1024.	**6.** 6561.	**7.** 5625.	**8.** 9216.
9. 27225.	**10.** 54756.	**11.** 49284.	**12.** 1[illegible]225.
13. 119025.	**14.** 193600.	**15.** [illegible].	**16** [illegible]1,409.
17. 4937284.	**18.** 2819041.	**19.** 1002001.	**20.** 1522756.
21. 82264900.	**22.** 62504836.	**23.** 97535376.	**24.** 21224449.

25. 3226694416. **26.** 6407522209. **27.** 236144689.

28. 360117609604. **29.** 2930662400[illegible]0. **30.** 15241578750190521.

31. A certain number of men spent R1681, each spending as many rupees as there were men; how many men were there?

32. A certain number of persons agree to subscribe as many pies each as there are subscribers; the whole subscription being R33. 5a. 4p. How many subscribers were there?

33. A gardener plants an orchard with 5776 trees and arranges them so that the number of rows of the trees equals the number of trees in each row. How many rows were there?

34. A general having 11025 men under him, arranges them into a solid square. Find the number of men in the front.

35. A general wishing to arrange his men, who were 63510 in number, into a solid square, found that there were 6 men over. How many men were there in the front?

36. Find the least integer which must be subtracted from 4230 in order to become a perfect square.

216. To find the square root of a Decimal Fraction.

To find the square root of a decimal fraction we proceed as in the case of a whole number. In pointing, the first point must be

placed or supposed to be placed on the units' figure. In the root the decimal point must be placed immediately aft root-figures corresponding to the integral part of the number.

We observe that if any decimal be squared there wi l be a n even number of decimal places in the result. Consequently a decimal fraction (in its simplest form) to be a perfect square must have an even number of decimal places, and the number of decimal places in the root mus be one-half of the number in the square.

If the given decimal is not a perfect square (which is a ways the case when the decimal in its simplest form contains an odd number of decimal places) the square root will be a non-terminating decimal; and we can find the square root to any number of decimal places we like.

In finding the square root of a decimal, the number of decimal places in it must be made *even*, by annexing ciphers, if necessary.

Example 1. Find the square roots of 11·9025 and ·5625.

```
   11·9025 ( 3·45   Ans.              ·5625 ( ·75   Ans.
    9                                  49
 64 ) 290                        145 ) 725
      256                              725
685 ) 3425
      3425
```

Example 2. Find the square root of ·045 to three places of decimals.

```
 ·045000 ( ·212...   Ans.
  4
 41 ) 50
      41
422 ) 900
      844
       56
```

Here, we are to have three decimal places in the root; therefore in the given number, we make the decimal places *six*.

Example 3. Find the square root of 3 to two places of decimals.

```
     3·0000 ( 1·73...   Ans.
     1
 27 ) 200
      189
343 ) 1100
      1029
        71
```

EXAMPLES. 126.

(*Examples* 1—8 *should to be taken orally.*)

Find the square root of

1. ·01.	**2.** ·09.	**3.** ·49.	**4.** 1·21.
5. 1·69.	**6.** 2·25.	**7.** ·0025.	**8.** ·0196.
9. 11·56.	**10.** 4·7089.	**11.** 39·0625.	**12.** 82·4464.
13. ·0064.	**14.** ·005329.	**15.** 1082·41.	**16.** 5·774409
17. ·00053361.	**18.** ·00002025.	**19.** 236·144689.	**20.** ·804609.

21. ·000003418801. **22.** 1·002001. **23.** 938703·06991561.

Find to four places of decimals square root of

24. 761·9.	**25.** 17.	**26.** 237·615.	**27.** 5.
28. 876·535.	**29.** ·1.	**30.** ·5.	**31.** 23·1.
32. ·9.	**33.** 20.	**34.** ·016.	**35.** ·00064.
36. 7.	**37.** 66.	**38.** 13.	

217. To find the square root of a Vulgar Fraction.

The square root of a vulgar fraction is the square root of its numerator divided by the square root of its denominator.

Example 1. $\sqrt{\frac{16}{25}}=\frac{\sqrt{16}}{\sqrt{25}}=\frac{4}{5}.$

Example 2. $\sqrt{2\frac{1}{4}}=\sqrt{\frac{9}{4}}=\frac{\sqrt{9}}{\sqrt{4}}=\frac{3}{2}=1\frac{1}{2}.$

Example 3. $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{\sqrt{4}}=\frac{1·73...}{2}=·86...$

If the denominator be not a perfect square it is advantageous make it so by multiplication.

Example 4. $\sqrt{\frac{1}{6}}=\sqrt{\frac{1\times6}{6\times6}}=\frac{\sqrt{6}}{\sqrt{36}}=\frac{2·449...}{6}=·408...$

Example 5. $\sqrt{\frac{5}{18}}=\sqrt{\frac{10}{36}}=\frac{\sqrt{10}}{\sqrt{36}}=\frac{3·1622...}{6}=·5270...$

Note. The square root of a fraction can also be found by reducing the fraction to a decimal and then extracting the square root of the decimal.

EXAMPLES. 127.

(*Examples* 1—8 *should be taken orally.*)

1. $\frac{1}{4}$. 2. $\frac{4}{9}$. 3. $\frac{1}{49}$. 4. $\frac{25}{36}$.
5. $\frac{9}{121}$. 6. $\frac{81}{169}$. 7. $\frac{144}{289}$. 8. $\frac{441}{10000}$.
9. $\frac{196}{2025}$. 10. $5513\frac{1}{16}$. 11. $32\frac{32}{49}$. 12. $101\frac{1}{400}$. 13. $\frac{8}{4\frac{1}{2}}$.
14. $2\cdot\dot{7}$. 15. $23\cdot\dot{4}$. 16. $3\cdot3\dot{6}\dot{1}$. 17. $8\cdot02\dot{7}$. 18. $\cdot07\dot{1}$.

Find to 3 places of decimals the square root of

19. $\frac{7}{4}$. 20. $\frac{5}{7}$. 21. $\frac{2}{3}$. 22. $\frac{5}{8}$. 23. $\frac{7}{12}$. 24. $\cdot\dot{3}$.

25. $\cdot41\dot{6}$. 26. $\frac{1\cdot23}{\cdot5}$. 27. $\frac{1}{2\cdot5}$. 28. $\frac{5\cdot04}{\cdot012}$.

29. Simplify $\sqrt{(75\frac{1}{9})}\times\sqrt{(1\cdot\dot{7})}\div\sqrt{(2\frac{34}{81})}$.

218. When *more than half* the number of figures of a square root has been obtained by the ordinary method, the remaining figures may be obtained by division only.

Example 1. To find the square root of 189475225.

Here we find the first three figures in the ordinary way. To find the remaining two figures by division, we take twice the part of the root already found, as the divisor; we bring down one figure to the last remainder and divide; then to the new remainder bring down the next figure and divide. The quotient thus obtained gives the two remaining figures of the root.

```
1̇8̇9̇4̇7̇'5̇2̇2̇5̇ ( 13765   Ans.
 1
 23 ) 89
      69
267 ) 2047
      1869
274 )  1785 ( 65
       1644
        1412
        1370
          42
```

Note. Of course this process does not show whether the given number is a perfect square or not, but it is very useful in cases like the following.

Example 2. Find the square root of 2 to seven places of decimals.

Here we find 5 figures of the root by the ordinary method and the remaining three by division.

```
         2 (1·4142135...Ans.
         1
   24 ) 100
         96
  281 )   400
          281
 2824 )  11900
         11296
28282 )   60400
          56564
   28284 ) 38360 ( 135
           28284
           100760
            84852
            159080
            141420
             17660
```

Example 3. Find the value of $\sqrt{32}-\sqrt{128}+\sqrt{50}$ to three places. [C. U. 1928.]

$$\sqrt{32}-\sqrt{128}+\sqrt{50}=\sqrt{16\times2}-\sqrt{64\times2}+\sqrt{25\times2}$$
$$=4\sqrt{2}-8\sqrt{2}+5\sqrt{2}$$
$$=9\sqrt{2}-8\sqrt{2}=\sqrt{2}=1·414. \quad \text{(See } Ex. \text{ 2.)}$$

Example 4. Simplify $\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$ to 2 places of decimals.

[C. U. 1925.]

$$\frac{\sqrt{2}+1}{\sqrt{2}-1}=\frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{(\sqrt{2}+1)^2}{2-1}=(\sqrt{2}+1)^2$$

$$\therefore \quad \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}=\sqrt{2}+1=1·41+1=2·41. \quad \text{(See } Ex. \text{ 2.)}$$

Example 5. Find the value of $\sqrt{6+2\sqrt{5}}$ correct to 3 places of decimals.

Since $(a+b)^2=a^2+b^2+2ab$, we have

$$6+2\sqrt{5}=5+1+2\sqrt{5}=(\sqrt{5})^2+1^2+2\sqrt{5}=(\sqrt{5}+1)^2$$

$$\therefore \quad \sqrt{6+2\sqrt{5}}=\sqrt{5}+1=2·236+1=3·236.$$

EXAMPLES. 128.

Find, correct to 6 places of decimals, the square root of

1. 5. **2.** 17. **3.** 761·9. **4.** ·0003841. **5.** $\frac{3}{5}$.
6. 3. **7.** ·07. **8.** ·85. **9.** 7619. **10.** $\frac{3}{8}$.
11. 237·615. **12.** 1·7. **13.** $\frac{5}{7}$. **14.** 23·8369.
15. ·000943. **16.** 10.

17. Find to 4 places of decimals the value of

$$2\sqrt{3}-\tfrac{1}{2}\sqrt{12}+\sqrt{27}.$$

18. Extract the square root of

$1-(\cdot 00135)^2$ correct to 5 places of decimals.

19. Find the value of $\dfrac{\sqrt{(2-\sqrt{2})}}{\sqrt{(2+\sqrt{2})}}$ correct to 5 places of decimals. [P. U. 1893.]

20. Simplify $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\dfrac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$. [P. U. 1891.]

21. Find the value of $\sqrt{8+2\sqrt{7}}$ correct to 3 places of decimals.

XXXII. CUBE ROOT.

219. A number is called the **cube root** of its cube. Thus 2 is the cube root of 8 ; 3 is the cube root of 27.

The cube root of a number is indicated by the symbol $\sqrt[3]{\ }$ placed before it. Thus $\sqrt[3]{8}$ indicates the *cube root of* 8, *i.e.*, 2.

A number whose cube root can be expressed exactly either by a whole number or by a fraction is called a **perfect cube**.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,
are respectively	1,	8,	27,	64,	125,	216,	343,	512,	729.

[These results should be committed to memory.]

220. When a number, which is a perfect cube, can be easily separated into prime factors, its cube root may be found by inspection.

Thus $\sqrt[3]{13824}=\sqrt[3]{2^3\times 2^3\times 2^3\times 3^3}=2\times 2\times 2\times 3=24.$

221. The cube root of a vulgar fraction is the cube root of its numerator divided by the cube root of its denominator.

Example 1. $\sqrt[3]{\dfrac{8}{125}}=\dfrac{\sqrt[3]{8}}{\sqrt[3]{125}}=\dfrac{2}{5}.$

Example 2. $\sqrt[3]{2\dfrac{10}{27}}=\sqrt[3]{\dfrac{64}{27}}=\dfrac{\sqrt[3]{64}}{\sqrt[3]{27}}=\dfrac{4}{3}=1\dfrac{1}{3}.$

222. The **fourth root** of a number is found by taking the square root of the square root of the number.

The **sixth root** of a number is found by taking the cube root of the square root of the number.

EXAMPLES. 129.

Find, by factors, the cube root of

1. 1331. **2.** 1728. **3.** 2744. **4.** 15625.

5. $\frac{216}{343}$. **6.** $\frac{125}{729}$. **7.** $18\frac{26}{27}$. **8.** $11\frac{25}{64}$.

Find the fourth root of

9. 256. **10.** 234256. **11.** 1679616. **12.** 1575·2961.

XXXIII. MEASUREMENT OF AREA.

223. In Arithmetic we consider the areas of **rectangles** only.

Example. The floor, the ceiling and each wall of an ordinary room; a sheet of paper; each side of an ordinary box or brick; all these are rectangular surfaces.

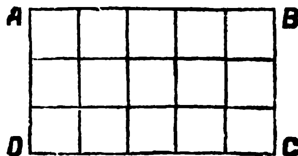
The length and breadth of a rectangle are called its **dimensions.**

224. The **unit of area** is a square whose side is the unit of length.

Area or **Surface** is measured by the number of units of area which it contains; just as a length is measured by the number of units of length which it contains.

225. To find the area of a rectangle.

Let *ABCD* be a rectangle, of which the length *AB* is 1 yd. 2 ft., and the breadth *AD* is 3 ft. Then, if the unit of length be a foot, the measure of *AB* is 5 and of *AD* is 3.



Divide *AB* and *AD* into 5 and 3 equal parts respectively, and through the points of division draw lines parallel to *AD*, *AB* respectively. Then the rectangle *ABCD* is divided into 5×3 equal squares, the side of each of which is a foot in length.

Now, each of these squares is the unit of area; therefore the *measure* of the area *ABCD* (which is the same as the number of these squares) is 5×3 or 15.

$\therefore$ Area of $ABCD = 15$ sq. ft.

And generally, in any rectangle,
measure of area = measure of length × measure of breadth ;
or, more briefly,

area = length × breadth.

Whence,

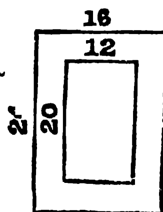
length = area ÷ breadth ;
breadth = area ÷ length.

Note. A square foot is a square whose side is a foot. Note the difference between "3 **square feet**" and "3 **feet square.**" *Three square feet* denotes an area 3 times as large as a square foot ; *three feet square* denotes the area of a square whose side is 3 feet.

Example 1. Find the area of the floor of a room 10 ft. 6 in. long and 6 ft. 4 in. broad.

Length of room $= 10\frac{1}{2}$ ft. ;
breadth „ „ $= 6\frac{1}{3}$ ft. ;
∴ area „ „ $= 10\frac{1}{2} \times 6\frac{1}{3}$ sq. ft.
$= \frac{21}{2} \times \frac{19}{3}$ sq. ft.
$= \frac{133}{2}$ sq. ft.
$= 66$ sq. ft. 72 sq. in.

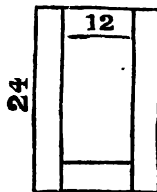
Example 2. A rectangular court, 24 yards long and 16 yards broad, has within it a path of uniform breadth of 2 yards running round it ; find the area of the path.



Area of court $= 24 \times 16$ sq. yd.
$= 384$ sq. yd.

The path takes off (2 + 2) yd. from the length, and (2 + 2) yd. from the breadth ;

∴ length of inner court = 20 yd.,
and breadth... ... ... = 12 yd. ;
∴ area ... ... = 20×12 sq. yd.
= 240 sq. yd.
∴ area of path = (384 − 240) sq. yd.
= 144 sq. yd.



Or thus :

Length of the path
$= (24 \times 2 + 12 \times 2)$ yd.
= 72 yd. ;
∴ the area of the path $= 72 \times 2$ sq. yd.
= 144 sq. yd.

Example 3. Find the breadth of a courtyard 41 sq. ft. 80 sq. in. in area, and 7 ft. 4 in. in length.

$$\text{Area}=(41+\tfrac{80}{144})\text{ sq. ft.}$$
$$=41\tfrac{5}{9}\text{ sq. ft.};$$
$$\text{length}=7\tfrac{1}{3}\text{ ft.}$$
$$\therefore\ \text{breadth}=\frac{41\frac{5}{9}}{7\frac{1}{3}}\text{ ft.}=\frac{374}{9}\times\frac{3}{22}\text{ ft.}=5\tfrac{2}{3}\text{ ft.}$$
$$=5\text{ ft. }8\text{ in.}$$

Example 4. How many paving stones, each 2 ft. 8 in. long and 17 in. wide, will cover the courtyard in *Ex.* 3 ?

$$\text{Area of court}=41\tfrac{5}{9}\text{ sq. ft.};$$
$$\text{area of a stone}=2\tfrac{2}{3}\times\tfrac{17}{12}\text{ sq. ft.}=\tfrac{34}{9}\text{ sq. ft.};$$
$$\therefore\ \text{number of stones reqd.}=\frac{41\frac{5}{9}}{\frac{34}{9}}=\frac{374}{9}\times\frac{9}{34}=11.$$

Example 5. Find the cost of matting the room in *Ex.* 1, at 3 annas per sq. ft.

The cost may be found by Practice or by Compound Multiplication.

EXAMPLES. 130.

Find the area of the rectangles having the following dimensions.

1. 15 ft. by 12 ft.
2. 20 ft. by 16 ft.
3. 13 ft. 6 in. by 8 ft. 8 in.
4. 9 ft. 10 in. by 6 ft. 7 in.
5. 10 ft. $7\frac{1}{2}$ in. by 7 ft. $4\frac{1}{2}$ in.
6. 9 yd. 2 ft. by 7 yd. 1 ft.

Find the breadth of a room whose

7. area = 363 sq. ft., and length = 33 ft.
8. area = 6 sq. ft. 60 sq. in., and length = 2 ft. 9 in.
9. area = 5 ac. 1 ro. 36 po., and length = 267 yd. 2 ft.
10. area = 94 sq. yd. 8 ft. 84 in., and length = 32 yd. 1 ft. 8 in.
11. Find the area of a square field whose side is 32 ft. 8 in.
12. Find the area of a square room whose side is 3 yd. 2 ft. 3 in.
13. How many paving stones, each $1\frac{1}{2}$ ft. by 9 in., would be required to pave a square courtyard whose side is 21 ft. ?
14. How many pieces of carpet, each 5 ft. long and 3 ft. wide, will cover the floor of a room 20 ft. by 13 ft. 6 in. ?
15. Find the cost of carpeting a room, 10 ft. 6 in. by 6 ft. 6 in., at R2 per sq. ft.

16. Find the cost of polishing a marble slab, 3 ft. 3 in. by 2 ft. 6 in., at *2d.* per sq. in.

17. A room, 20 ft. long, 16 ft. broad, has a stained border all round it 2 ft. wide; what is the area of the stained part?

18. A rectangular piece of ground is 88 yards long and contains an acre; it consists of a walk 6 ft. wide surrounding a grass-plot: find the area of the walk.

19. How many stone slabs, 3 ft. long, 1 ft. wide, are requisite for paving a path which encloses a rectangular garden half a mile long and quarter of a mile wide, the path being 6 ft. wide?

20. A gravel path 5 ft. wide runs round a rectangular garden 100 yd. by 75 yd.; find the cost of making it at *4a. 6p.* per sq. yd.

21. How many sq. yd. of matting will be wanted to cover a room 31 ft. 6 in. by 22 ft. 6 in.? What will be the cost at *4d.* per sq. yd.?

22. If 1200 stones, each 2 feet square, will pave a court, find the area of the court.

23. The cost of varnishing the floor of a room, 24 ft. long, at *1s. 6d.* per sq. yd., is £5; find the breadth of the room.

24. A garden roller is 3 ft. 3 in. wide, and its circumference is 6 ft. 9 in.; how many sq. ft. of ground does it pass over in one complete revolution?

25. A sheet of paper is 20 in. long and 18 in. wide; by how much must the width be narrowed to leave a surface of 2¼ sq. ft.?

26. What length must be cut off a plank which is 5¼ in. broad, that the area may be a sq. foot?

27. A factory has 100 windows, 60 of which severally contain 8 panes, each 9 in. by 6 in.; and the remainder severally contain 10 panes, each 2 ft. square; find the cost of glazing the whole at 10 annas per sq. ft.

28. What must be the length of a piece of land, 15 yards wide, that can be exchanged for a piece of the same quality, measuring 20 yards each way?

29. Find the area of the square which has the same perimeter as a rectangle whose length is 48 ft. and is 3 times its breadth.

30. How many flag-stones, each 5·76 ft. long and 4·15 ft. wide, are requisite for paving a cloister, which encloses a rectangular court, 45·77 yd. long and 41·93 yd. wide, the cloister being 12·45 ft. wide?

31. A room measuring 42 ft. 6 in. by 22 ft. 9 in. inside, with walls 2 ft. 3 in. thick, is surrounded by a verandah 10 ft. 6 in. wide. Find the cost of paving this verandah with tiles measuring 4½ in. by 3 in., and costing 6 pies each.

226. *Example* 1. Find the length of the side of a square which contains 91 sq. ft. 121 sq. in.

Area $=91$ sq. ft. 121 sq. in. $=13225$ sq. in.;

$\therefore$ length of side $=\sqrt{13225}$ in. $=115$ in. $=9$ ft. 7 in.

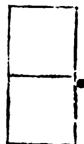
Example 2. Find the diagonal of a rectangular field, 16 yd. long and 12 yd. wide.

By Euclid I. 47,

the diagonal $=\sqrt{16^2+12^2}$ yd. $=\sqrt{256+144}$ yd. $=\sqrt{400}$ yd. $=20$ yd.

Example 3. The area of a room which is twice as long as it is broad is 26 sq. yd. 8 sq. ft.; how long is it?

The room can be divided into two equal squares whose side is equal to the breadth of the room.



Area of each square $=13$ sq. yd. 4 sq. ft.
$=121$ sq. ft.;

$\therefore$ side of each square $=\sqrt{121}$ ft. $=11$ ft.;

$\therefore$ breadth of room $=11$ ft. $=3$ yd. 2 ft.;

$\therefore$ length of room $=7$ yd. 1 ft.

EXAMPLES. 131.

1. The area of a square field is 10 acres; find the length of its side.

2. The area of a square room is 502 sq. ft. 73 sq. in.; find the length of each side.

3. How many yards of fencing are required to enclose a square garden containing 4 ro. 1 po. 29 yd. $6\frac{3}{4}$ ft.?

4. A rectangular field is 40 yards long and 30 yards broad; find the distance from corner to corner.

5. What is the length of the diagonal of a square whose side is 4 yards?

6. The area of a square is 900 sq. ft.; what is the length of its diagonal?

7. The area of the floor of a room is 162 sq. ft.; its length is twice its breadth; find its length.

8. Find the length of a rectangular field which is 3 times as long as it is broad and which contains 768 sq. yd.

9. A room is half as long again as it is broad and its area is 69·36 sq. yd.; find the perimeter.

10. The sides of two squares contain 77 yd. 1 ft. 9 in. and 7 yd. 2 ft. 4 in. respectively; find the side of a square whose area is equal to the sum of the areas of the two squares.

227. Carpeting the floor of a room.—Carpets are usually made in rectangular pieces or in rolls of uniform width. In the latter case lengths are cut according to the size of the floor. These are placed side by side and stitched together. Rectangular carpets are usually sold at so much per piece and rolls are sold at so much per *linear* yard and not per square yard.

Example. Find the length of carpet $2\frac{1}{3}$ ft. wide, required for a room 28 ft. long, 20 ft. broad.

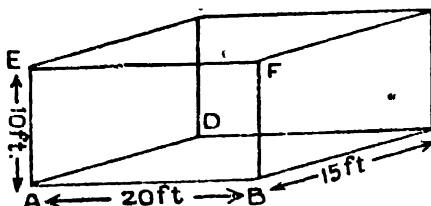
The carpet which will cover the floor of a room has the same area as the floor.

Area of floor $= 28 \times 20$ sq. ft.;

length of carpet reqd. $= \frac{28 \times 20}{2\frac{1}{3}}$ ft. $= \frac{28 \times 20 \times 3}{7}$ ft.

$= 240$ ft. $= 80$ yd.

228. Area of the four walls of a rectangular room.—Consider a room whose length, breadth, and height are respectively 20 ft., 15 ft. and 10 ft.



Let the above figure represent a room of which the floor is *ABCD* and the rectangles *ABFE*, *BCGF*, *CDHG* and *DAEH* are its four walls.

Since the areas of *ABFE* and *DCGH*, and of *BCGF* and *DAEH* are equal,

∴ the area of the four walls

$= 2 \times$ area of *ABFE* $+ 2 \times$ area of *BCGF*

$= (2 \times 20 \times 10 + 2 \times 15 \times 10)$ sq. ft.

$= 2 \times 10(20 + 15)$ sq. ft. ....(1)

$= 700$ sq. ft.

From (1) we see that the area of the four walls of the room can be stated as *twice the height multiplied by the sum of the length and breadth*; hence if h is the height and l and b the length and breadth respectively, then

the area of the four walls $=2h(l+b)$.

Again, since $2h(l+b)=h(2l+2b)$,

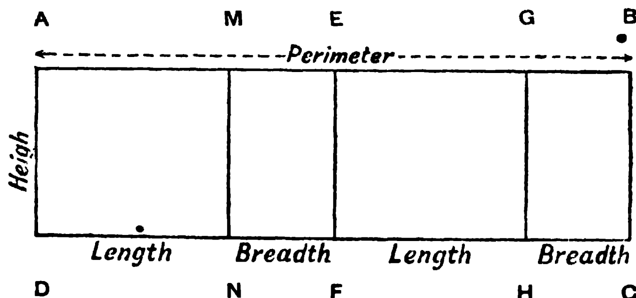
and $(2l+2b)=2AB+2BC$,

$=AB+DC+BC+AD$

=the perimeter of the room,

∴ the area of the four walls can also be stated as *height multiplied by the perimeter of the room.*

This result may also be obtained thus: Suppose the four walls of a room are spread out flat on the ground one after another. Then the position of the walls would be as shewn in the following diagram. That is, a continuous rectangle would be formed.



It is clear from the diagram that

the area of the four walls of a room = perimeter × height.

Note. It will be an interesting exercise for the student to make a model of the walls of a room thus: Take a rectangular piece of paper *ABCD*. Fold it along *EF*, dividing it into two equal parts. Then open it and again fold it along *GH* and also fold the part *AGHD* so that *AD* may coincide with *BC* in the folded position. The crease *MN* will now be formed and it will be found that $DN=FH$ and $NF=HC$. By bending the paper along the creases a model of the four walls of a room will be obtained.

Example. Find the area of the four walls of a rectangular room 16 ft. long, 12 ft. broad and 11 ft. high.

The perimeter $=(16+12)\times 2$ ft. $=56$ ft.;

∴ the area of walls $=56\times 11$ sq. ft. $=616$ sq. ft.

228a. Papering the walls of a room.

To find the length of paper required to cover the walls, proceed as in the *Example* in Art. 227.

Note. In estimating the length of paper required, deductions for doors, windows and fireplaces must be made.

N B. The cost of carpet or paper may be found by Practice or by Compound Multiplication.

EXAMPLES. 132.

Find the length of carpet required for rooms having the following dimensions :

1. Room, 25 ft. long, 18 ft. broad ; carpet, 2 ft. 6 in. wide.

2. Room, 20 ft. long, 12 ft. 6 in. broad ; carpet, 27 in. wide.

3. Room, $30\frac{3}{4}$ ft. long, $20\frac{1}{2}$ ft. broad ; carpet, 42 in. wide.

Find the expense of carpeting a room,

4. 16 ft. by 10 ft., with carpet 3 ft. wide, at R2. 8*a.* a yard.

5. 30 ft. 9 in. by 25 ft., with carpet 30 in. wide, at 4*s.* 6*d.* a yard.

Find the area of the walls of the following rectangular rooms :

6. Length 20 ft., breadth 16 ft., height 9 ft.

7. Length 15 ft. 6 in., breadth 12 ft., height 9 ft.

8. Length 21 ft. 7 in , breadth 16 ft. 5 in., height $3\frac{1}{2}$ yd.

Find the length of wall paper required for the following rooms :

9. 25 ft. long, 20 ft. wide, 12 ft. high ; paper 15 in. wide.

10. 14 ft. long, 10 ft. wide, 7 ft. high ; paper 14 in. wide.

11. 27 ft. long, 18 ft. wide, 10 ft. high ; with paper 16 in. wide, allowing for 2 doors each 7 ft. by 4 ft.

12. 28 ft. long, 20 ft. broad, $9\frac{1}{2}$ ft. high ; with paper 20 in. wide, allowing for a door 6 ft. by $3\frac{1}{2}$ ft. and a window 3 ft. by $2\frac{1}{2}$ ft.

Find the expense of papering rooms whose dimensions are :

13. Length 21 ft., breadth 16 ft., height 10 ft. ; with paper 16 in. wide, at 4*a.* a yard.

14. Length 50 ft., breadth 35 ft., height 15 ft. ; with paper 15 in. wide, at 6*d.* a yard.

15. Length 18 ft., breadth 16 ft., height 9 ft. ; with paper 15 in. wide, at 9*d.* a yard, allowing for 3 doors each 6 ft. by $3\frac{1}{2}$ ft., 2 windows 4 ft. by $2\frac{1}{2}$ ft., and a fireplace 6 ft. by 4 ft. 6 in.

16. How many yards will remain out of 300 yards of matting 2 ft. 6 in. wide, after covering two floors, each 25 ft. 6 in. by 21 ft. ?

17. A square room whose floor measures 56 sq. yd. 2 sq. ft. 36 sq. in., is 10 ft. 4 in. high ; find the expense of whitewashing its ceiling and walls at 2*p*. per sq. yd.

18. The cost of covering the floor of a room, $12\frac{1}{2}$ yd. by $8\frac{3}{4}$ yd., with carpet $2\frac{1}{2}$ ft wide, is £30 . 14 . $7\frac{1}{2}$; find the price of carpet per yard.

19. It costs £2. 5*s*. to paper a room 10 yd. long and 8 yd. wide, with paper $1\frac{1}{2}$ ft. wide, at 3*d*. per yard ; find the height of the room.

20. The cost of carpeting a room $16\frac{1}{2}$ ft. long and $12\frac{1}{4}$ ft. broad, with carpet at 6*s*. per yard, is £14. 17*s*. ; find the width of the carpet.

21. If a postage stamp be $\frac{8}{9}$ of an inch long and $\frac{3}{4}$ of an inch broad, what will be the cost of covering the walls of a room which is 15 ft. long, 12 ft. wide and 9 ft. high, with postage stamps, 9 pies each ?

22. What will be the cost of papering a room, 24 ft. long by 20 ft. broad and 8 ft. high, which has 2 doors each 7 ft. by 4 ft., with paper 2 ft. wide, at ₨4 a piece ; the cost of putting it on, being 4*a*. per piece, and each piece being 4 yards long ?

23. The matting of a room, 3 times as long as broad, at 4 annas per sq. ft. cost ₨75 ; and the painting of the walls at 2 annas per sq. yd. cost ₨6. 6*a*. $2\frac{2}{3}$*p*. ; what is the height of the room ?

24. Find the expense of lining a cistern 10 ft. long, 8 ft. broad and 3 ft. deep, with lead at ₨10 per cwt., which weighs 5 lb. per sq. ft.

25. Find the cost of papering a room, 18 ft. long, 12 ft. broad and 10 ft. high, with paper 32 in. wide, at 6 annas a yard, allowing for a door 7 ft. by 4 ft , 3 windows each 4 ft. by 3 ft. and a panelling 2 ft. high round the floor.

26. A box with a lid is to be made of plank, one inch thick ; the external dimensions are to be 18 in., 12 in. and 9 in. : how many sq. ft. of plank will be required ?

27. The length of a room is $32\frac{1}{2}$ ft. The cost of papering the walls at ₨1. 14*a*. per sq. yd. is ₨308. 2*a*. ; and the cost of carpeting the floor at ₨2. 4*a*. per sq. yd. is ₨150. 5*a*. Find the height and width of the room.

28. Find the cost of whitewashing the ceiling and the inner and outer sides of the walls of a room, 20 ft. long, 12 ft. wide and 15 ft. high, at 1 pie per sq. ft. ; the walls being $1\frac{1}{2}$ ft. thick and 3 ft. higher at the outside.

XXXIV. MEASUREMENT OF SOLIDITY.

229. In Arithmetic we consider the volumes of **rectangular solids** only.

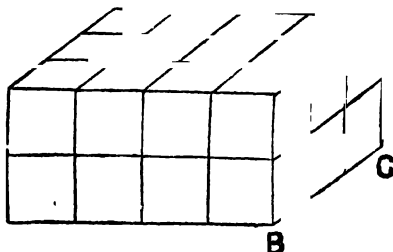
Example. A rectangular box, a brick, are rectangular solids.

The length, breadth and thickness (or height or depth) of a rectangular solid are called its **dimensions.**

230. The **unit of volume** is a cube each of whose edges is the unit of length.

Volume or **cubic content** is measured by the number of units of volume which it contains.

231. To find the volume of a rectangular solid or rectangular parallelopiped.



Let the annexed figure represent a rectangular parallelopiped, of which the length *AB* is 4ft., breadth *BC* is 3 ft. and thickness *AD* is 2 ft Divide *AB BC*, *AD* respectively into 4, 3 and 2 equal parts and through the points of division draw planes parallel to the sides. Then the solid will be divided into a number of equal blocks, each of which is a *cubic foot*; and since there are two layers, in each of which there are 4×3 blocks, we see that there are $4 \times 3 \times 2$ blocks altogether, and the solid therefore contains $4 \times 3 \times 2$ cubic feet.

$\therefore$ The volume of the solid $= 4 \times 3 \times 2$ cu. ft.

And generally, in any rectangular solid,

the measure of volume = measure of length × measure of breadth × measure of thickness.

Or, more briefly,

Volume = length × breadth × thickness.

Whence, thickness = volume ÷ (length × breadth) : etc.

Example 1. Find the cubic content of a rectangular block of marble whose dimensions are 3 ft. 2 in., 2 ft. 3 in. and 1 ft. 6 in.

Volume $= 3\frac{1}{6} \times 2\frac{1}{4} \times 1\frac{1}{2}$ cu. ft. $= 10\frac{11}{16}$ cu. ft.

Example 2. How many bricks will be required to build a wall 20 ft. long, 10 ft. high and 2 ft. thick; each brick with its share of the mortar being 6 in. long, 3 in. wide and 2 in. deep?

$$\text{Number of bricks} = \frac{\text{volume of the wall}}{\text{volume of each brick}} = \frac{20 \times 10 \times 2}{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6}} = 19200.$$

Example 3. A rectangular cistern is 6 ft. long and 4 ft. broad; what is the depth of water in it, when it contains 72 cubic feet of water?

$$\text{Depth} = \frac{\text{volume of water}}{\text{area of the base}} = \frac{72}{6 \times 4} \text{ ft.} = 3 \text{ ft.}$$

Example 4. A box with a lid is to be made of half-an-inch plank; its internal dimensions are to be 20 in., 15 in. and 9 in. How many cu. in. of wood will be required?

The external dimensions of the box are 21 in., 16 in. and 10 in.: ∴ its external volume $= 21 \times 16 \times 10$ cu. in. $= 3360$ cu. in.: and its internal volume $= 20 \times 15 \times 9$ cu. in. $= 2700$ cu. in. ∴ Volume of wood required for the box $= (3360 - 2700)$ cu. in. $= 660$ cu. in.

We may obtain the area of the plank required by dividing the volume of the wood by the thickness of the plank.

EXAMPLES. 133.

Find the cubic contents of the rectangular solids having the following dimensions:

1. 10 ft., 8 ft., 5 ft.

2. $7\frac{1}{2}$ ft., $5\frac{1}{4}$ ft., $4\frac{2}{3}$ ft.

3. 3 yd., 7 ft., 30 in.

4. 5 ft. 10 in., 3 ft., 6 in.

5. 7 yd. 2 ft. 9 in., 6 yd. 1 ft. 3 in., 10 ft. 10 in.

6. Find the cubic content of a cube whose edge is $3\frac{1}{2}$ ft.

7. How many pounds of water will fill a cistern 2 yd. long, 3 ft. broad and 9 in. deep, having given that a cu. ft. of water weighs 1000 oz.?

8. How many bricks, each 9 in. by 6 in. by 4 in., are required for a wall 22 yd. long, 8 ft. high and 2 ft. 6 in. thick, leaving in it a doorway 6 ft. by 4 ft.?

9. How many times can a bucket, holding 2 cu. ft. of water, be filled from a tank 30 ft. long, 25 ft. wide and 10 ft. deep?

10. In what time will a cistern 16 ft. by 12 ft. by 10 ft., be filled by a pipe which discharges 40 cu. ft. of water per minute?

11. How many sheets, each 4 ft. long, 2 ft. broad and $\frac{1}{4}$ of an inch thick, can be made from 4 cu. ft. of iron?

12. Find the total weight of 27 sheets of copper, each 6 ft. long, 4 ft. broad and $\frac{1}{8}$ of an inch thick, a cubic foot of copper weighing 2 cwt.

13. How many times can a pint-bottle be filled from a cistern 138·637 in. by 70 in. by 10 in., having given that a gallon contains 277·274 cubic inches ?

14. A cu. inch of gold is hammered into a plate 6 in. square ; find the thickness of the plate as the decimal of an inch.

15. Water is flowing into a reservoir which is 5 ft. square ; how many cu. ft. of water will have flown in when the depth of water is $2\frac{1}{2}$ ft. ?

16. A cistern, 12 ft. long and 8 ft. 6 in. broad, contains water ; how many cu. ft. of water must be drawn off to make the surface sink half an inch ?

17. A room, 40 ft. $10\frac{1}{2}$ in. by 25 ft. 8 in., accommodates 100 persons ; what must be the height of the room if each person has $175\frac{2331}{3200}$ cu. ft. of air ?

18. What length must be cut off a rectangular marble slab, $1\frac{1}{2}$ ft. broad and 8 in. thick, in order that it may contain 2 cu. ft. ?

19. Find the cost of digging a canal 1 mile long, 6 ft. wide and 5 ft. deep, at 4 annas per cu. yd.

20. A lake, whose area is 30 acres, is covered with ice 6 inches thick ; find the weight of the ice in tons, if a cubic foot of ice weigh 900 oz. Avoir.

21. There are 1530 cu. ft. of air in a room 9 ft. high ; find the cost of carpeting it at ℞1 per sq. ft.

22. A square room, 10 ft. high, contains 4000 cu. ft. of air ; how many yards of paper, 2 ft. wide, will be required for covering its walls ?

23. A solid stack, 41 ft. 8 in. by 16 ft. 8 in. by 14 ft. 7 in., contains 125000 bricks, each 10 in. long and $3\frac{1}{2}$ in. thick ; find the width of each brick.

24. A piece of ground is 100 yd. long and 75 yd. wide. To what uniform depth must it be excavated that the earth taken out may form an embankment of 25000 cubic yards, supposing the earth to be increased one-ninth in volume by removal ?

25. A box (with cover) is made of an-inch-and-a-half plank ; its external dimensions are 4 ft., 3 ft. 6 in. and 2 ft. 3 in. : find the weight of the box, supposing a cu. ft. of the wood to weigh 36 lb.

26. The roof of a verandah is supported by 16 teak beams, each 9 ft. long, 3 in. broad and 5 in. deep. If the weight of a cubic inch of teak is $\frac{18}{25}$ of that of a cubic inch of water, and if a cubic foot of water weighs 1000 oz., find the weight in lbs. of the timber in the verandah.

27. A crow wishing to quench its thirst came to a vessel which contained 28 cu. in. of water. The crow being unable to

reach the water, picked up several small stones, each three quarters of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel. If the size of the vessel was such that it would exactly hold 73 cubic inches of water, find the number of stones dropped in by the crow.

28. The top of a tank is a rectangle whose sides are 15 ft. and 9 ft.; it is of the same horizontal section throughout its depth. What must be its depth in order that it may contain 12960 gallons of water, one gallon containing 277·274 cubic inches?

29. A moat is to be dug all round a rectangular fort, 200 yd. long and 150 yd. broad; it is to have vertical sides and to be 27 ft. wide and 10 ft. deep throughout. Find the cost of digging it at 4 annas per cubic yard.

30. A room, 21 ft. long by $13\frac{1}{2}$ ft. wide, is surrounded by walls $1\frac{1}{2}$ ft. thick and 14 ft. high. There are two doors each $4\frac{1}{2}$ ft. by 6 ft., and one window 3 ft. by $4\frac{1}{2}$ feet. Find (i) the cost of building the walls at the rate of Rs. 1*a.* per cubic yard, and (ii) the number of bricks, each measuring 9 in. by 4 in. by $2\frac{1}{4}$ in., required for the work.

XXXV. PROBLEMS AND THE UNITARY METHOD.

232. When the value, weight or length, etc., of any number of units is given, we can, by *Compound Division*, obtain the value, weight or length, etc., of one of the units. And when the value, weight or length, etc., of one unit is given, we can, by *Compound Multiplication*, obtain the value, weight or length, etc., of any number of units of the same kind.

The solution by the application of the two above principles is called the **Unitary Method** or the **Method of Reduction to the Unit.** The method will be fully explained by the following examples.

233. *Example* 1. If 9 articles cost R36, what is the cost of 1 article?

The cost of 9 articles = R36,

∴ .... 1 article = R$\frac{36}{9}$

= R4. *Ans.*

Example 2. If 1 lb. of tea costs 2*s.* 6*d.*, what will 8 lb. cost?

The cost of 1 lb. = 2*s.* 6*d.*,

∴ .... 8 ... = (2*s.* 6*d.*) × 8

= £1. *Ans.*

EXAMPLES. 134.

(*Most of the following examples should be taken orally.*)

1. If 7 articles cost R2. 10*a.*, what is the cost of 1 article?

2. If 12 maunds of wheat cost R30, what will 1 maund cost?

3. If 7½ yards of cloth cost R1. 14*a.*, how much will 1 yd. cost?

4. If the weight of 16 equal bags of rice be 40 maunds, what is the weight of 1 bag?

5. If the length of a piece of cloth worth 18*s.* be 12 yards, what is the length of a piece of the same cloth worth 1*s.*?

6. If the rent of 13 acres of land is £4. 17*s.*, what is the rent of 1 acre?

7. If the income-tax on R200 be R5 . 3 . 4, what is the tax on R1?

8. If 1 chair costs R2. 12*a.*, how much will 13 chairs cost?

9. If 1 lb. of sugar costs 7*d.*, what will 10 lb. cost?

10. If 1 bullock can plough 3⅓ bighas in a day, how many bighas can 11 bullocks plough in a day?

11. If a man walk 3¾ miles in 1 hour, how far does he walk in 9¼ hours?

12. A servant's wages being 7*s.* 6*d.* per week, how much ought he to receive for 7 weeks?

13. If the railway fare for 1 mile is 2½*p.*, what is the fare for 24 miles?

14. If the carriage of 1 maund for 150 miles cost R2, what will be the cost of the carriage of 10½ maunds for the same distance?

Example 3. If 5 men can do a piece of work in 3 days, how long will it take 1 man to do it?

5 men can do the work in 3 days,

∴ 1 man..............................(3 × 5) days,

i.e., 15 days. *Ans.*

N. B. **Each column represents the work done by 5 men in 1 day and each cell represents the work done by 1 man in 1 day.**

Example 4. If 1 man can do a piece of work in 21 days, in how many days can 3 men do it ?

1 man can do the work in 21 days,

∴ 3 men............................$\frac{21}{3}$ days,

i.e., 7 days. *Ans.*

Note. In questions such as the two above, it should be noticed that to an **increase** in the number of workmen corresponds a **diminution** in the number of days, and *vice versa*.

EXAMPLES. 135. (*Oral.*)

1. If 10 men can do a piece of work in 3 days, how long will it take one man to do it ?

2. If 12 men finish a piece of work in 5 days, in how many days could one man finish it ?

3. If 3 maunds of rice last 9 persons 30 days, how long would they last 1 person ?

4. If 7 cwt. can be carried 100 miles for 3s., how far can 1 cwt. be carried for the same sum ?

5. If 13 acres can be rented for 7 months for a certain sum, for how many months can 1 acre be rented for the same sum ?

6. If 1 man can do a piece of work in 40½ days, how long will it take 9 men to do it ?

7. If 30 bushels feed 28 horses for a week, how many horses would they keep for 4 weeks ?

8. If 1 man reap a field in 18 days, how long will 4 men be doing it ?

9. A ship performs a voyage in 55 days, sailing 1 knot an hour, how many days would she take to perform the same voyage sailing 5 knots an hour ?

10. If the carriages of 56 maunds for 1 mile cost a certain sum, how much will be carried 14 miles for the same money ?

11. If 18 horses plough a field in 15 days, how many horses will plough it in 1 day ?

12. If 18 horses plough a field in 15 days, in how many days will 1 horse plough it ?

13. If 1 horse can be kept 8 days for ₨2. 8*a.*, for how many days can 4 horses be kept for the same sum ?

234. Each of the above questions requires either multiplication or division for its solution. In the following questions the two processes are combined.

Example 1. If 3 yards of cloth cost R4. 8*a*., what will be the cost of 35 yards ?

The cost of 3 yards = R4. 8*a*,

∴ .... 1 yard = R4. 8*a*. $\times \frac{1}{3}$,

∴ ....35 yards = R4. 8*a*. $\times \frac{35}{3}$

= R1. 8*a*. $\times$ 35

= R52. 8*a*. *Ans.*

[R1. 8*a*. $\times$ 35 = R35 + R17. 8*a*. = R52. 8*a*.]

Example 2. How much must be paid for 17 maunds of sugar, when 8 maunds cost R74 ?

The cost of 8 maunds = R74,..................(1)

∴ .... 1 maund = R74 $\times \frac{1}{8}$,

∴ .... 9 maunds = R74 $\times \frac{9}{8}$

= R83. 4*a*. ;.........(2)

∴ ....17 maunds = R157. 4*a*. (by addition).

Here we avoid the multiplication by 17 which cannot be factorised.

Example 3. If 6 maunds of wheat cost R7. 8*a*., how much can be purchased for R12. 8*a*. ?

R7. 8*a*. = 120*a*.

R12. 8*a*. = 200*a*.

120*a*. is the cost of 6 maunds,

∴ 40*a*. .... 2 ....,

∴ 200*a*. ....10 .... *Ans.*

The artifice employed in this example should be carefully noted. We use here 40a. as the unit common to 120a. and 200a.

Note. The student will here observe that we have obtained the result more quickly than if we had proceeded as in the foregoing examples. As a general rule it will be found more convenient to pass from what is given to what is wanted by way of the *greatest* unit common to both of them. Here we have passed from 120*a*. to 200*a*. by way of 40*a*., which is the G. C. M. of 120*a*. and 200*a*.

Example 4. If $\frac{9}{8}$ of an estate be worth R90, what is the value of $\frac{8}{9}$ of it ?

$\frac{9}{8}$ of the estate is worth R90,

∴ the estate is worth R90 $\times \frac{8}{9}$,

∴ $\frac{8}{9}$ of the estate is worth R90 $\times \frac{8}{9} \times \frac{8}{9}$ or R80. *Ans.*

Example 5. Express 1 mile in metres, 32 metres being equal to 35 yards.

35 yards = 32 metres,

∴ 5 yards = $\frac{32}{7}$ metres,

∴ 1760 yards = $\frac{32 \times 352}{7}$ metres or $1609\frac{1}{7}$ metres.

Here we have passed from 35 yd. to 1760 yd. by way of 5 yd., the G. C. M. of the two quantities.

EXAMPLES. 136.

1. If 30 bullocks cost R810, what is the cost of 77 bullocks ?

2. If 5 cwt. cost R6. 4*a.*, what is the cost of 16 cwt. ?

3. Find the value of 21 yd. of cloth when 44 yd. cost R33.

4. If 7 pieces of cloth cost R350, what will 13 pieces cost ?

5. If 13 reams of paper cost £6. 10*s.*, what is the price of 21 reams ?

6. If 23 copies of a book cost R35. 15*a.*, how much will 31 copies cost ?

7. If the cost of 60 eggs be 1*s.* 3*d.*, how many can be purchased for 5*s.* ?

8. How many oranges can be bought for R2. 3*a.*, at the rate of 8*a.* 9*p.* a dozen ?

9. If 4 cwt. cost £1. 1*s.* 1*d.*, what will 2 tons 8 cwt. cost ?

10. If 35 sheep produce 20 lb. of wool, what would 63 sheep produce ?

11. If 42 men earn R3 . 4 . 6 for a day's work, what would 112 men earn ?

12. If the railway fare for 100 miles be R3 . 8 . 6, what is the fare for 275 miles ?

13. If 8 persons can be boarded for £3, how many can be boarded for £7. 10*s.* ?

14. What is the value of 600 pins at the rate of 2*d.* per gross ?

15. If $7\frac{3}{4}$ lb. cost 2*s.* 7*d.*, what will $1\frac{1}{4}$ cwt. cost ?

16. If $\frac{3}{4}$ of a maund cost R3. 12*a.*, find the cost of $3\frac{3}{4}$ seers.

17. If $\frac{3}{7}$ of an estate be worth R2700, what is the value of $\frac{5}{8}$ of the estate ?

18. If $\frac{7}{24}$ of a cargo be worth £357. 7*s.*, what is the value of $\frac{3}{8}$ of the cargo ?

19. The owner of ·375 of a ship sold $\frac{3}{5}$ of his share for R5040 ; find the value of ·875 of the ship at the same rate.

20. A man lost $\frac{2}{5}$ of his money, and then spent $\frac{2}{3}$ of the remainder; after which he had R120 left: how much did he lose?

21. A gentleman possessing $\frac{3}{14}$ of an estate sold $\frac{2}{7}$ of $\frac{5}{13}$ of his share for R241. 4*a.*; what would ·2 of $\frac{3}{16}$ of the estate sell for at the same rate?

22. If a man walk 46 miles in 3 days, in how many days will he walk 115 miles?

23. If the rent of 34 acres is R21. 4*a.*, what is the rent of 51 acres?

24. A servant's wages being £10. 8*s.* per annum, how much ought she to receive for 7 weeks? [1 year = 52 weeks.]

25. A man's annual income is R4088; what does he receive for 15 days? [1 year = 365 days.]

26. If 27 bus. $2\frac{1}{2}$ pk. cost £10 . 7 . $2\frac{1}{4}$, what is the cost of a bushel and a half?

27. If 3 cwt. 3 qr. cost £6. 15*s.*, what will be the cost of 2 cwt.?

28. A sack of potatoes weighs 89 seers; if 6 such sacks cost R22. 4*a.*, what will be the cost of 22 seers?

29. If 17 ac. 2 ro. 38 po. supply 3 horses, how many acres will supply 16 horses?

30. If the carriage of 25 maunds for 500 miles cost R9. 6*a.*, what weight can be carried the same distance for R8?

31. If a piece of land worth R375 yield an income of R7. 8*a.*, what should be the value of a piece of land which yields an income of R18. 12*a.*?

32. If $3\frac{2}{3}$ acres can be mown in 7 days, how long will it take to mow $9\frac{1}{3}$ acres?

33. If 350 rupees weigh 9 lb., how many pounds will 625 rupees weigh?

34. In a certain time the population of a town increased from 78960 to 82908; find by how many the population of another town of 92360 inhabitants would have increased at the same rate in the same time.

35. A man walks 4 miles in an hour; how many yards does he walk in a minute?

36. A railway train travels at the rate of 20 miles in $1\frac{1}{2}$ hours; find the rate per minute.

37. An express train goes 10 times as fast as a man who walks 6 ft. in a second; how many miles per hour does it go?

38. Express $7\frac{1}{2}$ miles in kilometres, 5 kilometres being equal to 5456 yards.

39. If $6\frac{1}{2}$ grammes be equal to 105 grains, express a pound Avoir. in grammes.

40. Convert £3 . 7 . 6 to Indian money, given ₨8 = 15*s*.

41. Convert 7 tons to maunds, given 35 seers = 72 lb.

42. Express $3\frac{1}{3}$ dollars in Indian money, 9 dollars being equal to 20 rupees.

43. If 8 horses eat as much as 6 oxen, how many oxen will eat as much as 20 horses ?

44. If 4 men do as much work as 6 boys, how many men will do the work of 18 boys ?

45. If the price of 7 horses and 5 oxen is ₨520, and that of an ox is ₨20, find the price of a horse.

46. If the weight of 5 rupees and 3 pice is 1200 grains, and that of a rupee is 180 grains, find the weight of a pice.

47. If 8 horses and 20 sheep eat the grass of 7 acres in a certain time, how many acres will feed 10 horses and 24 sheep for the same time, supposing a horse to eat as much as 4 sheep ?

48. If 15 chairs and 2 tables cost ₨400, find the cost of 12 chairs and 3 tables, the cost of 10 chairs being equal to that of 4 tables.

49. If the wages of 4 men be equal to those of 5 women, what will 8 women earn in a day, the daily earnings of 10 men being ₨1. 9*a*. ?

50. If a shop-keeper uses a weight of 15 oz. for 1 lb., how much will a customer lose in buying 24 lb. ?

Example 6. If 35 men finish a piece of work in 8 days, how many men will finish it in 10 days ?

In 8 days the work is done by 35 men,
∴ ... 2 .... 35×4......,
∴ ...10 .... $\frac{35 \times 4}{5}$ ......,
or 28 men. *Ans.*

Here we have passed from 8 *da. to* 10 *da. by way of* 2 *da., the G. C. M. of the two quantities.*

Example 7. If the penny loaf weighs 12 oz. when wheat is £4 a quarter, what should it weigh when wheat is £4. 16*s*. a quarter ?

£4 = 80*s*. ; £4. 16*s*. = 96*s*.

When wheat is 80*s*. a qr. the loaf weighs 12 oz.,
∴ .... 16*s*. .... (12×5) oz.
∴ .... 96*s*. .... $\frac{12 \times 5}{6}$ oz.,
or 10 oz. *Ans.*

Here we have passed from 80*s. to* 96*s. by way of* 16*s., the G. C. M. of the two quantities.*

Example 8. A garrison of 1200 men is provisioned for 60 days; if after 15 days 300 men leave the garrison, how long will the remaining provisions last the men left?

The provisions left would last 1200 men 45 days,

∴ they would last 300 men (45 × 4) days,

∴ they would last 900 men $\frac{45 \times 4}{3}$ days, or 60 days. *Ans.*

EXAMPLES. 137.

1. If 9 men can mow a field in 4 days, in how many days could 6 men mow the same field?

2. If 12 horses can plough a field in 7 days, in how many days could 14 horses plough it?

3. If 16 men can finish a piece of work in 5 days, in how many days could 10 men do it?

4. If 25 men reap a field in 12 days, how many men could reap it in 20 days?

5. If 7 cwt. feed 15 horses for 8 days, how many horses would they feed 12 days?

6. If 28 maunds can be carried 50 miles for a certain sum, what weight can be carried 125 miles for the same sum?

7. If 16 bighas can be rented for 9 months for R10, for how many months can 36 bighas be rented for the same sum?

8. A man walks from Calcutta to Hugly in 6 hours, walking 4 miles an hour; how long would he take if he rode at the rate of 9 miles an hour?

9. If the twopenny loaf weighs 20 oz. when wheat is £4. 16*s.* a quarter, what should it weigh when wheat is £8 a quarter?

10. If the sixpenny loaf weighs 64 oz. when wheat is 6*s.* 9*d.* a bushel, what is the price of wheat per bushel when the sixpenny loaf weighs 48 oz.?

11. From a mass of silver I can make 64 plates weighing 3 oz. each, how many 4 oz. plates could I make from the same?

12. A garrison of 1200 men has provisions for 75 days; how long would they last if the garrison were reduced to 500 men?

13. A fortress is provisioned for 4 weeks at the rate of 20 oz. a day for each man: if only 12 oz. be served out daily for each man, how long can the place hold out?

14. A garrison of 1000 men is provisioned for 70 days: if after 20 days the garrison is re-enforced by 200 men, how long will the remaining provisions last?

15. If 7 men can mow a meadow in 7 days, working 10 hours a day, how many additional hours a day must they work to do it in 5 days?

16. If I borrow R300 for 8 months, for how long should I lend R400 in return?

17. If it requires $27\frac{1}{2}$ yd. of carpet 9 in. wide to cover a room, how many yards of carpet 7 in. wide will be necessary to cover the same room?

EXAMPLES. 138.

1. If 30 seers of corn feed 6 horses for 4 days, how many horses would they feed for 12 days?

2. If 30 seers of corn feed 6 horses for 4 days, how many horses would 25 seers feed for the same time?

3. If 30 seers of corn feed 6 horses for 4 days, for how many days would they feed 8 horses?

4. If 30 seers of corn feed 6 horses for 4 days, for how many days would $52\frac{1}{2}$ seers feed the same number of horses?

5. If 30 seers of corn feed 6 horses for 4 days, how many seers will feed 10 horses for the same time?

6. If 30 seers of corn feed 6 horses for 4 days, how many seers will feed the same number of horses for 9 days?

7. If 20 men reap a field of 6 acres in 40 hours, in how many hours will 35 men reap the same field?

8. If 20 men reap a field of 6 acres in 40 hours, how many men will reap the same field in 25 hours?

9. If 20 men reap a field of 6 acres in 40 hours, how many acres will 35 men reap in the same time?

10. If 20 men reap a field of 6 acres in 40 hours, how many men will reap 15 acres in the same time?

11. If 20 men reap a field of 6 acres in 40 hours, how many acres will they reap in 55 hours?

12. If 20 men reap a field of 6 acres in 40 hours, in how many hours will they reap a field of 8 acres?

13. When rice is R3 per md., how many people can be fed for the same sum that would feed 90 people when rice is R2. 8*a.* per md.?

14. If 1 lb. of flour cost 9*p.* when wheat is R3 per md., what should be the price of a md. of wheat when 1 lb. of flour costs 1*a.*?

15. How many yards of cloth worth 4*a.* 6*p.* per yard must be given in exchange for 30 yards at 3*a.* 6*p.* per yard ?

16. Find the length of a strip of land 20 yd. wide, that should be given in exchange for a piece measuring 40 yd. by 30 yd.

17. If 3 lb. of tea cost as much as 10 lb. of sugar, how much tea should be given in exchange for 25 lb. of sugar ?

18. A brewer receives 10 doz. of brandy in exchange for 4 barrels of ale worth £3. 10*s.* a barrel ; what does the brandy cost him per bottle ?

19. A man contracts to perform a piece of work in 20 days and immediately employs upon it 16 men. At the end of 12 days the work is only half done ; what additional number of men must he employ to fulfil the contract ?

20. A merchant of Calcutta indented from London goods worth £640, and paid £10 for freight. If a rupee is equal to 1*s.* 9*d.*, for how many annas must he sell goods, for which he paid 1*s.* to the London manufacturer, in order to gain £50 on the whole outlay ?

***21.** If a quantity of flour serve 36 men for 15 days at the rate of 12 oz. a day for each man, how many ounces a day will each man get, when the same quantity of flour serves 42 men for the same time ?

Example 9. If 10 men can do a piece of work in 12 days, working 7 hours a day, how many hours a day must 6 men work to do the same in 14 days ?

10 men can do the work in (12×7) hours,

$\therefore$ 2 .... $(12 \times 7 \times 5)$ ......,

$\therefore$ 6 .... $\frac{12 \times 7 \times 5}{3}$ ...... ;

$\therefore$ to complete the work in 14 days, they must work $\frac{12 \times 7 \times 5}{3 \times 14}$ hours, or 10 hours a day.

Example 10. If a number of men can dig a trench 210 yd. long, 3 wide and 2 deep, in 5 days of 11 hours each, in how many days of 10 hours each, will they dig a trench 420 yd. long, 6 wide and 3 deep ?

$(210 \times 3 \times 2)$ cu. yd. is dug in 55 hours.

$\therefore$ 1 .... $\frac{55}{210 \times 3 \times 2}$ hours,

$\therefore$ $(420 \times 6 \times 3)$ .... $\frac{55 \times 420 \times 6 \times 3}{210 \times 3 \times 2}$ hours,

or 330 hours ;

$\therefore$ the number of days required $= \frac{330}{10} = 33$.

Example 11. If 8 oxen or 6 horses eat the grass of a field in 10 days, in how many days will 5 oxen and 4 horses eat it?

8 oxen eat as much as 6 horses,

$\therefore$ 1 ox eats.................. $\frac{6}{8}$ horses,

$\therefore$ 5 oxen eat.................. $\frac{6\times5}{8}$ horses, or $\frac{15}{4}$ horses;

$\therefore$ 5 oxen and 4 horses eat as much as $(\frac{15}{4}+4)$ horses, or $\frac{31}{4}$ horses.

Now, 6 horses eat the grass in 10 days,

$\therefore$ 1 horse will eat............ 10×6 ....,

$\therefore$ $\frac{31}{4}$ horses .... $\frac{10\times6\times4}{31}$ ... ...,

or $7\frac{23}{31}$ days.

EXAMPLES. 139.

1. If 5 men can do a piece of work in 8 days, working 7 hours a day, how many men will do the same piece of work in $4\frac{2}{3}$ days, working 10 hours a day?

2. If 9 men can do a piece of work in 7 days working 10 hours a day, how many hours a day must 6 men work to do the same in 30 days?

3. If 12 men can do a piece of work in 8 days of 7 hours each, in how many days of 6 hours each can 10 men do the same?

4. If 20 masons build a wall, 50 ft. long, 2 ft. thick and 14 ft. high, in 12 days, in how many days will they build a wall, 55 ft. long, 4 thick and 16 high?

5. If 20 men dig a trench, 100 yd. long, 5 wide and 3 deep, in 3 days, how many men will dig a trench 150 yd. long, 6 wide and 2 deep, in the same time?

6. If 5 men reap a rectangular field, 200 ft. by 50 ft., in 2 days of 10 hours each, in how many days of 8 hours each can they reap another, 300 ft. by 40 ft?

7. If 6 men and 8 boys can do a piece of work in 18 days, in how many days will 3 men and 5 boys do it?

8. If 5 men, 7 women or 9 boys can dig a ditch in 15 days, in how many days can 1 man, 1 woman and 1 boy dig it?

9. 4 men do as much work as 6 boys in the same time, and a piece of work in which 20 men and 15 boys are engaged take 25 days; how many days would it take if 15 men and 20 boys were employed upon it?

10. If 10 gas-burners, which are lighted 4 hours every evening for 15 days, consume a quantity of gas which costs ₨3, for how many days can 12 burners be lighted 5 hours every evening at the same cost?

11. If a piece of matting, measuring 7 ft. 4 in. by 5 ft., cost R6. 14*a.*, what will be the cost of a piece of the same matting, measuring 10 ft. by 6 ft. 6 in.?

12. If the cost of printing a book of 250 pages, with 21 lines on each page, and on an average 10 words in each line, be R125, find the cost of printing a book of 300 pages, with 14 lines on each page and 8 words in each line.

13. If 8 men, working 7 hours a day, take 12 days to complete a piece of work, how long will 14 boys, working 6 hours a day, take to do the same work, the work of one man being equal to that of two boys in the same time?

14. If the feeding of 8 horses and 20 sheep for a month cost R100, what will be the cost of feeding 6 horses and 50 sheep for a month, supposing that 2 horses eat as much as 15 sheep?

BANKRUPTCIES, RATING, TAXING, ETC.

235. Rates are certain payments which local bodies such as Municipal Boards, District Boards, etc., are empowered by law to levy in order to meet local expenses. The amount which a man pays depends upon the annual value of his property. This value is called the *assessment* or *rateable value* of his property. Rates are usually calculated as "so much in the rupee or pound." Suppose the rate is 1*a.* 6*p.* in the rupee. It means that 1*a.* 6*p.* has to be paid on every R1 of the rateable value.

Taxes are sums of money fixed by and paid to the Government of the country. Taxes are uniform throughout the country, while rates vary from place to place. In India 'rates levied by local bodies are frequently called taxes.

Income-tax is a tax levied on a man's annual income and is also payable to the Government of the country. It is levied in such a way that a poor man pays nothing, and its rate increases as income increases. It is also reckoned as "so many pies in the rupee", or as "so many pence in the pound."

A bankrupt is a man who has run into debt and whose debts exceed the amount of money and the value of the property he possesses.

A debtor is a man who owes money.

A creditor is a man to whom money is owing.

Liabilities or **Debts** are the sum total of money which a man owes. His **assets** consist of the money which he has in cash or which could be realised from his various debtors, and the money which would be obtained by the sale of his property.

Dividends are the payments which a bankrupt can make to his creditors. They are reckoned as "so much on each rupee or pound" of his liabilities.

236. *Example* 1. A bankrupt's debts are R7240, and his assets are R5430; how much can he pay in the rupee?

In the place of R7240 he can pay R5430,

∴ ....R1 ....R$\frac{5430}{7240}$, or R$\frac{3}{4}$,

or 12 annas;

∴ he can pay 12*a.* in the rupee.

Example 2. A bankrupt's debts amount to £3720, and he pays 18*s.* in the pound; what are his assets?

First Method:

In the place of £1 he pays 18*s.*,

∴ .... £3720..............(3720 × 18)*s.*,

∴ his assets are (3720 × 18)*s.*, or £3348.

Second Method:

He pays 18*s.* in £1,

∴ ....£$\frac{9}{10}$ in £1;

∴ his assets = £3720 × $\frac{9}{10}$ = £3348.

Third Method:

$18s. = 20s. - 2s. = £(1 - \cdot1)$,

∴ his assets = £3720 × (1 − ·1) = £(3720 − 372)

= £3348.

Example 3. A man pays an income-tax of R125 at the rate of 5*p.* in the rupee; find his income.

R125 = 24000*p.*

He pays 5*p.* on R1,

∴ ....24000*p.* on R4800;

∴ his income is R4800.

Example 4. After paying an income-tax of 6*d.* in the pound a man has £780 left; find his gross income.

He has 19*s.* 6*d.* left out of £1,

∴ .... 1*s.* ....£$\frac{2}{39}$;

∴ ....(780 × 20)*s* ....£$\frac{2 \times 780 \times 20}{39}$, or £800;

∴ his gross income is £800.

Example 5. A man pays an income-tax of 6*p.* in the rupee on $\frac{2}{3}$ of his income ; how much in the rupee does he pay on his whole income ?

He pays 6*p.* in the rupee on $\frac{2}{3}$ of his income, *i.e.*, he pays $\frac{6}{16 \times 12}$ of $\frac{2}{3}$ of his income, or $\frac{1}{48}$ of his income. But $\frac{1}{48}$ of R1 = 4*p.* ; ∴ he pays 4*p.* in the rupee on his whole income.

Example 6. When income-tax is 5*p.* in the rupee, a person has to pay R20 more than when the tax was 4*p.* in the rupee ; find his income.

Difference of tax is 1*p.* when the income is R1,

∴ .... (20 × 16 × 12)*p*.............. R(20 × 16 × 12),

or R3840 ;

∴ his income is R3840.

EXAMPLES. 140.

1. Find the income tax on R3600 at 5*p.* in the R.

2. How much will a poor-rate of 2*s.* 6*d.* in the £ produce in a parish where the whole property is rated at £3768. 8*s.* ?

3. Find the amount of road-cess, at 6*p.* in the R, on a rental of R5500.

4. A bankrupt's debts are R7880, and his assets R4925 ; how much in the rupee can he pay ?

5. A bankrupt's effects amount to R6131 . 5 . 4, and his debts are R36788 ; how much can he pay in the rupee ?

6. If a man has to pay £9 . 7 . 6 for income-tax on an income of £750, what is the rate of tax per £ ?

7. A bankrupt's debts are £3798, and he pays 12*a.* 6*p.* in the rupee ; what are his assets ?

8. A bankrupt's assets are £2900, and he pays his creditors 14*s.* 6*d.* in the £ ; what do his debts amount to ?

9. A man pays an income-tax of R40 at the rate of 4*p.* in the rupee ; find his income.

10. If I pay £16. 10*s.* 6*d.* for income-tax, being at the rate of 10*d.* in the £, what is my income ?

11. After paying an income-tax of 5*p.* in the rupee a man has R2805 left ; find his gross income ?

12. A person after paying 7*d.* in the £ for income-tax has £174. 15*s.* left ; what was his gross income ?

13. A creditor received 16*s.* 3*d.* in the £, and thereby lost £135. 10*s.* ; how much was due to him ?

14. A man pays an income-tax of $4p$. in the rupee on $\frac{3}{4}$ of his income; what rate per rupee does he pay on his whole income?

15. A man pays an income-tax of $8p$. in the rupee on $\frac{3}{8}$ of his income; what fraction of his whole income is paid as income-tax?

16. When the income-tax is $9d$. in the pound a person has to pay £40 less than when the tax was $1s$. in the pound; find his income.

17. When the income-tax is $7d$. in the pound a person has to pay £25 more than when the tax was $5d$. in the pound; find his income.

PROBLEMS RELATING TO WORK DONE IN A CERTAIN TIME.

237. The following examples will explain the methods to be adopted in solving problems relating to work done in a certain time. In dealing with the work of different agents, the amount of work they can do *in the same time* (one day or one hour or one minute, etc., according to the data given) should first be ascertained.

238. *Example* 1. *A* can do a piece of work in 7 days, and *B* can do it in 9 days; how long will *A* and *B*, working together, take to do the work?

A can do the work in 7 days, ∴ *A* can do $\frac{1}{7}$ of it in 1 day;
B .... 9........, ∴ *B*......... $\frac{1}{9}$ ....;
∴ *A* and *B* together can do $(\frac{1}{7}+\frac{1}{9})$ of it in one day;
i.e., ... $\frac{16}{63}$ ....;
∴ .... the whole in $\frac{63}{16}$ days;
∴ the time required $=\frac{63}{16}$ days $=3\frac{15}{16}$ days.

Example 2. *A* and *B* together can perform a piece of work in 5 days, and *A* alone can do it in 8 days; what time will it take *B* to do it alone?

A and *B* can do the work in 5 days, ∴ they can do $\frac{1}{5}$ of it in 1 day;
A alone .... 8 ......, ∴ he............ $\frac{1}{8}$ ....;
∴ *B* alone can do $(\frac{1}{5}-\frac{1}{8})$ of it in 1 day,
i.e., ... $\frac{3}{40}$ ....,
∴ .... the whole in $\frac{40}{3}$ days, or $13\frac{1}{3}$ days. *Ans.*

Example 3. A vessel can be filled by a pipe in 25 minutes, and it can be emptied by a waste-pipe in 20 minutes; if both the pipes be opened when the vessel is full, how soon will it be empty?

1st pipe fills $\frac{1}{25}$ of the vessel in 1 minute,
2nd pipe empties $\frac{1}{20}$ ....;
∴ when both pipes are open
$(\frac{1}{20}-\frac{1}{25})$ of the vessel is emptied in 1 minute,
i.e., $\frac{1}{100}$ ....;
∴ the whole will be emptied in 100 minutes.

Example 4. *A* and *B* can do a piece of work in 5 hours; *A* and *C* in 4 hours; *B* and *C* in $3\frac{1}{2}$ hours. In what time can *A* alone do it?

A and *B* can do $\frac{1}{5}$ in 1 hour;
A and *C* .... $\frac{1}{4}$ ....;
∴ two men of *A*'s strength, and *B* and *C* can do $\frac{1}{5}+\frac{1}{4}$ in 1 hour;
but *B* and *C* can do $\frac{2}{7}$ in 1 hour;
∴ two men of *A*'s strength can do $\frac{1}{5}+\frac{1}{4}-\frac{2}{7}$ in 1 hour;
or $\frac{23}{140}$ in 1 hour;
∴ *A* can do $\frac{23}{280}$ in 1 hour;
∴ *A* can do the whole in $\frac{280}{23}$ hours, or $12\frac{4}{23}$ hours. *Ans.*

Example 5. *A* does $\frac{4}{5}$ of a piece of work in 20 days; he then calls in *B*, and they finish the work in 3 days; how long would *B* take to do the whole work by himself?

A does $\frac{4}{5}$ of the work in 20 days,
∴ *A* can do $\frac{1}{25}$ of the work in 1 day,
but *A* and *B* do $\frac{1}{5}$ of the work in 3 days,
∴ *A* and *B* do $\frac{1}{15}$ of the work in 1 day,
∴ *B* does $(\frac{1}{15}-\frac{1}{25})$ of the work in 1 day,
i.e., *B* can do $\frac{2}{75}$ ....,
∴ *B* can do the whole work in $\frac{75}{2}$ days, or $37\frac{1}{2}$ days. *Ans.*

EXAMPLES. 141.

(*Examples* 1—7 *should be taken orally.*)

1. *A* can do a piece of work in 1 day, *B* can also do it in the same time; how long will they take if both work together?

2. *A* can do a piece of work in 1 day, *B* can do it in 2 days; in what time will they do it if they work together?

3. *A* can do a piece of work in 2 hours, and *B* can do it in 3 hours ; how long will they take if both work together ?

4. If *A* does a piece of work in $1\frac{2}{3}$ hours, which *B* can do in $2\frac{1}{2}$ hours ; in what time will they do it if they work together ?

5. A cistern can be filled by a pipe in 2 hours, and it can be emptied by a waste-pipe in 3 hours ; in what time will it be filled if both the pipes are turned on together when the cistern is empty ?

6. A cistern can be filled by a pipe in 3 hours, and it can be emptied by a waste-pipe in 2 hours ; if both the pipes are opened when the cistern is full, how soon will it be empty ?

7. *A* and *B* can do a piece of work in half-an-hour, and *B* alone can do it in one hour ; in what time can *A* alone do it ?

8. *A* can do a piece of work in 10 hours ; *B* can do it in 8 hours. In what time will they do it if they work together ?

9. If *A* does a piece of work in 4 days, which *B* can do in 5, and *C* can do in 6 ; in what time will they do it, all working together ?

10. A cistern can be filled by one pipe in $3\frac{1}{2}$ hours, by a second in $3\frac{1}{3}$ hours, and by a third in $5\frac{1}{4}$ hours ; in what time will it be filled by all the three in action together ?

11. *A* can reap a field in 10 days ; *B* can reap it in 12 days ; *C* can reap it in 15 days ; how long will it take them all together to reap it, and what part of the work will be done by each ?

12. *A* and *B* together can dig a trench in 4 days, and *A* alone can dig it in 6 days ; in how many days can *B* alone dig it ?

13. Two pipes, *P* and *Q*, together can fill a cistern in 20 minutes, and *P* alone in 30 minutes : how long would *Q* alone take ?

14. A vessel can be filled by one pipe in 8 minutes, by a second pipe in 10 minutes ; it can be emptied by a waste-pipe in 12 minutes : in what time will the vessel be filled if all the three be opened at once ?

15. A vessel has 3 pipes connected with it, 2 to supply and 1 to draw off. The first alone can fill the vessel in $4\frac{1}{2}$ hours, the second in 3 hours, and the third can empty it in $1\frac{1}{2}$ hours. If all the pipes be opened when the vessel is half-full, how soon will it be empty ?

16. *A* and *B* can do a piece of work in 6 days ; *A* and *C* in $5\frac{1}{2}$ days ; *B* and *C* in 4 days. In what time could each do it ?

17. *A* and *B* can mow a field in $3\frac{1}{2}$ days ; *A* and *C* in 4 days ; *B* and *C* in 5 days. In what time could they mow it, all working together ?

18. *A* does $\frac{3}{8}$ of a piece of work in 9 days ; he then calls in *B*, and they finish the work in 6 days. How long would *B* take to do the whole work by himself ?

19. *A* does $\frac{7}{10}$ of a piece of work in 15 days; he does the remainder with the assistance of *B* in 4 days. In what time could *A* and *B* together do it?

20. *A* can do a piece of work in 16 days, *B* in 10 days; *A* and *B* work at it together for 6 days, and then *C* finishes it in 3 days: in how many days could *C* have done it alone?

21. *A* and *B* together can do a piece of work in 6 days, *B* alone could do it in 16 days. If *B* stops after 3 days, how long afterwards will *A* have finished the work?

22. *A* and *B* can reap a field in 30 days, working together. After 11 days, however, *B* is called off, and *A* finishes it by himself in 38 days more. In what time could each alone do the whole?

23. *A*, *B* and *C* together can do a piece of work in 6 days, which *B* alone can do in 16 days, and *B* and *C* together can do in 10 days; in how many days can *A* and *B* together do it?

24. Five men can do a piece of work in 2 hours, which 7 women could do in 3 hours, or 9 children in 4 hours. How long would 1 man, 1 woman and 1 child together take to do the work?

25. *A* can do a piece of work in 4 hours, *B* and *C* can do it in 3 hours, *A* and *C* can do it in 2 hours. How long would *B* alone take to do it?

26. *A* and *B* together can do a piece of work in 8 days; *B* alone can do it in 12 days; supposing *B* alone works at it for 4 days, in how many more days could *A* alone finish it?

27. Three taps, *A*, *B* and *C*, can fill a cistern in 10, 15 and 20 minutes respectively. They are all turned on at once, but after 3 minutes *C* is turned off. How many minutes longer will *A* and *B* take to fill the cistern?

28. Three taps, *A*, *B* and *C*, can fill a cistern in 10 min., 12 min. and 15 min. respectively. They are all turned on at once, but after $1\frac{1}{4}$ min. *B* and *C* are turned off. How many minutes longer will *A* take then to fill the cistern?

29. Two pipes, *A* and *B*, can fill a cistern in 3 hours and 4 hours respectively; a waste-pipe *C* can empty it in 2 hours; if these pipes be opened in order at 7, 8 and 9 o'clock, find when the cistern will be filled.

30. A piece of work was to be completed in 40 days; a number of men employed upon it did only half the work in 24 days; 16 more men were then set on, and the work was completed in the specified time: how many men were employed at first?

31. *A* can do a certain work in the same time in which *B* and *C* together can do it. If *A* and *B* together could do it in 10 days, and *C* alone in 50 days, in what time could *B* alone do it?

32. *A* and *B* can do a piece of work in 10 days, *B* and *C* in 15 days, and *A* and *C* in 25 days ; they all work at it together for 4 days ; *A* then leaves, and *B* and *C* go on together for 5 days more, and then *B* leaves : in how many more days will *C* complete the work ?

33. *A* takes twice as much time as *B* and thrice as much time as *C* to finish a piece of work ; working together they can finish the work in 2 days ; find the time each will take to finish the work.

34. A cistern can be filled by two pipes in 30 and 40 minutes respectively ; both the pipes were opened at once, but after some time the first was shut up, and the cistern was filled in 10 minutes more. How long after the pipes had been opened was the first pipe shut up ?

35. A cistern has 3 pipes, *A*, *B* and *C* ; *A* and *B* can fill it in 2 and 3 hours respectively ; *C* is a waste-pipe. If all the three pipes be opened at once $\frac{7}{24}$ of the cistern will be filled up in 30 minutes. In what time can *C* empty the full cistern ?

36. Forty men can finish a piece of work in 40 days ; but if 5 men leave the work after every tenth day, in what time will the whole work be completed ?

37. *A* is engaged to do a piece of work ; after working for 2 days he leaves, and *B* who is called in finishes the remaining part of the work in 9 days. Had *A* left the work after working for 3 days, *B* would have finished the remainder of the work in 6 days. In how many days can each, working alone, finish the whole work ?

38. A vessel has 2 pipes attached to it, one to supply and one to draw off. The supply-pipe can fill the vessel in 40 minutes, and the waste-pipe can empty it in an hour. If the supply-pipe and waste-pipe are kept open in alternate mintutes, in what time will the vessel be filled ?

39. A boy and a girl begin to fill a cistern ; the boy brings a quart at the end of every 2 minutes and the girl brings a pint every 3 minutes In what time will the cistern be filled, if it holds $4\frac{1}{2}$ gallons ?

PROBLEMS RELATING TO CLOCKS.

239. The clock dial is divided into 60 equal parts and in one minute the minute-hand travels through one such part. Each of these parts is called a "minute-division" or "minute-space". While the minute-hand travels all round the dial, that is, through 60 minute-divisions, the hour-hand travels through only 5 minute-divisions. Therefore while the minute-hand moves over 1 division,

the hour-hand moves over $\frac{5}{60}$ or $\frac{1}{12}$ of a division. It thus follows that in one minute the minute-hand gains $(1-\frac{1}{12})$ or $\frac{11}{12}$ of a minute-division over the hour-hand. Consequently in any number of minutes the minute-hand will gain $\frac{11}{12}$ of that number of minute-divisions over the hour-hand. Hence whatever may have been the positions of the hands, in 24 minutes, for example, the minute-hand will gain $\frac{11}{12}\times 24$ or 22 minute-divisions over the hour-hand.

Example. Two clocks are at 12 noon; one gains 40 seconds and the other loses 50 seconds in 24 hours; after what interval will the one have gained 16 minutes on the other, and what time will each then show? What will be the true time when the first clock indicates 3 P. M. on the following day?

(i) The one clock gains on the other (40 + 50) sec. in 24 hours;

i.e., it gains $\frac{3}{2}$ min. in 1 day,

∴ .... 1 .... $\frac{2}{3}$ ....,

∴ .... 16 ....$\frac{2\times 16}{3}$ days, or $\frac{32}{3}$ days,

or 10 days 16 hours (true time).

(ii) Now in $\frac{32}{3}$ days the first clock gains $\frac{32}{3}\times 40$ sec. or $7\frac{1}{9}$ min., and the second loses $\frac{32}{3}\times 50$ sec. or $8\frac{8}{9}$ min.

But the correct clock, at the end of the interval (*i.e.*, 10 days 16 hours) will show 4 A. M.

Therefore the first will show 4 h. $7\frac{1}{9}$ min. A. M.;

and the second will show 3 h. $51\frac{1}{9}$ min. A. M.

(iii) From 12 noon to 3 P. M. on the following day there are 27 hours.

24 hr. 40 sec. of the first clock = 1 day of the correct clock,

i.e., $\frac{2161}{90}$ hr. .... = 1 day ....,

∴ 1 hr. .... = $\frac{90}{2161}$ day ....,

∴ 27 hr. .... = $\frac{90\times 27}{2161}$ day ....,

Now $\frac{90\times 27}{2161}$ day = 1 da. 2 hr. $59\frac{541}{2161}$ min.

∴ When the first clock indicates 3 P. M. on the following day, the true time will be 2 h. $59\frac{541}{2161}$ min. P. M.

EXAMPLES. 142.

1. A watch which is 5 minutes too fast at 12 o'clock on Sunday gains 2 min. 15 sec. per day; what time will it indicate at half past 2 P. M. on the following Tuesday?

2. A clock which is 10 minutes too fast at 9 A. M. on Monday loses 3 min. per day; what time will it show at a quarter to 3 P. M. on the following Wednesday?

3. One clock gains 2 minutes, and a second gains 3 minutes in 24 hours; the first is put right at 12 o'clock on Tuesday, the second at 3 P. M. on the following Wednesday: when will they indicate the same time?

4. Two clocks are exactly together at 8 A. M. on a certain day; one loses 6 seconds and the other gains 10 seconds in 24 hours; when will the one be half an hour before the other, and what time will each clock then show?

5. A watch which shows correct time at noon on Tuesday gains $2\frac{1}{2}$ min. a day: what is the correct time on the following Sunday when it is 9 A. M. by the watch?

6. Two clocks strike 9 together on Monday morning; on Tuesday morning one wants 10 minutes to 11, when the other strikes 11. How much must the slower be put on, or the faster put back, that they may strike 9 together in the evening?

7. A clock which was 1·4 min. fast at a quarter to 11 P. M. on Dec. 2, was 8 min. slow at 9 A. M. on Dec. 7; when was it exactly right?

8. A clock which was 1·2 min. fast at a quarter to 11 P. M. on Nov. 28, was exactly right at 11-30 P. M. the following day. How many minutes was it slow at a quarter to 2 P. M. on Dec. 7?

9. A clock which is $7\frac{1}{2}$ min. fast on Tuesday at noon, is $4\frac{1}{2}$ min. fast at midnight on the following Monday; how much did it lose in a day?

10. A watch which gains $7\frac{1}{2}$ min. in a day is 12 minutes fast at midnight on Sunday. What will be the true time when the watch indicates 4-32 P. M. on Wednesday?

11. Two clocks, of which one gains $3\frac{1}{2}$ min. and the other loses $2\frac{1}{2}$ min. in 24 hours, were both within 1 min. of the true time, the former fast and the latter slow, at noon on Sunday last; they now differ from one another by 15 min.: find the day of the week and the hour of the day.

12. A clock loses $2\frac{1}{2}$ minutes a day; how must the hands be placed at 9 A. M. so as to point to true time at noon?

13. One clock gains $12\frac{1}{2}$ minutes, and another gains $7\frac{1}{2}$ minutes in 12 hours. They are set right at noon on Sunday. Determine the time indicated by each clock, when the one appears to have gained $21\frac{2}{3}$ minutes on the other.

14. A clock set accurately at 1 o'clock indicates 10 minutes to 6 at 6 o'clock; what is the true time when the clock indicates 6 o'clock?

15. A watch is 73 seconds slow at noon on January 1st 1927; how much must it gain daily that it may be $17\frac{1}{2}$ seconds fast at noon on July 1st?

16 A watch is set right at 10 P. M. on Sunday ; at 10 A. M. on Wednesday it is 5 minutes too fast ; what will be the true time when it is 2 P. M. by the watch on Friday ?

17. A watch which gains 5 minutes in 12 hours is put right on January 1st 1928 ; when will it again show correct time ?

18. A church-clock was 15 minutes too fast 10 days ago, and to-day at the same hour it is 15 minutes too slow : when did it show true time ? When will it again show true time ?

19. Two clocks, of which one gains and the other loses one minute in an hour, strike one o'clock together ; what will be the interval, measured by a correct clock, between their respective striking 2 ?

240. In connection with the relative positions of the two hands of a clock the following points should be noted :

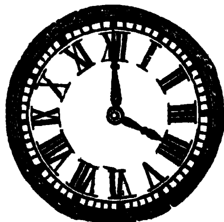
(i) While the minute-hand passes over 60 minute-divisions the hour-hand passes over only 5. Therefore in 60 minutes the minute-hand gains 55 divisions on the hour-hand ; and therefore in 12 minutes the minute-hand gains 11 divisions on the hour-hand.

(ii) When the hands are coincident, the number of minute-divisions between them is *zero*.

(iii) When the hands are opposite to each other, the number of minute-divisions between them is 30.

(iv) The minute-hand in making one complete revolution round the dial describes four right angles or 360 degrees. Therefore one minute-division corresponds to $\frac{360}{60}$ or 6 degrees of angular measurement. Hence the hands are at right angles to each other when they are 15 divisions apart, since $6° \times 15 = 90°$ = a right angle. When the hands are at right angles, the minute-hand will be either *ahead* of or *behind* the hour-hand by 15 divisions. This occurs twice every hour when the number of divisions between them, *measured in the same direction*, is either 15 or 45.

Example. Find the time between 4 and 5 o'clock when the hands of a clock are (i) together, (ii) at right angles, (iii) opposite to each other.



At 4 o'clock the minute-hand is 20 divisions behind the other.

(i) The two hands to be together between 4 and 5, the minute-hand has to gain 20 divisions on the hour-hand.

The minute-hand gains 11 divisions in 12 minutes,

∴ .... 1 division in $\frac{12}{11}$ ....,

∴ .... 20 divisions in $\frac{12 \times 20}{11}$ .... ;

∴ the time required is $\frac{12 \times 20}{11}$ min. or $21\frac{9}{11}$ min. past 4.

(ii) When the hands are at right angles there is a space of 15 minute-divisions between them. Between 4 and 5 this will happen twice ; first, when the minute-hand has gained 5 (*i.e.*, 20 − 15) divisions ; and secondly, when it has gained 35 (*i.e.*, 20 + 15) divisions.

The minute-hand gains 11 divisions in 12 minutes,

∴ .... 1 division in $\frac{12}{11}$ ....,

∴ .... 5 divisions in $\frac{12 \times 5}{11}$ .... ;

and .... 35 divisions in $\frac{12 \times 35}{11}$ ....

∴ The two hands will be at right angles at $\frac{12 \times 5}{11}$ min. or $5\frac{5}{11}$ min. past 4 ; and also at $\frac{12 \times 35}{11}$ min. or $38\frac{2}{11}$ min. past 4.

(ii) When the hands are opposite to each other, there is a space of 30 divisions between them. This will happen when the minute-hand has gained 50 (*i.e.*, 20 + 30) divisions.

The process will be similar to that in the preceding cases. The time is $54\frac{6}{11}$ min. past 4.

EXAMPLES. 143.

At what time are the hands of a clock (i) coincident, (ii) at right angles, (iii) opposite each other, (iv) 12 divisions apart, (v) 22 divisions apart, between the hours of

1. 2 and 3 ? 2. 3 and 4 ? 3. 6 and 7 ?

4. 12 and 1 ? 5. 7 and 8 ? 6. 10 and 11 ?

7. A watch is 10 minutes too fast at noon ; it loses 2 min. in one hour : find the true time when its hands are at right angles between 2 and 3 o'clock.

8. A clock is 5 minutes too slow at 1 ; it gains 1 min. in an hour : what is the true time when its hands are together for the fifth time after 1 o'clock ?

9. A clock is put right at 4 P. M.; it gains $1\frac{1}{2}$ min. in an hour : what is the true time when its hands are at right angles for the fourth time after 4 ?

10. A clock indicates correct time when its hands are together between 2 and 3 o'clock ; if it had been losing 2 min. every hour, what time did it indicate at 12 noon ?

11. A clock, in which the hour-hand has been displaced, shows the time to be 16 minutes past 3, and the two hands are together; the time is between 3 and 4 o'clock. Find by how many minute-divisions the hand has been displaced.

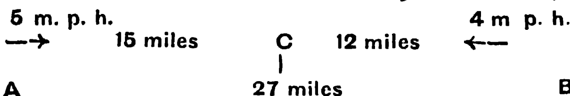
12. If the hands of a clock come together every 63 minutes (true time), how much does the clock gain or lose in a day?

PROBLEMS CONCERNING TIME AND DISTANCE.

241. The following simple illustrations will explain the principles involved in solving problems on Time and Distance.

The abbreviation "m. p. h." is used for "miles per hour".

(i) **Simple Motion.**—Suppose the distance between two places *A* and *B* is 27 miles and that two persons walk towards each other from *A* and *B* at the rates of 5 miles and 4 miles



per hour respectively. Of the whole distance the two together cover (5+4) or 9 miles per hour, and since the whole distance is 27 miles, the time required to cover it will be $\frac{27}{9}$ hr.=3 hr.

In 3 hr. the man from *A* will walk (3×5) or 15 miles and the man from *B* will walk (3×4) or 12 miles, *i.e.*, they will meet at a place *C*, 15 miles from *A* and 12 miles from *B*.

Again suppose that both walk from *A* and *B* in the same direction towards *D*. As the man from *A* walks at the rate of 5 m. p. h. and the other man from *B* walks at the rate of 4 m. p. h., it is clear the first man gains 1 mile in 1 hour on the second man.

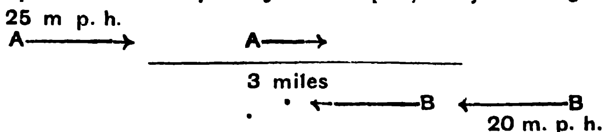


Since the distance between the two places is 27 miles, the first man will overtake the second when he (the first man) gains 27 miles on the other (the second man). Therefore the man from *A* will overtake the man from *B* in 27 hours.

In 27 hours the first man will walk (27×5) or 135 miles and the second man will walk (27×4) or 108 miles. Hence they will meet at a place *D*, 135 miles from *A* and 108 miles from *B*.

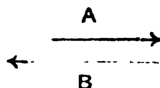
(ii) **Motion of Trains.**—Suppose that two trains *A* and *B*, 77 yd. and 99 yd. long, are running at the rates of 25 and 20 miles an hour respectively. If they are running in opposite

directions, they are approaching each other at (20+25) or 45 miles per hour. If they are 3 miles apart, they will together



cover 3 miles in $\frac{3}{45}$ or $\frac{1}{15}$ hr. = 4 minutes, *i.e.*, they will meet in 4 minutes.

Suppose that after meeting, the train *A* comes from the position shown in the above diagram to the position shown below



where the head of *A* and the end of *B*, are in a line. It is evident from the diagram that the two trains together have travelled a distance equal to the length of the train *B*. (To get a simpler idea imagine the train *A* to be momentarily stationery, then to be in the above position the train *B* must travel a distance equal to its own length).

Since the two trains together cover a distance of 45 miles = 45 × 1760 yd. in 1 hour, therefore they will cover a distance of 99 yd. in $\frac{99}{45 \times 1760}$ hr. or $\frac{99 \times 60 \times 60}{45 \times 1760}$ sec. or $4\frac{1}{2}$ sec., *i. e.*, after meeting, the head of the train *A* will pass the other train in $4\frac{1}{2}$ sec. Similarly it may be shewn that a person sitting anywhere in the first train will pass the other train in the same time.

Again, suppose that after meeting, the two trains come to the following position where their ends are in a line. Here the trains have just passed each other. It is evident from the

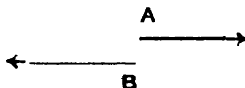
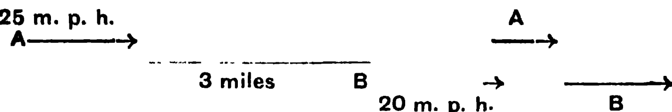


diagram that the two trains together have travelled a distance equal to the lengths of the two trains, *i.e.*, (77+99) or 176 yd.

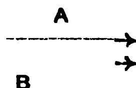
Since the two trains together cover a distance of 45 miles (= 45 × 1760 yd.) in 1 hr., therefore they will cover a distance of 176 yd. in $\frac{176}{45 \times 1760}$ hr. = $\frac{176 \times 60 \times 60}{45 \times 1760}$ sec. = 8 sec., *i.e.*, after meeting, the two trains will pass each other in 8 sec.

Now, again suppose that the two trains *A* and *B*, which are 3 miles apart are running in the same direction. Then the



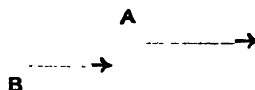
faster train *A* gains (25 − 20) or 5 miles in an hour on *B*. Therefore it will gain 3 miles in $\frac{3}{5}$ hr. or 36 minutes, *i.e.*, *A* will overtake *B* in 36 minutes.

Suppose that after meeting, the train *A* comes from the position shown in the above diagram to the position given below where their heads are in a line. It is clear that *A* has gained a distance equal to the length of *B*.



Since *A* gains 5 miles (= 5 × 1760 yd.) in 1 hr., therefore it will gain 99 yd. in $\frac{99}{5 \times 1760}$ hr., or, $\frac{99 \times 60 \times 60}{5 \times 1760}$ sec., or, $40\frac{1}{2}$ sec. *i.e.*, the head of the train *A* will pass the head of the other train in $40\frac{1}{2}$ sec. Similarly it may be shewn that a person sitting anywhere in the first train will pass the other train in the same time.

Again, suppose that after meeting, the two trains come to the following position where the end of *A* is in a line with the



head of *B*. Here *A* has just completely passed *B*. It is evident from the diagram that *A* has gained a distance equal to the lengths of the two trains, *i.e.*, (77 + 99) or 176 yd.

Since *A* gains 5 miles (= 5 × 1760 yd.) in 1 hr., therefore it will gain 176 yd. in $\frac{176}{5 \times 1760}$ hr., or, $\frac{176 \times 60 \times 60}{5 \times 1760}$ sec., or, 72 sec., *i.e.*, after meeting, the two trains will pass each other in 72 sec.

From the foregoing illustrations we learn that the *effective* rate of two trains is (1) the *sum* of their rates if they are running in *opposite* directions, or, (2) the *difference* of their rates if they are running in the *same* direction.

(iii) **Motion up and down a river.**—(*a*) When a man rows down a river, the rate at which he advances = (rate of his own pull) + (rate of the current). (*b*) When a man rows up a river, the rate at which he advances = (rate of his own pull) − (rate of the current).

Example 1. A passenger train leaves Calcutta at 4 P. M. and travels at the rate of 20 miles an hour ; the mail train leaves Calcutta at 9 P. M. and travels, on a parallel line of rails, at the rate of 30 miles an hour : when and where will the second train overtake the first ?

The first train has started 5 hours before the second ; and is therefore (20×5) or 100 miles away when the second train starts. Therefore the second train has to gain 100 miles on the first, at the rate of 10 (*i.e.*, $30 - 20$) miles an hour.

Second train gains 10 miles in 1 hour on the first,

$\therefore$ .... 100 .... 10 hours............... ;

$\therefore$ the time required is 10 hours after the second train starts ; and $\therefore$ the second overtakes the first (30×10) or 300 miles from Calcutta.

Example 2. A hare, pursued by a greyhound, is 30 yards before him at starting ; whilst the hare takes 4 leaps the dog takes 3 ; in one leap the hare goes $1\frac{1}{2}$ yards, and the dog, $2\frac{1}{2}$ yards : how far will the hare have gone when she is caught by the hound ?

Whilst the hare runs $(4 \times 1\frac{1}{2})$ yd., or 6 yd., the dog runs $(3 \times 2\frac{1}{2})$ yd., or $7\frac{1}{2}$ yd. Hence

The dog gains $1\frac{1}{2}$ yd. whilst the hare runs 6 yd.,

$\therefore$ .... 3 yd. .... 12 yd.,

$\therefore$ ....30 yd. .... 120 yd. ;

$\therefore$ the required distance is 120 yd.

Example 3. *A* starts from *P* to walk to *Q*, a distance of $51\frac{3}{4}$ miles, at the rate of $3\frac{3}{4}$ miles an hour ; an hour later *B* starts from *Q* for *P* and walks at the rate of $4\frac{1}{4}$ miles an hour : when and where will *A* meet *B* ?

A has already gone $3\frac{3}{4}$ miles when *B* starts. Of the remaining 48 miles, *A* walks $3\frac{3}{4}$ and *B* walks $4\frac{1}{4}$ in one hour ; that is, they together pass over $(3\frac{3}{4} + 4\frac{1}{4})$ or 8 miles in one hour. Therefore 48 miles are passed over in $\frac{48}{8}$ or 6 hours. Therefore *A* meets *B* in 6 hours after *B* started. And therefore they meet at a distance of $4\frac{1}{4} \times 6$ or $25\frac{1}{2}$ miles from *Q*.

Example 4. Two trains, 50 yd. and 60 yd. long respectively, run at the rates of 45 and 30 miles an hour respectively on parallel rails in opposite directions: how long do they take to pass each other? How long would they take to pass each other if they were running in the same direction? How long would a person sitting in the first train take to pass the other?

(i) The two trains running in opposite directions will pass each other in the time in which (50+60) or 110 yards are passed over at the rate of (45+30) or 75 miles an hour.

Now, 75 miles are passed over in 1 hour,

i.e., 75×1760 yd. .... 1 hour,

$\therefore$ 110 yd. .... $\frac{1}{1200}$ hour;

$\therefore$ the time required $=\frac{1}{1200}$ hr., or 3 seconds.

(ii) When the trains run in the same direction they pass each other in the time in which (50+60) or 110 yards are passed over at the rate of (45−30) or 15 miles an hour. The time required will be found to be 15 seconds.

(iii) First, when the trains are running in opposite directions, a person sitting in the first train will pass the other in the time in which 60 yd. (*i.e.*, the length of the second train) are passed over at the rate of (45+30) or 75 miles an hour. The required time will be found to be $1\frac{7}{11}$ seconds.

Secondly, when the trains run in the same direction, 60 yd. are to be passed over at the rate of (45−30) or 15 miles an hour. The required time will be found to be $8\frac{2}{11}$ seconds.

Example 5. A man rows down a river 18 miles in 4 hours with the stream, and returns in 12 hours; find the rate at which he rows, and the rate at which the stream flows.

He rows 18 miles in 4 hours down the stream; therefore he rows $\frac{18}{4}$ or $4\frac{1}{2}$ miles an hour down the stream.

Again, he rows 18 miles in 12 hours up the stream; therefore he rows $\frac{18}{12}$ or $1\frac{1}{2}$ miles an hour up the stream.

$\therefore$ $4\frac{1}{2}$ miles an hour is the sum of the rate at which the man rows and the rate at which the stream flows; and $1\frac{1}{2}$ miles an hour is their difference. Hence the rates are 3 miles and $1\frac{1}{2}$ miles an hour respectively.

Example 6. If a snail, on the average, creep 31 inches up a pole during 12 hours in the night, and slip down 16 inches during the 12 hours in the day, how many hours will he be in getting to the top of a pole 35 feet high?

Length of the pole = 420 in. Now in 24 hours the snail creeps up (31−16) in. or 15 in.; therefore in (24×26) hr. the snail creeps

up (15×26) in. or 390 in.; therefore he has $(420 - 390)$ in. or 30 in more to get up. And he goes over 31 in. in 12 hr., and therefore over 30 in. in $\frac{12 \times 30}{31}$ hr. Therefore he reaches the top in $(24 \times 26 + \frac{12 \times 30}{31})$ hr., or in $635\frac{19}{31}$ hours. [The number of days (26) has been so determined that (420 in. $-$ 15 in. $\times$ 26) may be equal to 31 in. or just less than 31 in.]

EXAMPLES. 144.

1. One man takes 100 steps a minute, each 2 ft. long; another walks 4 miles an hour; if they start together, how soon will one of them be 38 yards ahead of the other?

2. A person wishing to go from A to B walked for $4\frac{1}{4}$ hours at the rate of 1 mile in $21\frac{3}{4}$ min., he then rode for $16\frac{1}{4}$ hours three times as fast as he walked, and then had to travel by rail for $10\frac{3}{4}$ hours three times as fast as he rode; find the distance from A to B.

3. A train leaves Calcutta at 7-30 A. M. and travels 25 miles an hour: another train leaves Calcutta at noon and travels 40 miles an hour: when and where will the second train overtake the first?

4. A train going 30 miles an hour leaves Calcutta for Allahabad (600 miles) at 9 P. M.; another train going 40 miles an hour leaves Allahabad for Calcutta at the same time; when and where will they pass each other?

5. Two trains, each 88 yards long, are running in opposite directions on parallel rails, the first at 40 miles an hour, the other at 35 miles an hour; how long will they take to pass each other?

6. In the above example, if the trains run in the same direction, how long will a person sitting in the faster train take to pass the other?

7. A man rows down a river 15 miles in 3 hours with the stream and returns in $7\frac{1}{2}$ hours: find the rate at which he rows, and the rate at which the stream flows.

8. A man rows 12 miles in 5 hours against the stream, the rate of which is 4 miles an hour; how long will he be rowing 15 miles with the stream?

9. A policeman goes after a thief who has 100 yards' start; if the policeman run a mile in 6 minutes, and the thief a mile in 10 minutes, how far will the thief have gone before he is overtaken?

10. A man starts at 7 A. M. and travels at the rate of $4\frac{3}{4}$ miles an hour; at 8-15 A. M. a coach starts from the same place and follows the man, travelling at the rate of $6\frac{1}{2}$ miles an hour; at what o'clock will the coach overtake the man?

11. *A* starts from Allahabad to Cawnpore and walks at the rate of 5 miles an hour; *B* starts from Cawnpore 3 hours later and walks towards Allahabad at the rate of $4\frac{1}{2}$ miles an hour; if they meet in 11 hours after *B* started, find the distance from Allahabad to Cawnpore.

12. *A* starts from Calcutta to Hughly (24 miles) at 6 A. M. walking 4 miles an hour; *B* starts from Calcutta an hour later and reaches Hughly one hour before *A*; where did they meet?

13. A man walks to a town at the rate of $3\frac{1}{2}$ miles an hour and rides back at the rate of 6 miles an hour; how far has he walked, the whole time occupied having been 3 hours 10 minutes?

14. *A* and *B* run a mile in opposite directions; while *A* runs 6 yards *B* runs 5; *B* gets 9 seconds' start, during which time he runs $22\frac{1}{2}$ yards; find when he will pass *A*.

15. A train leaves Calcutta at 7 A. M. and reaches Burdwan at 11 A. M.; another train leaves Burdwan at 8 A. M. and reaches Calcutta at 10-30 A. M.: at what hour do they meet?

16. A train starts from *P* for *Q* travelling 20 miles an hour; $1\frac{1}{2}$ hours later another train starts from *P* and travelling at the rate of 30 miles an hour reaches *Q* $2\frac{1}{2}$ hours before the first train: find the distance from *P* to *Q*.

17. A horseman leaves Madras at 10 A. M. and in 5 hours overtakes a coach which left Madras at 9 A. M. If the coach had been 2 miles farther on the road when the horseman started, it would have been overtaken in 7 hours. Find the rates of the horseman and the coach.

18. *A* and *B* start at the same time from Patna and Bankipore, and proceed towards each other at the rates of 3 and 4 miles per hour respectively. They meet when *B* has walked one mile farther than *A*. Find the distance between Patna and Bankipore.

19. *A*, *B* and *C* start from the same place at intervals of an hour and walk at the rate of 3, 4 and 5 miles an hour respectively. *A* starts first, but when he is overtaken by *B* he returns towards the starting-place: find the distance from the starting-place where he would meet *C*.

20. A man rides at the rate of 11 miles an hour, but stops 5 minutes to change horses at the end of every 7th mile; how long will he take to go a distance of 94 miles?

21. A man rides at the rate of 10 miles an hour, but stops 10 minutes to change horses at the end of every 12th mile; how long will he take to go a distance of 96 miles?

22. If a gun fire 7 shots every 9 minutes, how many will it fire in an hour?

23. A monkey, climbing up a greased pole, ascends 10 ft. and slips down 3 ft. in alternate minutes. If the pole is 63 ft. high, how long will it take him to reach the top?

24. A party of tourists set out for a station 3 miles distant and go at the rate of 3 miles an hour. After going half a mile one of them has to return to the starting point; at what rate must he now walk in order to reach the station at the same time as the others?

25. A motor car, travelling at the rate of 40 miles an hour, starts from a place *P* to travel, on a road running beside a railway, to a place *Q*, a distance of 150 miles. Fifteen minutes after the motor car has started, a train, travelling at the rate of 60 miles an hour, leaves *P* to go to *Q*. If the train stops for five minutes at a station 35 miles from *P*, find:

(1) At what distances from *P* the train and motor car are together.

(2) The length of time between the arrivals at *Q* of the train and motor car.

26. The Sara-Sirajganj Railway is 55 miles long, its two terminii being Ishurdi and Sirajganj. It is a single line railway and has stations at every 5 miles, where it is possible for two trains to pass one another. A fast train starts at noon from Ishurdi, running at the rate of 50 miles an hour, and stops 2 minutes at the fifth station out. A slow train starts 5 minutes later from Sirajganj, running at the rate of 20 miles an hour, and stops 2 minutes at every station. At what station must the slow train lie by for the fast train to pass it? The slow train must be out of the way at least a minute before the fast train is due.

UNIFORM CIRCULAR MOTION.

242. *Example.* *A*, *B* and *C* start from the same point and travel round an island 30 miles in circumference, *A* and *B* travelling in the same direction and *C* in the opposite direction. If *A* travels at the rate of 5, *B* at the rate of 7 and *C* at the rate of 8 miles an hour, in how many hours will they all come together again?

B gains 2 miles on *A* in 1 hour; $\therefore$ he gains 30 miles or a complete circuit in $\frac{30}{2}$ hr., that is, *A* and *B* are together at the end of every 15 hours. *A* and *C* together pass over 13 miles in 1 hr; $\therefore$ they come together every $\frac{30}{13}$ hours. And therefore *A*, *B* and *C* will come together at the end of any number of hours which is a common multiple of 15 and $\frac{30}{13}$; but the L. C. M. of 15 and $\frac{30}{13}$ is 30: therefore *A*, *B*, *C* are first together at the end of 30 hours.

EXAMPLES. 145.

1. A and B start together from the same point to walk round a circular course, 10 miles long; A walks 4 miles and B 3 miles an hour. When will they next meet, (i) if they walk in the same direction, (ii) if they walk in opposite directions?

2. A takes 3 hours and B takes 5 to walk round a park. If they start together, when will they next meet, supposing (i) that they walk in the same direction, (ii) that they walk in opposite directions?

3. A, B, C start from the same point and travel in the same direction round an island 63 miles in circumference, A at the rate of 10, B at the rate of 12, and C at the rate of 16 miles a day; and how many days will they come together again?

4. A can go round an island in 15 days, B can go round it in 20 days and C in 25 days. If they start simultaneously from the same point, A and B travelling in one direction and C in the opposite direction, in how many days will they come together again? In how many days will they come together again at the starting point?

5. Three boys agree to start together from the same point and run round a circular park 6 miles in circumference; they run at the rates of 3, 5 and 7 miles per hour respectively; in how many hours will they come together again? In what time will they come together again at the point from which they started?

RACES AND GAMES OF SKILL.

243. *Example* 1. A can beat B by 40 yards in a mile race; B can beat C by 20 yards in a mile race; if A and C run a mile, by how much will A win?

A can run 1760 yards while B runs 1720,

∴ A .... $\frac{1760}{43}$ .... B 40,

∴ A .... $\frac{1760 \times 44}{43}$ .... B 1760,

[but B .... 1760 .... C .... 1740,]

∴ A .... $\frac{1760 \times 44}{43}$ .... C 1740,

∴ A .... 1760 .... C .... $\frac{1740 \times 43}{44}$ or $1700\frac{5}{11}$ yd.*

∴ A will win by $(1760 - 1700\frac{5}{11})$ or $59\frac{6}{11}$ yards.

[*$1740 \times \frac{43}{44} = 1740 \times (1 - \frac{1}{44}) = 1740 - \frac{435}{11} = 1740 - 39\frac{6}{11} = 1700\frac{5}{11}$.]

Example 2. *A* can give *B* 20 yards and *C* 30 yards in a race of 200 yards ; how many yards can *B* give *C* in 300 yards ?

[**Note.**—"*A* can give *B* 20 yards in 200 yards" means that in a race of 200 yards *A* can give *B* 20 yards' start. Consequently while *A* runs 200 yards *B* runs 180 yards.]

While *A* runs 200 yards *B* runs 180,

and ....*A* ...... 200 ....*C* 170,

∴ ....*B* ...... 180 ....*C* 170,

∴ ....*B* ...... 60 ....*C* ...... $\frac{170}{3}$,

∴ ....*B* ...... 300 ....*C* ...... $\frac{1700\cdot5}{3}$ or $283\frac{1}{3}$ yards.

∴ *B* can give *C* $(300-283\frac{1}{3})$ or $16\frac{2}{3}$ yards in 300.

Example 3. In a game of skill *A* can give *B* and *B* can give *C*, 10 points out of a game of 50 ; how many should *A* give *C* ?

[**Note.**—"*A* can give *B* 10 points out of a game of 50" means that while *A* makes 50 points *B* can make $(50-10)$ or 40 points.]

C can make 40 points while *B* makes 50,

∴ *C*.............. 4................. *B*........... 5,

∴ *C*.............. 32................. *B*...........40 ;

but *A*..............50................. *B*...........40 ;

∴ *C*.............. 32................. *A*...........50.

∴ *A* can give *C* $(50-32)$ or 18 points in 50.

EXAMPLES. 146.

1. In a mile race *A* gives *B* 60 yard's start, and beats him by 28 yards. If *A* runs the mile in 5 minutes, how long will *B* take ?

2. In a mile race *A* can beat *B* by 40 yards, and *B* can beat *C* by 40 yards : how many yards' start can *A* give *C* that there may be a dead heat ?

3. *A* can give *B* 60 yards, and *C* 80 yards in a race of 500 yards ; by how much could *B* beat *C* in a mile race ?

4. *A* runs 15 yards while *B* runs 12 ; *B* runs 10 miles while *C* runs 12 : if *C* runs a mile in 10 minutes, what time will *A* take to do it ?

5. At a game of skill *A* can give *B* 15 points out of 50, and *A* can give *C* 10 points out of 40 : which is the better player, *B* or *C*, and how many points can he give the other in 75 ?

6. *A* and *B* run a mile race ; *A* runs the whole course at the rate of 100 yards per minute ; *B* running at the rate of 80 yards per minute for 5 minutes, quickens his speed to 120 yards per minute : which wins ? by how much ? and by what time ?

7. In a game of billiards A can give B 10 points, and C 14 points in 50 ; how many can B give C so as to make an even match ?

8. A can give B 300 yards in 1 mile, and C can give B 700 yards in 2 miles ; if A and C run a mile, which will win and by how much ?

9. A can give B 100 yards' and C 150 yards' start in a mile ; B can give C a start of 5 seconds in a mile : how long does each take to run half a mile ?

10. In a mile race A gives B 50 yards' start, and beats him by 38 yards ; B giving C 40 yards' start is beaten by 60 yards ; if A and C run over the same course, which will win and by how much ?

11. At a game of rackets A can give B 8 points in 40, and B can give C 10 points in 50 ; how many points could A give C in 25 ?

12. A can give B 20 yards' and C 30 yards' start, while B can give C 2 seconds' start in a race of 250 yards ; how long does each take to run 100 yards ?

13. One boy runs 200 yards and another 180 yards in a minute. How many yards' start must the second have that they may run a dead heat in a mile race ?

14. In a game at fives A can give B 3 points out of 15, and A can give C 7 points ; how many points can B give C so as to make an even match ?

15. A and B run a mile and A wins by half a minute. A and C run a mile and A beats C by 88 yards. B and C run and B wins by 20 seconds. In what time can each run a mile ?

16. A beats B by 20 yards, C beats D by 60 yards, and B beats D by 40 yards, in a mile race. If A and C run, which will win and by how much ?

CHAIN RULE.

244. *Example* 1. If 8 rupees are worth 15 shillings, and 25 shillings are worth 6 dollars, how many dollars are equal to 45 rupees ?

$$R8 = 15s., \qquad \therefore \quad R1 = \tfrac{15}{8}s.$$

$$25s. = 6 \text{ dollars}, \qquad \therefore \quad 1s. = \tfrac{6}{25} \text{ dollars}.$$

$$\therefore \quad R45 = 45 \times \tfrac{15}{8}s.$$

$$= 45 \times \tfrac{15}{8} \times \tfrac{6}{25} \text{ dollars, or } 20\tfrac{1}{4} \text{ dollars}.$$

Example 2. If A in 3 days can do as much work as B in 4 days, and B in 5 days can do as much as C in 6 days, how long will A require to do a piece of work which C can do in 16 days?

What C can do in 6 da. B can do in 5 da.,
∴ .... C .... 1 ... B .... $\frac{5}{6}$ ...,
and .... B .... 4 ..., A .... 3 ...,
∴ .... B .... 1 ... A .... $\frac{3}{4}$ ...
∴ What C can do in 16 days B can do in $16\times\frac{5}{6}$ days,
∴ .... C .... A .... $16\times\frac{5}{6}\times\frac{3}{4}$ days
or 10 days.

EXAMPLES. 147.

1. If 25 rupees are worth 46 shillings, 20 shillings are worth 25 francs, and 240 francs are worth 47 dollars, how many dollars are equivalent to 40 rupees?

2. If R8 = 15*s*., £3 = 20 thalers, and 25 thalers = 93 francs, express a franc in Indian money.

3. If 72 carlini = 25 shillings, 4 shillings = 5 francs, and 8 scudi = 45 francs, how many scudi are equal to 1296 carlini?

4. If 5 chickens cost as much as 4 ducks, 6 ducks cost as much as 3 geese, and 7 geese cost as much as 5 turkeys, what is the price of a chicken when a turkey costs R8?

5. If 5 lb. of tea be worth 3 lb. of coffee, 5 lb. of coffee be worth 2 lb. of sugar, and 7 lb. of sugar be worth 30 lb. of rice, how many pounds of tea must be given in exchange for 20 lb. of rice?

6. If 12 oxen eat as much as 29 sheep, 15 sheep eat as much as 25 hogs, 17 hogs eat as much as 3 camels, and 8 camels eat as much as 13 horses, how many horses will eat as much as 1632 oxen?

7. If A can do as much work in 4 days as B can do in 5, and B can do as much in 6 days as C in 7; in what time will C do a piece of work which A can do in a week?

8. If A can do as much work in $1\frac{1}{2}$ days as B can do in 2, and B can do as much in $2\frac{1}{2}$ days as C in 3; in what time will A and B together do a piece of work which C can do in 10 days?

9. While A does $\frac{1}{3}$ of a piece of work B does $\frac{1}{4}$, and while B does $\frac{1}{5}$, C does $\frac{1}{6}$; in how many hours will C finish a piece of work which A finishes in 20 hours?

10. If 3 ducks are worth 4 chickens, and 3 geese are worth 10 ducks, find the value of a goose, a pair of chickens being worth 4*a*. 6*p*.

XXXVI. COMPLEX PROBLEMS.

245. In the problems in the preceding section we have found the change in one quantity corresponding to the change in *one* other. In the following examples we shall have to find the change in one quantity correspoinding to the changes in *two* others.

Example 1. If 15 horses can plough 12 acres in 10 days, in how many days can 9 horses plough 18 acres ?

	15 horses can plough	12 acres in 10 days,
$\therefore$	1 horse	12 acres in (10×15) days,
$\therefore$	1 horse	1 acre in $\frac{10\times 15}{12}$ days,
$\therefore$	9 horses	1 acre in $\frac{10\times 15}{12\times 9}$ days,
$\therefore$	9 horses	18 acres in $\frac{10\times 15\times 18}{12\times 9}$ days,
		or 25 days. *Ans.*

Note. We might use 3 horses and 6 acres as common units with advantage. Thus :

	15 horses can plough	12 acres in 10 days,
$\therefore$	3 horses	12 acres in 10×5 days,
$\therefore$	3 horses	6 acres in $\frac{10\times 5}{2}$ days,
$\therefore$	9 horses	6 acres in $\frac{10\times 5}{2\times 3}$ days,
$\therefore$	9 horses	18 acres in $\frac{10\times 5\times 3}{2\times 3}$ days,
		or 25 days. *Ans.*

Example 2. If 6 men earn R15 in 10 days, how much do 8 men earn in 7 days ?

In 10 days 6 men earn R15,

$\therefore$ In 1 day 6 men earn R$\frac{15}{10}$ or R$\frac{3}{2}$,

$\therefore$ In 1 day 1 man earns R$\frac{3}{2\times 6}$ or R$\frac{1}{4}$,

$\therefore$ In 7 days 1 man earns R$\frac{7}{4}$,

$\therefore$ In 7 days 8 men earn R$\frac{7\times 8}{4}$ or R14. *Ans.*

Example 3. If 6 men can do a piece of work in 8 days, how many men can do a piece of work 4 times as great in $\frac{1}{3}$ of the time ?

	The work can be done in 8 days by		6 men,
$\therefore$	....	$\frac{8}{3}$	18 men,
$\therefore$	4 times the work....................	$\frac{8}{3}$	72 men. *Ans.*

Example 4. If the sixpenny loaf weigh 8 oz. when wheat is 15*s*. a bushel, what ought a bushel of wheat to be when the fourpenny loaf weighs 12 oz.?

	Sixpenny	loaf weighs 8 oz.	when wheat is	15*s*.	a bushel,
∴	penny	loaf weighs 8 oz.	....	$\frac{5}{2}$*s*.	............,
∴	penny	loaf weighs 1 oz.	....	20*s*.	............,
∴	fourpenny	loaf weighs 1 oz.	....	80*s*.	............,
∴	fourpenny	loaf weighs 12 oz.	....	$\frac{20}{3}$*s*.	............,
				or 6*s*. 8*d*.	a bushel.

Example 5. If 5 cannon, which fire 3 rounds in 5 minutes, kill 135 men in $1\frac{1}{2}$ hours, how many cannon, which fire 5 rounds in 6 minutes, will kill 250 men in 1 hour?

	In 54 rounds 135 men are killed by	5	cannon,
∴	... 1 round 135	5×54	..........,
∴	... 1 round 1 man is	$\frac{5\times54}{135}$	..........,
∴	... 50 rounds 1	$\frac{5\times54}{135\times50}$	..........,
	... 50 rounds 250 men are	$\frac{5\times54\times250}{135\times50}$	....
		or 10 cannon.	

EXAMPLES. 148.

1. If 5 men earn £3 in 12 days, in how many days will 8 men earn £4?

2. If 10 horses can plough 50 acres in 20 days, how many acres will 12 horses plough in 15 days?

3. If 24 horses eat 9 bushels of corn in 21 days, for how many days will 33 bushels feed 7 horses?

4. If 30 men can build a wall 20 ft. high in 15 days, how many men will it take to build one 25 ft. high in $7\frac{1}{2}$ days?

5. If 12 horses are fed for 17 days at a cost of R110. 8*a*., how many horses can be fed for 27 days at a cost of R117?

6. If 10 fires consume 75 maunds of coal in 14 days, in how many days will 18 fires consume 100 maunds?

7. If the carriage of 10 md. 20 seers for 250 miles be R41. 0*a*. 3*p*., what should be paid for the carriage of 12 md. for 200 miles?

8. If the wages of 13 men for 25 days amount to R203. 2*a*., how many men must work for 16 days to receive R300?

9. What is a month's rent for $116\frac{2}{3}$ bighas of land, if R22. 8*a*. per annum be given for 9 bighas?

10. If 14 persons can live on R1400 for 28 months, how long can 18 persons live on R1350 ?

11. If 5 men dig a trench $7\frac{1}{2}$ yd. long in 21 days, how many men can dig a similar trench 20 yd. long in 35 days ?

12. If 20 pumps can raise 1250 maunds of water in 5 hours, how many pumps can raise 750 maunds of water in 10 hours ?

13. If 20 men do a piece of work in 13 days, in what time can 15 men do another piece of work $2\frac{1}{2}$ times as great ?

14. If 10 men do a piece of work in 8 days, how many men will do a piece of work, 4 times as great, in $\frac{1}{3}$ of the time ?

15. If the fourpenny loaf weighs 10 oz. when wheat is 50*s.* a quarter, what should a threepenny loaf weigh when wheat is 55*s.* a quarter ?

16. If the 3 lb. loaf cost 8*d.* when corn is 30*s.* per bushel, how much ought the 5 lb. loaf to cost when corn is 36*s.* per bushel ?

17. If I get 1 lb. weight of bread for $7\frac{1}{2}$*d.* when wheat is 15*s.* a bushel, what ought a bushel of wheat to be when I get 12 oz. of bread for 4*d.* ?

18. If 14 men in 20 days of $12\frac{1}{2}$ hours each earn R456. 4*a.*, how many hours a day should 24 men work to earn R547. 8*a.* in 21 days, at the same rate ?

19. If 15 men can do a piece of work in 12 days of 6 hours each, how many men will it take to do 5 times the amount if they work 20 days of 10 hours each ?

20. If a man complete a journey of 1980 miles in 18 days, travelling 11 hours a day, in how many days would he travel 540 miles, going 6 hours a day at the same rate ?

21. When rice is R2. 8*a.* a maund, 10 men can be fed for 12 days at a certain cost ; how many men can be fed for 4 days at the same cost, when rice is R3 a maund ?

22. When flour is R4 a maund, 16 men can be fed for 5 days at a cost of R8 ; for how many days can 12 men be fed at a cost of R10. 8*a.*, when flour is R3. 8*a.* per maund ?

23. If 15 men can build a wall 270 ft. long, 5 high and 2 thick, in 18 days, in how many days will 16 men build a wall 180 ft. long, 4 high and 3 thick ?

24. If 10 men working 6 hours a day dig a trench 105 ft. long, 4 wide and 2 deep, in 6 days, how many hours a day must 264 men work in order to dig a trench 126 ft. long, 20 wide and 11 deep in 10 days ?

25. A garrison of 1200 men is provisioned for 50 days, allowing 10 oz. per man per day ; if it is reinforced by 300 men,

to what must the daily allowance be reduced that the provisions may last the increased number of men 60 days ?

26. If the carriage of goods weighing 2 cwt. 3 qr. 6 lb. for 300 miles cost £6 . 10 . 10, what will be the charge for carrying 2 wagon-loads of the same, each weighing 14 cwt. 0 qr. 4 lb., 450 miles ?

27. If the gas for 6 burners, 6 hours every day, for 8 days cost R4. 8*a.*, how many burners may be lighted 5 hours every evening for 10 days at the cost of R6. 4*a.* ?

28. If 3 cannon, firing 4 rounds in 6 minutes, kill 250 men in half an hour, how many cannon, firing 3 rounds in 5 minutes, will kill 600 men in an hour ?

29. If 15 men can make an embankment, 966 yd. long, in 8 days, working 10½ hours daily, how many men would be required to make an embankment, 575 yd. long, in 12 days, working 7½ hours daily, 8 extra men being taken on during the last 2 days ?

30. If 50 men, working 8 hours a day, dig in 5 days, a trench of 275 cu. yd. ; in how many days of 10 hours each could 40 men dig a trench of 330 cu. yd., when the hardness of the ground in the first case is twice that in the second, and 3 men of the former company can do the work of 4 men of the latter ?

31. If 6 men, working 8 hours a day, can mow 60 acres in 4 days ; in how many days will 4 men, two of whom work 10 hours and two 7 hours a day, mow 85 acres ?

32. If 6 men and 8 boys can reap a field of 15 acres in 4 days, how many acres will 7 men and 4 boys reap in 9 days, two boys reaping as much as a man in the same time ?

33. If 4 horses eat as much as 18 sheep, and if 5 horses and 30 sheep can be kept for 15 days at a cost of R51 . 3 . 6, at what cost can 7 horses and 15 sheep be kept for 20 days ?

34. The rent of a farm of 41¼ acres for 39 months was R89. 6*a.* ; what would be the area of another farm, the rent of which for 33 months was R103. 2*a.*, 4 acres of the latter being worth as much as 3 acres of the former ?

35. A vessel with a crew of 27 men, provisioned for 90 days at the rate of 22 oz. a day per man, was, after 27 days, forced by stress of weather to lie at anchor for a fortnight, at the end of which time 3 men died ; how must the provisions be apportioned that they may hold out the extra time ?

36. If 10 men or 16 boys, working 6 hours a day, can do a piece of work in 20 days, how many hours a day must 7 men and 8 boys work to do another piece of work 3 times as great in 15 days ?

37. If 5 men, 8 women or 12 boys can do a piece of work in 16 days, working 7 hours a day, how many men, with the assistance of 4 women and 6 boys, will be able to do another piece of work $2\frac{1}{2}$ times as great in 35 days, working 5 hours a day?

246. The following problems are of a different class.

Example 1. The price of 5 horses and 6 oxen is ₨680, that of 4 horses and 7 oxen is ₨610; find the price of an ox.

The price of 5 horses and 6 oxen = ₨680,

∴ .... 20 .... 24 ...... = ₨2720. ......(i)

Again .... 4 .... 7 ...... = ₨610,

∴ .... 20 .... 35 ...... = ₨3050. ......(ii)

∴ The price of 11 oxen = ₨3050 − ₨2720 [subtracting (i) from (ii)

= ₨330;

∴ the price of 1 ox = ₨30.

Example 2. 3 men and 5 boys can do $\frac{19}{20}$ of a piece of work in 3 days; 4 men and 8 boys can do $\frac{14}{15}$ of it in 2 days; in what time can a boy do the whole work?

In 3 days 3 men and 5 boys can do $\frac{19}{20}$,

∴ ... 1 day 3 .... 5 .... $\frac{19}{60}$,

∴ ... 1 day 12 .... 20 .... $\frac{19}{15}$. ......(i)

Again ... 2 days 4 .... 8 .... $\frac{14}{15}$,

∴ ... 1 day 4 .... 8 .... $\frac{7}{15}$,

∴ ... 1 day 12 .... 24 .... $\frac{7}{5}$. ......(ii)

∴ In 1 day 4 boys can do $(\frac{7}{5}-\frac{19}{15})$ of the work,
[subtracting (i) from (ii)]

i.e., ... 4 boys can do $\frac{2}{15}$ of the work,

∴ .... 1 boy can do $\frac{1}{30}$ of the work,

∴ 1 boy can do the whole work in 30 days.

EXAMPLES. 149.

1. If 9 horses and 7 cows cost ₨770, and 5 horses and 9 cows cost ₨530; find the price of a cow.

2. The price of 5 maunds of flour and 6 maunds of rice is ₨39, and that of 7 maunds of flour and 4 maunds of rice is ₨37; find the price of one maund of flour and of one maund of rice.

3. If 10 rupees and 11 shillings weigh 2760 grains, and 8 rupees and 10 shillings weigh $2312\frac{8}{11}$ grains, find the weight of a rupee and of a shilling.

4. If 7 sheep and 9 pigs cost ₹107, and 9 sheep and 7 pigs cost ₹101, how much will 1 sheep and 1 pig cost ?

5. The cost of 4 chairs and 5 tables is ₹120, and that of 5 chairs and 4 tables ₹105 ; find the price of a chair and of a table.

6. 2 men and 3 boys can do $\frac{3}{4}$ of a piece of work in 6 days ; 3 men and 5 boys can do $\frac{19}{24}$ of it in 4 days. In what time can a boy do the whole work ?

7. 7 men and 8 boys can do a piece of work in 2 days : 4 men and 12 boys can do $\frac{29}{65}$ of the work in 1 day. In what time can a man do the work ?

8. 5 men and 6 boys can do $\frac{3}{5}$ of a piece of work in 3 days ; 10 men and 18 boys can do the whole work in 2 days. In what time will a man and a boy be able to do double the work ?

9. If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days, how long will it take 2 men and 2 boys to reap 10 acres ?

10. If 2 boys and 1 man can do a piece of work in 4 hours, and 2 men and 1 boy can do the same in 3 hours, find in what time a man, a boy, and a man and a boy together, respectively, could do the same.

11. On a piece of work 4 men and 5 boys are employed, who do $\frac{1}{2}$ of it in 6 days ; after this, 1 man and 2 boys more are put on, and $\frac{1}{3}$ more is done in 3 days ; how many more men must be put on to finish the work in one more day ?

12. A cistern containing 210 buckets may be filled by two pipes. When the first pipe has been open 4 and the second 5 hours, 90 buckets of water were obtained. When the 1st was open 7 and the 2nd $3\frac{1}{2}$ hours, 126 buckets were obtained. In what time will the cistern be full, if both pipes work ?

XXXVII. RATIO AND PROPORTION.

247. The relation which one quantity bears to another of the same kind with respect to magnitude may be expressed in two ways. Thus if A has ₹5 and B, ₹10, we may say that

(i) A has ₹5 less than B,

or (ii) A has half of the money that B has.

The first result is obtained by subtracting A's money from B's money. The second result is obtained by dividing the number of rupees that A has by the number of rupees that B has, *i.e.*, by forming a fraction whose numerator is the number of rupees that A has and whose denominator is the number of rupees that B has.

Again, suppose *A* has ₹500000 and *B*, has ₹500005. The difference of the amounts is still ₹5, but is a negligible quantity compared with ₹500000, for the fraction obtained by dividing 500000 by 500005 is very nearly equal to 1.

It is evident, therefore, that the second form of comparison expresses more clearly the relation which *A*'s amount bears to *B*'s amount with respect to magnitude. Hence in comparing two quantities of the same kind with respect to magnitude, *e.g.*, two sums of money, or two lengths, or two weights, a fraction is formed whose numerator and denominator show the number of units of the two quantities compared. Such a fraction is called the **ratio** of the two quantities. Hence **ratio** is the relation which one quantity bears to another with respect to magnitude, the comparison being made by considering what *multiple*, *part or parts* the first quantity is of the second.

As has been stated above the ratio of one quantity to another (of the same kind) is determined by the *fraction* whose numerator is the measure of the first quantity and whose denominator is the measure of the second quantity, both the quantities being expressed in terms of the same unit.

Thus, the ratio of 3*s*. to 5*s*. is determined by the fraction $\frac{3}{5}$; of 2 yd. to 5 ft. by the fraction $\frac{6}{5}$.

The first of the two quantities forming a ratio is called the **antecodent** and the second is called the **consequent** of the ratio; the two together are called the **terms** of the ratio. The ratio of 3*s*. to 5*s*. is written 3*s*. : 5*s*. and may be expressed either in the form 3 : 5 or in the fractional form $\frac{3}{5}$.

A ratio is of *greater* inequality when the antecedent is greater than the consequent; and is of *less* inequality when the antecedent is less than the consequent. A ratio of greater inequality is, therefore, greater than unity and is expressed by an improper fraction; while a ratio of less inequality is less than unity and is expressed by a proper fraction.

Noto. The *inverse* ratio of 3*s*. to 5*s*. is the ratio of 5*s*. to 3*s*.

248. The value of a ratio does not depend upon the nature of the quantities involved. Thus, the ratios, 2 yd. : 5 yd., 2*s*. : 5*s*., 2 lb. : 5 lb., are all equal, each of these being determined by the fraction $\frac{2}{5}$. Hence, in investigating the properties of ratios, we usually consider the terms to be numbers, because numbers measure quantities of all kinds.

249. As ratios are fractions, the properties of the latter apply equally to the former. Therefore the value of a ratio is not altered by multiplying or dividing both its terms by the same number. Thus the ratios, 2 : 3, 4 : 6, 80 : 120, are all equal.

250. Ratios are **compounded** by taking the product of the antecedents for a new antecedent and the product of the consequents for a new consequent. Thus the ratio compounded of the ratios, 2 : 3 and 6 : 7 is $2\times6 : 3\times7$ or 4 : 7.

251. Four quantities are said to be **in proportion** or **proportionals** when the ratio of the first to the second is equal to the ratio of the third to the fourth.

Thus, 3, 4, 9, 12 are in proportion ; since the ratio of 3 to 4 is equal to the ratio of 9 to 12.

N. B. When four quantities are in proportion, it is not necessary than all of them should be of the same kind ; it is only necessary that the first two should be of the same kind, as also the second two.

The existence of proportion among the numbers is denoted thus :—

$$3 : 4 = 9 : 12,$$

which is read "3 to 4 equals 9 to 12" ;

or thus :— $3 : 4 :: 9 : 12,$

which is rea "3 is to 4 as 9 is to 12" ;

or thus :— $\frac{3}{4}=\frac{9}{12},$

which is read "3 over 4 is equal to 9 over 12".

Of this proportion 3 and 12 are called the **extremes**, and 4 and 9, the means ; 12 is called a **fourth proportional** to 3, 4 and 9.

252. When four quantities are in proportion so that

	first	: second	:: third	: fourth ;
then also,	second	: first	:: fourth	: third ;
and	fourth	: third	:: second	: first.

Also, if the quantities are all the same kind,

first	: third	:: second	: fourth.

253. When four numbers are in proportion, the product of the expremes is equal to the product of the means.

For example, 3 : 4 = 6 : 8, and we have $3\times8=4\times6$.

Hence also, an extreme = product of the means ÷ the other extreme ; and, a mean = product of the extremes ÷ the other mean.

254. Three quantities of the same kind are said to be **in continued proportion** when the ratio of the first to the second is equal to the ratio of the second to the third. The second quantity is called a **mean proportional** between the first and third ; and the third quantity is called a **third proportional** to the first and second.

Thus, 2, 4 and 8 are in continued proportion ; for $2:4=4:8$; 4 is a mean proportional between 2 and 8 ; and 8 is a third proportional to 2 and 4.

It is obvious that the square of the mean proportional between two numbers is equal to their product.

255. *Example* 1. Find a fourth propotional to 3, 9 and 4.

$$3:9=4:\text{number required},$$

$$\therefore \quad \text{number required}=\frac{9\times4}{3}=12.$$

Example 2. Find the number which has the same ratio to 20 that 3 has to 5.

$$3:5=\text{number required}:20,$$

$$\therefore \quad \text{number required}=\frac{3\times20}{5}=12.$$

Example 3. Find a mean proportional between 3 and 12.

Square of the number required $=3\times12=36$;

$$\therefore \quad \text{the number required}=\sqrt{36}=6.$$

Example 4. A, B, C, D are quantities of the same kind ; and the ratio of A to B is $3:4$, of B to C is $5:7$, and of C to D is $8 . 9$. Find the ratio of A to D.

$$\text{Now,}\quad \frac{A}{B}=\frac{3}{4},\quad \frac{B}{C}=\frac{5}{7}\text{ and }\frac{C}{D}=\frac{8}{9};$$

$$\therefore \quad \frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}=\frac{3}{4}\times\frac{5}{7}\times\frac{8}{9},\text{ or }\frac{A}{D}=\frac{10}{21};$$

that is, $A:D::10:21$.

Note. We find the **continued ratio** of A, B, C, and D, that is, we compare A, B, C and D thus :

$$A:B=3:4,$$

$$B:C=5:7=1:\tfrac{7}{5}=4:\tfrac{28}{5},$$

$$C:D=8:9=1:\tfrac{9}{8}=\tfrac{28}{5}:\tfrac{63}{10},$$

We change the terms of the ratios in such a way that each antecedent may be equal to the preceding consequent.

$$\therefore A:B:C:D=3:4:\tfrac{28}{5}:\tfrac{63}{10}$$

$$=30:40:56:63;$$

which is read "A is to B is to C is to D as 30 is to 40 is to 56 is to 63".

And A, B, C, D are said to be **in proportion of** or **proportional** to 30, 40, 56, 63

Example 5. A mixture (42 gallons) contains wine and water in the ratio of 5 to 2 ; find the quantities of wine and water in the mixture.

If the mixture be divided into 7 (*i.e.*, $5+2$) equal parts, 5 of the parts will be wine and 2 water.

$\therefore$ The quantity of wine $= \frac{42}{7} \times 5$ gallons $= 30$ gallons ;

and the quantity of water $= \frac{42}{7} \times 2$ gallons $= 12$ gallons.

Example 6. A mixture (40 gallons) contains wine and water in the ratio of 3 to 1 ; how much water must be added to it that the ratio of wine to water may be $5 : 2$?

We find, as in the preceding example, that the mixture contains 30 gall. wine and 10 gall. water. Now while the wine remains the same 30 gallons, the water is to be increased so that the ratio of wine to water may be $5 : 2$; but $5 : 2 = 30 : 12$; $\therefore$ $(12 - 10)$ gall. or 2 gall. of water must be added.

EXAMPLES. 150.

Find the value of each of the following ratios in its simplest form :

1. $15 : 21$. **2.** R39 : R65. **3.** £3 : £5. 10*s*.

4. 360 in. : 270 in. **5.** 350 lb. : 725 lb. **6.** 2°. 5′ : 3°.

7. $3\frac{3}{4} : 5\frac{5}{8}$. **8.** $2\frac{3}{5} : 4\cdot\dot{3}$. **9.** 3 yd. : 7 ft. 6 in.

Express in its simplest form the ratio compounded of the ratios

10. $7 : 9$ and $45 : 28$. **11.** $1 : 2$, $2 : 3$ and $3 : 4$.

12. $2\frac{1}{2} : 3\frac{1}{3}$ and $\cdot\dot{3} : \cdot 25$. **13.** $4 : 7$, $5 : 8$ and $21 : 30$.

Compare the ratios,

14. $3 : 5$ and $7 : 8$. **15.** $13 : 21$ and $18 : 29$.

16. $2 : 3$, $3 : 4$ and $4 : 5$. **17.** $3 : 7$, $5 : 9$ and $7 : 11$.

Are the following in proportion ?

18. 6, 11, 18, 33. **19.** 5, 7, 20, 27. **20.** R3, R2. 4*a*., 4, 3.

Find a fourth proportional to

21. 7, 9 and 8. **22.** $2\frac{1}{2}$, 3 and $4\frac{1}{3}$. **23.** ·2, ·02 and ·002.

24. R380, R570 and 12 lb. **25.** 4 yd., 2 yd. 2 ft. and £2.

26. 12 acres, 27 ac. and 20 men. **27.** 12 men, 9 men and £3.

28. 6 miles, 20 mi. and 9 hours. **29.** 3 cwt., 84 lb. and £1. 8*s*.

Find a mean proportional between

30. 7 and 28. **31.** 13 and 117. **32.** 9464 and 5600.

33. $\frac{5}{28}$ and $\frac{20}{63}$. **34.** $2\frac{1}{2}$ and $5\frac{5}{8}$. **35.** ·3 and ·012.

Find a third proportional to

36. $2\frac{1}{4}$ and $7\frac{1}{2}$. **37.** 7 and $5\frac{3}{8}$. **38.** R2 and R1. 4*a*.

39. Compare the rates of two trains, one of which runs 17 miles in 2 hours and the other $12\frac{1}{2}$ miles in $2\frac{1}{2}$ hours.

40. $A : B = 3 : 4$, $B : C = \frac{3}{4} : \frac{4}{5}$; find the ratio of A to C.

41. If $A = \frac{4}{5}$ of B, and $B = 2\frac{1}{2}$ of C, find the ratio of A to C.

42. If, when A earns R4, B earns R5 ; and when B earns R6, C earns R7 ; and when C earns R8, D earns R9 ; compare the earnings of A, B, C and D.

43. Two sums of money are proportional to 7 and 8 ; the first is £2 ; what is the other ?

44. The weights of equal volumes of gold and water are as 37 is to 2. If a cu. ft. of water weigh 1000 oz., find the weight of a cu. ft. of gold.

45. The ratio of the circumference of a circle to its diameter is 22 : 7 ; find the circumference of a circle 10 ft. 6 in. in diameter.

46. One man adds 5 seers of water to 15 seers of milk, and another 3 seers of water to 12 seers of milk ; compare the amount of milk in the two mixtures.

47. While A makes a profit of £3, B makes £4 ; and while B makes a profit of £5, C makes £6 ; if A makes a profit of £20, how much does C make in the same time ?

48. A mixture (50 gall.) contains wine and water in the ratio of 3 : 2 ; find the quantities of wine and water in the mixture.

49. A mixture (30 gall.) contains wine and water in the ratio of 7 to 3 ; how much water must be added to it that the ratio of wine to water may be 3 : 7 ?

50. A greyhound pursues a hare and takes 4 leaps for every 5 leaps of the hare, but 5 leaps of the hound are equal to 4 of the hare : compare the rates of hound and hare.

XXXVIII. SIMPLE PROPORTION. RULE OF THREE.

256. Problems which we have solved by the Unitary Method may also be solved by the method of finding a fourth proportional to three given quantities.

Example 1. Find the price of 12 maunds of sugar when the price of 5 maunds is R60.

Here we observed that if the weight be *increased* 2, 3...times, the price will also be *increased* 2, 3...times ; therefore the ratio of the two weights is equal to the ratio of the two corresponding prices.

Hence 5 md. : 12 md. :: R60 : the answer ;

$\therefore$ the answer = R$\frac{12 \times 60}{5}$ = R144.

Example 2. If 12 men can do a piece of work in 5 days, in how many days will 15 men do it ?

Here we observe that if the number of men be *increased* 2, 3... times, the number of days will be *decreased* 2, 3...times ; therefore the *inverse ratio* of the numbers of men is equal to the *ratio* of the corresponding numbers of days.

Hence 15 men : 12 men :: 5 days : the answer ;

$\therefore$ the answer $= \frac{12 \times 5}{15}$ days $= 4$ days.

257. The above method of solving a problem by finding a fourth proportional to three given quantities is commonly known by the name of **Rule of Three.**

In the first problem we have an example of what is called the Rule of Three **Direct**, because there the *direct* ration of the two weights is equal to the ratio of the corresponding prices.

In the second problem we have an example of what is called the Rule of Three **Inverse**, because there the *inverse* ratio of the numbers of men is equal to the ratio of the corresponding numbers of days.

258. It is obvious that the second term in a proportion is greater or less than the first according as the fourth is greater or less than the third. Hence we may lay down the following general rule for arranging the terms in a Rule of Three question.

Denote the answer by the letter x and place it for the 4th term ; and of the three given quantities place that which is of the same kind as the answer, for the 3rd term. Next from the nature of the question determine whether the answer will be greater or less than the third term, and place the greater or less of the two remaining quantities for the 2nd term according as the answer is greater or less than the 3rd term ; then place the remaining quantity for the first term.

Note. In working, the two first quantities in the proportion must be replaced by the numbers which measure them in terms of the same unit.

Example 1. If the third class railway fare for 110 miles is R1 . 11 . 6, what is the fare for 350 miles ?

	mi	mi.	R. a. p.	
	110 :	350 ::	1 . 11 . 6 :	x,
i.e.,	11 :	35 ::	1 . 11 . 6 :	x ;

$$\therefore \quad x = \frac{\text{R}1 . 11 . 6 \times 35}{11} = \frac{\text{R}60 . 2 . 6}{11}$$

$$= \text{R}5 . 7 . 6. \quad \textit{Ans.}$$

Or thus : $\because$ R1 . 11 . 6 = 330*p*.

$x = \frac{35 \times 330}{11} p. = 1050p.$

$= R5 . 7 . 6.$

The latter method is the one more generally adopted. The learner should observe that the 3rd term being expressed in pies the answer obtained at the first instance is also in pies.

Example 2. If a quantity of rice serve 100 men for 15 weeks, how many men will it serve 6 weeks ?

weeks weeks men

$6 : 15 :: 100 : x,$

i.e., $2 : 5 :: 100 : x;$

$\therefore\ x = \frac{5 \times 100}{2}$ men = 250 men. *Ans.*

Example 3. A bankrupt's debts amount to £1320, and his assets (*i.e.*, the value of his property) are £990, how much can he pay in the pound ?

£. £. £.

$1320 : 1 :: 990 : x,$

$\therefore\ x = £\frac{1 \times 990}{1320} = £\frac{3}{4} = 15s.$ *Ans.*

Example 4. A man, after paying an income-tax of 4*p*. in the rupee, has R4794 left ; what is his gross income ?

R1 = 192*p*. ; R1 − 4*p*. = R188*p*.

p. *p*. R.

$188 : 192 :: 4794 : x,$

i.e. $47 : 48 :: 4794 : x;$

$\therefore\ x = R\frac{48 \times 4794}{47} = R4896.$ *Ans.*

Example 5. If 8 oxen or 6 horses eat the grass of a field in 10 days, in how many days will 5 oxen and 4 horses eat it ?

oxen oxen horses

$8 : 5 :: 6 : x,$

$\therefore\ x = \frac{5 \times 6}{8}$ horses $= \frac{15}{4}$ horses.

$\therefore$ 5 oxen and 4 horses will eat as much as $(\frac{15}{4} + 4)$ or $\frac{31}{4}$ horses.

horses horses days

Now, $\frac{31}{4} : 6 :: 10 : x,$

$\therefore\ x = \frac{6 \times 10 \times 4}{31}$ days $= 7\frac{23}{31}$ days. *Ans.*

Example 6. *A* can do a piece of work in 7 days, and *B* can do it in 9 days ; how long will *A* and *B*, working together, take to do the work ?

A can do $\frac{1}{7}$ of the work and B can do $\frac{1}{9}$ of the work in 1 day ; $\therefore$ A and B together can do $(\frac{1}{7}+\frac{1}{9})$ or $\frac{16}{63}$ of the work in 1 day.

$$\begin{array}{ccccccc} \text{work} & & \text{work} & & \text{day} & & \\ \frac{16}{63} & : & 1 & :: & 1 & : & x, \end{array}$$

$$\therefore \quad x=\tfrac{63}{16} \text{ days}=3\tfrac{15}{16} \text{ days. } \textit{Ans.}$$

Example 7. At what time between 2 and 3 o'clock are the hands of a clock at right angles to each other ?

The minute-hand gains 11 divisions on the hour-hand in 12 minutes ; and here it has to gain (10+15) or 25 divisions.

$$\begin{array}{ccccccc} \text{div.} & & \text{div.} & & \text{min.} & & \\ 11 & : & 25 & :: & 12 & : & x, \end{array}$$

$$\therefore \quad x=\tfrac{25\times 12}{11} \text{ min.}=27\tfrac{3}{11} \text{ min. ;}$$

$\therefore$ the two hands will be at right angles to each other at $27\frac{3}{11}$ minutes past 2.

Example 8. A can beat B by 40 yards in a mile race ; B can beat C by 20 yards in a mile race ; if A and C run a mile, by how much will A win ?

While A runs 1760 yd., B runs 1720 ;

and.........B ...... 1760 yd., C ...... 1740.

$$\begin{array}{cccccccc} & 1760 & : & 1720 & :: & 1740 & : & x, \\ \textit{i.e.,} & 44 & : & 43 & :: & 1740 & : & x, \end{array}$$

$$\therefore \quad x=\tfrac{43\times 1740}{44} \text{ yd.}=1700\tfrac{5}{11} \text{ yd.}$$

$\therefore$ While B runs 1720 yd., C runs $1700\frac{5}{11}$ yd., but while B runs 1720 yd., A runs 1760 yd. ; $\therefore$ while A runs 1760 yd., C runs $1700\frac{5}{11}$ yd. $\therefore$ A will win by $(1760-1700\frac{5}{11})$ or $59\frac{6}{11}$ yd.

Example 9. A starts from P to walk to Q, a distance of $51\frac{3}{4}$ miles, at the rate of $3\frac{3}{4}$ miles an hour ; an hour later B starts from Q for P and walks at the rate of $4\frac{1}{4}$ miles an hour : when and where will A meet B ?

A has already gone $3\frac{3}{4}$ miles when B starts. Of the remaining 48 miles, A walks $3\frac{3}{4}$ and B walks $4\frac{1}{4}$ in one hour ; that is, they together pass over $(3\frac{3}{4}+4\frac{1}{4})$ or 8 miles in one hour.

$$\begin{array}{ccccccc} \text{miles} & & \text{miles} & & \text{hour} & & \\ 8 & : & 48 & :: & 1 & : & x, \end{array}$$

$$\therefore \quad x=\tfrac{48}{8} \text{ hours}=6 \text{ hours.}$$

$\therefore$ A meets B in 6 hours after B started. And therefore they meet at a distance of $4\frac{1}{4}\times 6$ or $25\frac{1}{2}$ miles from Q.

[For Examples for Exercise see Section XXXV.]

XXXIX. DOUBLE RULE OF THREE.

259. Complex problems which would require two or more applications of the Rule of Three are usually solved py a shorter method, commonly called the **Double Rule of Three.** The method will be best explained by means of examples.

***Example* 1.** If 9 men can reap 6 acres in 10 days, how many men will reap 12 acres in 15 days ?

The problem can be divided into two parts.

(i) If 9 men can reap 6 acres in 10 days, in how many days will 9 men reap 12 acres ?

Let x be the number of days. The number of days varies directly as the number of acres.

$$\therefore \frac{6 \text{ acres}}{12 \text{ acres}} = \frac{10 \text{ days}}{x \text{ days}}.$$

$$\therefore \quad x = \frac{12 \times 10}{6} = 20,$$

i.e., the number of days is 20.

(ii) If 9 men can reap 12 acres in 20 days, how many men will reap 12 acres in 15 days ?

Let x be the number of men. The number of men varies inversely as the number of days, hence

$$\frac{x \text{ men}}{9 \text{ men}} = \frac{20 \text{ days}}{15 \text{ days}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\therefore \quad x = \frac{20 \times 9}{15} = 12.$$

$\therefore$ The number of men reqd. in the original question = 12. *Ans.*

Now, putting $\frac{12 \times 10}{6}$ for 20, in (i) we get

$$\frac{x}{9} = \frac{12 \times 10}{6 \times 15}, \text{ or } \frac{6 \times 15}{12 \times 10} = \frac{9}{x};$$

$$i.e., \quad 6 \times 15 : 12 \times 10 :: 9 : x.$$

But the ratio $6 \times 15 : 12 \times 10$ is the ratio compounded of the ratios $6 : 12$ and $15 : 10$.

Hence we proceed as explained below in solving problems which require two or more applications of the Rule of Three.

We denote the answer by x and place it for the 4th term, and place 9 men (which is of the same kind as the answer) for the 3rd term. We next take 6 acres and 12 acres (a pair of quantities of the same kind), and consider whether the answer will be

greater or less than the 3rd term in the question "if 9 men can reap 6 acres, how many men will reap 12 acres, supposing the time to be the same in both cases ?" and we find that the answer will be greater ; we therefore place 12 acres for the 2nd and 6 acres for the 1st term. Then we take 10 days and 15 days (another pair of quantities of the same kind), and consider whether the answer will be greater or less than the 3rd term in the question "if 9 men can reap in 10 days, how many men will reap in 15 days, supposing the number of acres to be the same in both cases ?" and we find that the answer will be less ; we therefore place 10 days for the 2nd and 15 days for the 1st term, under those already obtained

$$\left.\begin{array}{lrcr} \text{acres} & 6 & : & 12 \\ \text{days} & 15 & : & 10 \end{array}\right\} :: 9 \text{ men} : x.$$

We now multiply the numbers in the 1st term for the final 1st term and the numbers in the 2nd term for the final 2nd term. Thus

$$6 \times 15 : 12 \times 10 :: 9 : x,$$

$$\therefore \; x = \frac{12 \times 10 \times 9}{6 \times 15} \text{ men} = 12 \text{ men. } \textit{Ans.}$$

Note. Each pair of quantities of the same kind should be replaced by their measures in terms of the same unit.

Remark. Each additional pair of quantities of the same kind would be treated in a like manner.

Example 2. If 72 men can dig a trench, 324 yd. long, 12 yd. wide and 8 ft. deep, in 9 days of 12 hours each ; how many men can dig a trench, 1458 yd. long, 40 ft. wide and 3 yd. deep, in 36 days of 9 hours each ?

$$\left.\begin{array}{lrcr} \text{ft. long} & 324 \times 3 & : & 1458 \times 3 \\ \text{ft. wide} & 12 \times 3 & : & 40 \\ \text{ft. deep} & 8 & : & 3 \times 3 \\ \text{days} & 36 & : & 9 \\ \text{hours} & 9 & : & 12 \end{array}\right\} :: 72 \text{ men} : x,$$

$$\therefore \; x = \frac{1458 \times 3 \times 40 \times 3 \times 3 \times 9 \times 12 \times 72}{324 \times 3 \times 12 \times 3 \times 8 \times 36 \times 9} \text{ men} = 135 \text{ men. } \textit{Ans.}$$

Or better thus :

$$\left.\begin{array}{ll} \text{cu. ft. } (324 \times 3) \times (12 \times 3) \times 8 & : (1458 \times 3) \times 40 \times (3 \times 3) \\ \text{hours } \; 36 \times 9 & : 9 \times 12 \end{array}\right\} :: 72 : x.$$

Example 3. If 10 men can perform a piece of work in 24 days, how many men will perform another piece of work 3 times as great in $\frac{1}{5}$ of the time ?

$$\left.\begin{array}{lrcr} \text{work} & 1 & : & 3 \\ \text{days} & \frac{24}{5} & : & 24 \end{array}\right\} :: 10 \text{ men} : x,$$

$$\therefore \; x = \frac{3 \times 24 \times 10}{\frac{24}{5}} \text{ men} = \frac{3 \times 24 \times 10 \times 5}{24} \text{ men} = 150 \text{ men. } \textit{Ans.}$$

Example 4. If the sixpenny loaf weigh 8 oz. when wheat is 15*s*. a bushel, what ought a bushel of wheat to be when the fourpenny loaf weighs 12 oz. ?

$$\left.\begin{array}{lll} \text{pence} & 6 & : 4 \\ \text{ounces} & 12 & : 8 \end{array}\right\} :: 15s. : x,$$

$$\therefore \quad x = \tfrac{4 \times 8 \times 15}{6 \times 12} s. = \tfrac{20}{3} s. = 6s.\ 8d. \quad \textit{Ans.}$$

Example 5. If 5 cannon, which fire 3 rounds in 5 minutes, kill 135 men in $1\frac{1}{2}$ hours, how many cannon, which fire 5 rounds in 6 minutes, will kill 250 men in 1 hour ?

[The first 5 cannon, each firing 54 rounds, kill 135 men ; it is required to find how many cannon, each firing 50 rounds, will kill 250 men.]

$$\left.\begin{array}{lll} \text{rounds} & 50 & : 54 \\ \text{men} & 135 & : 250 \end{array}\right\} :: 5 \text{ cannon} : x,$$

$$\therefore \quad x = \tfrac{54 \times 250 \times 5}{50 \times 135} \text{ cannon} = 10 \text{ cannon.} \quad \textit{Ans.}$$

260. Examples in Double Rule of Three can be worked more conveniently in a little different manner. In this method the first *work* and second *work* are respectively taken for the third and fourth terms of the proportion, and the first *cause* and second *cause* respectively for the first and second terms ; for, the ratio of the two *causes* is equal to the ratio of the corresponding *works*. We shall apply the method to the first two of the foregoing examples.

Example 1. 9 men in 10 days will do the same amount of work as (9 × 10) men will do in 1 day ; and x men in 15 days will do the same amount of work as (x × 15) men will do in 1 day.

$$\therefore \quad 9 \times 10 : x \times 15 :: 6 : 12,$$

$$\therefore \quad x \times 15 \times 6 = 9 \times 10 \times 12,$$

$$\therefore \quad x = \tfrac{9 \times 10 \times 12}{15 \times 6} \text{ men} = 12 \text{ men.} \quad \textit{Ans.}$$

Example 2.

$$72 \times 9 \times 12 : x \times 36 \times 9 :: (324 \times 3) \times (12 \times 3) \times 8 : (1458 \times 3) \times 40 \times (3 \times 3)$$

$$\therefore \quad x = \tfrac{72 \times 9 \times 12 \times 1458 \times 3 \times 40 \times 3 \times 3}{36 \times 9 \times 324 \times 3 \times 12 \times 3 \times 8} \text{ men}$$

$$= 135 \text{ men.} \quad \textit{Ans.}$$

[For Examples for Exercise see Section xxxvi.]

XL. APPLICATION OF ALGEBRA TO ARITHMETIC.

(*This Section may be taken at a later stage at the discretion of the teacher.*)

261. Application of Algebraical methods to the solution of Arithmetical questions is sometimes very convenient. The student should learn at as early a stage as possible that Algebra and Arithmetic are not two different subjects having no connection whatsoever. On the contrary, they are two correlated branches of the same science of Mathematics. Algebraic methods may be freely used in Arithmetic if their application leads to a simpler solution of any question.

262. Application of Formulæ.—When an algebraical formula is applicable, it may be used with great advantage in simplifying fractions.

Example 1. Simplify $\dfrac{\frac{4}{7}\times\frac{4}{7}-\frac{3}{7}\times\frac{3}{7}}{\frac{4}{7}-\frac{3}{7}}$.

Let $\frac{4}{7}=a$, and $\frac{3}{7}=b$, then the given fraction

$$=\frac{a^2-b^2}{a-b}=\frac{(a+b)(a-b)}{(a-b)}=a+b=\tfrac{4}{7}+\tfrac{3}{7}=1. \quad \textit{Ans.}$$

Example 2. Simplify $\dfrac{\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}+\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}}{\frac{3}{4}\times\frac{3}{4}-\frac{3}{4}\times\frac{2}{3}+\frac{2}{3}\times\frac{2}{3}}$.

Let $\frac{3}{4}=a$, and $\frac{2}{3}=b$, then the given fraction

$$=\frac{a^3+b^3}{a^2-ab+b^2}=\frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2}=a+b=\tfrac{3}{4}+\tfrac{2}{3}=\tfrac{17}{12}=1\tfrac{5}{12}. \quad \textit{Ans.}$$

EXAMPLES. 151.

Simplify

1. $\dfrac{\frac{1}{408}\times\frac{1}{408}-\frac{1}{459}\times\frac{1}{459}}{\frac{1}{408}-\frac{1}{459}}$.

2. $\dfrac{(\frac{2}{5}\text{ of }\frac{1}{5})-(\frac{2}{7}\text{ of }\frac{1}{7})}{(\frac{1}{5}\times\frac{1}{5}\times\frac{2}{7})-(\frac{1}{7}\times\frac{1}{7}\times\frac{2}{5})}$.

3. $(967+\frac{1}{967})(967+\frac{1}{967})-(967-\frac{1}{967})(967-\frac{1}{967})$.

4. $\{(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2})+(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3})\}\div\{(\frac{1}{2}\times\frac{1}{2})-(\frac{1}{2}\times\frac{1}{3})+(\frac{1}{3}\times\frac{1}{3})\}$.

5. $\frac{4}{3}\times\frac{4}{3}\times\frac{4}{3}-3\times\frac{4}{3}\times\frac{4}{3}\times\frac{1}{3}+3\times\frac{4}{3}\times\frac{1}{3}\times\frac{1}{3}-\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$.

6. $\dfrac{\frac{1}{2}\times\frac{1}{2}-2\times\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}-\frac{1}{4}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{2}-2\times\frac{1}{2}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}-\frac{1}{3}\times\frac{1}{3}}$.

7. $\dfrac{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{2}}$.

8. $\dfrac{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}+\frac{1}{4})}{(\frac{1}{2}+\frac{1}{4})(\frac{1}{2}+\frac{1}{4})-(\frac{1}{3}+\frac{1}{4})(\frac{1}{3}+\frac{1}{4})}\times\left(\frac{1}{\frac{1}{2}+\frac{1}{4}}\times\frac{1}{\frac{1}{3}+\frac{1}{4}}\right)$.

9. $\dfrac{(\frac{1}{2})^4+(\frac{1}{10})^2+(\frac{1}{5})^4}{(\frac{1}{2})^2-(\frac{1}{10})+(\frac{1}{5})^2}$.

10. $\dfrac{(\frac{1}{7}\times\frac{1}{7}\times\frac{1}{7}\times\frac{1}{7})+(\frac{1}{7}\times\frac{1}{7}\times\frac{1}{9}\times\frac{1}{9})+(\frac{1}{9}\times\frac{1}{9}\times\frac{1}{9}\times\frac{1}{9})}{\{(\frac{1}{7}\times\frac{1}{7})+(\frac{1}{7}\times\frac{1}{9})+(\frac{1}{9}\times\frac{1}{9})\}\times\{(\frac{1}{7}\times\frac{1}{7})-(\frac{1}{7}\times\frac{1}{9})+(\frac{1}{9}\times\frac{1}{9})\}}$.

11. $\dfrac{(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5})+(\frac{1}{7}\times\frac{1}{7}\times\frac{1}{7})+(\frac{1}{9}\times\frac{1}{9}\times\frac{1}{9})-3(\frac{1}{5}\times\frac{1}{7}\times\frac{1}{9})}{\{(\frac{1}{5}\times\frac{1}{5})+(\frac{1}{7}\times\frac{1}{7})+(\frac{1}{9}\times\frac{1}{9})\}-\{(\frac{1}{5}\times\frac{1}{7})+(\frac{1}{7}\times\frac{1}{9})+(\frac{1}{9}\times\frac{1}{5})\}}$.

12. $\dfrac{\frac{1}{3}\times\frac{1}{5}\times(\frac{1}{3}+\frac{1}{5})+\frac{1}{5}\times\frac{1}{7}\times(\frac{1}{5}+\frac{1}{7})+\frac{1}{7}\times\frac{1}{3}\times(\frac{1}{7}+\frac{1}{3})+2\times\frac{1}{3}\times\frac{1}{5}\times\frac{1}{7}}{\frac{1}{3}\text{ of }\frac{1}{3}+\frac{1}{3}\text{ of }\frac{1}{5}+\frac{1}{3}\text{ of }\frac{1}{7}+\frac{1}{5}\text{ of }\frac{1}{7}}$.

263. Application of Formulæ to the Simplification of Decimal Fractions.

Example 1. Simplify $\dfrac{\cdot704\times\cdot704-\cdot296\times\cdot296}{\cdot704-\cdot296}$.

Let $\cdot704=a$, and $\cdot296=b$, then the given fraction

$$=\frac{a^2-b^2}{a-b}=\frac{(a+b)(a-b)}{a-b}=a+b=\cdot704+\cdot296=1\cdot000=1.\quad \textit{Ans.}$$

Example 2. Simplify $(3\cdot2)^2+(2\cdot8)^2-(6\cdot4)(2\cdot8)$.

Let $3\cdot2=a$, and $2\cdot8=b$, then the given fraction

$$=a^2+b^2-2ab=(a-b)^2=(3\cdot2-2\cdot8)^2=(\cdot4)^2=\cdot16.\quad \textit{Ans.}$$

Example 3. Find the value of

$$(1\cdot25)^3+2\cdot25\times(1\cdot25)^2+3\cdot75\times(\cdot75)^2+(\cdot75)^3.$$

Let $1\cdot25=a$, and $\cdot75=b$, then the given fraction

$$=a^3+3b\times a^2+3a\times b^2+b^3=a^3+3a^2b+3ab^2+b^3=(a+b)^3$$
$$=(1\cdot25+\cdot75)^3=(2)^3=8.$$

Example 4. Simplify $\dfrac{\cdot89\times\cdot89\times\cdot89-\cdot64\times\cdot64\times\cdot64}{\cdot89\times\cdot89+\cdot89\times\cdot64+\cdot64\times\cdot64}$.

Let $\cdot89=a$, and $\cdot64=b$, then the given fraction

$$=\frac{a^3-b^3}{a^2+ab+b^2}=\frac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}=a-b=\cdot89-\cdot64=\cdot25.$$

Example 5. Simplify $\dfrac{\cdot835\times\cdot835\times\cdot835+\cdot165\times\cdot165\times\cdot165}{\cdot835\times\cdot835-\cdot835\times\cdot165+\cdot165\times\cdot165}$.

Let $\cdot835=a$, and $\cdot165=b$, then the given fraction

$$=\frac{a^3+b^3}{a^2-ab+b^2}=\frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2}=a+b=\cdot835+\cdot165=1.$$

EXAMPLES. 152.

Simplify

1. $1{\cdot}79 \times 1{\cdot}79 + 2{\cdot}42 \times 1{\cdot}79 + 1{\cdot}21 \times 1{\cdot}21.$

2. $\dfrac{1{\cdot}59 \times 15{\cdot}9 - {\cdot}41 \times 4{\cdot}1}{15{\cdot}9 - 4{\cdot}1}.$

3. $\dfrac{({\cdot}0179)^2 - ({\cdot}0178)^2}{({\cdot}0026)^2 - ({\cdot}0025)^2}.$

4. $\dfrac{{\cdot}727 \times {\cdot}727 - {\cdot}273 \times {\cdot}273}{{\cdot}727 - {\cdot}273}.$

5. ${\cdot}54 \times {\cdot}54 \times {\cdot}54 + {\cdot}46 \times {\cdot}46 \times {\cdot}46 + 3 \times {\cdot}54 \times {\cdot}46 \times {\cdot}46 + 3 \times {\cdot}54 \times {\cdot}54 \times {\cdot}46$

6. $\dfrac{{\cdot}727 \times {\cdot}727 \times {\cdot}727 + {\cdot}273 \times {\cdot}273 \times {\cdot}273}{{\cdot}727 \times {\cdot}727 - {\cdot}727 \times {\cdot}273 + {\cdot}273 \times {\cdot}273}.$

7. $\dfrac{(3{\cdot}2)^2 + (2{\cdot}8)^2 - (6{\cdot}4)(2{\cdot}8)}{(1{\cdot}6)^2 + (2{\cdot}4)^2 + (3{\cdot}2)(2{\cdot}4)}.$

8. $\dfrac{{\cdot}1 \times {\cdot}1 \times {\cdot}1 + {\cdot}01 \times {\cdot}01 \times {\cdot}01}{{\cdot}2 \times {\cdot}2 \times {\cdot}2 + {\cdot}02 \times {\cdot}02 \times {\cdot}02}.$

9. $\dfrac{2{\cdot}31 \times 2{\cdot}31 \times 2{\cdot}31 - 1{\cdot}69 \times 1{\cdot}69 \times 1{\cdot}69}{2{\cdot}31 \times 2{\cdot}31 - 1{\cdot}69 \times 1{\cdot}69}.$

10. $\dfrac{3{\cdot}2 \times 3{\cdot}2 + 2{\cdot}8 \times 2{\cdot}8 - 6{\cdot}4 \times 2{\cdot}8}{1{\cdot}6 \times 1{\cdot}6 + 2{\cdot}4 \times 2{\cdot}4 + 3{\cdot}2 \times 2{\cdot}4}.$

11. $\dfrac{{\cdot}07 \times \{({\cdot}07)^2 + 1\}}{({\cdot}07)^4 - 1} \times \dfrac{\{({\cdot}07)^2 - {\cdot}07\} \times ({\cdot}07 + 1)}{({\cdot}07)^2}.$

12. ${\cdot}54 \times {\cdot}54 \times {\cdot}54 + {\cdot}46 \times {\cdot}46 \times {\cdot}46 + 3 \times {\cdot}54 \times {\cdot}46.$

264. Application of Equations.

Example 1. The sum of two numbers is 45 and their difference is 21 ; find the numbers.

Let x denote the smaller number ; then $x+21$ denotes the greater.

$$\text{Hence } x + (x+21) = 45,$$
$$\therefore \quad 2x + 21 = 45,$$
$$\therefore \quad 2x = 45 - 21 = 24,$$
$$\therefore \quad x = 12.$$

Therefore 12 is the smaller number and (12+21) or 33, the greater.

Example 2. Divide R1000 among A, B and C, in such a way that A may have R120 more than B, and B R110 more than C.

Let x denote the number of rupees that C has ; then B has $(x+110)$ rupees, and A has $(x+110+120)$ rupees.

Then, by the question,

$$x+(x+110)+(x+110+120)=1000,$$
$$\therefore \quad 3x+340=1000,$$
$$\therefore \quad 3x=1000-340=660,$$
$$\therefore \quad 3x=220.$$

Hence C gets R220 ; B, R330 ; and A, R450. *Ans.*

Example 3. Divide R100 among 3 men, 5 women and 6 boys, so that each man may receive three times as much, and each woman twice as much as a boy.

Let x denote the number of rupees that each boy gets ; then each woman gets $2x$ rupees ; and each man gets $3x$ rupees.

Therefore	6 boys	get	$6x$ rupees,
	5 women	get	$10x$ rupees,
and	3 men	get	$9x$ rupees.

Then, by the question, $6x+10x+9x=100,$

$$\therefore \quad 25x=100,$$
$$\therefore \quad x=4.$$

Hence each boy gets R4 ; each woman, R8 ; and each man, R12.

Example 4. Divide R28 into an equal number of rupees, half-rupees and quarter-rupees.

Let x denote the number of coins of each kind. Then we have x rupees ; x half-rupees or $\frac{x}{2}$ rupees ; and x quarter-rupees or $\frac{x}{4}$ rupees.

By the question, $x+\frac{x}{2}+\frac{x}{4}=28,$

$$\therefore \quad \tfrac{7}{4}x=28,$$
$$\therefore \quad x=28\times\tfrac{4}{7}=16.$$

$\therefore$ The number of each kind of coin $=16$.

Example 5. A purse contains R27. 4*a*. in half-rupees and quarter-rupees. If the number of coins be 70, how many are there of each kind ?

Let x represent the number of half-rupees. Then $(70-x)$ represents the number of quarter-rupees.

Now, x half-rupees $= 8x$ annas,

and $(70-x)$ quarter-rupees $=(70-x)\times 4$ annas $=(280-4x)$ annas.

By the question, $8x+(280-4x)=27\times 16+4=436$,

$$\therefore \quad 4x=436-280=156,$$

$$\therefore \quad x=39.$$

$\therefore$ The number of half-rupees $=39$.

$\therefore$ The number of quarter-rupees $=70-39=31$.

Example 6. The number of spectators at a football match was 15568, the sum of sixpence was charged for admission, and 1245 of the persons admitted paid, in addition, one shilling each for grand-stand tickets. The total receipts were £423. 7*s*. 6*d*. How many persons were admitted without payment?

£423. 7*s*. 6*d*. = 101610*d*.

Now suppose x persons were admitted without payment.

Then $(15568-x)$ persons paid 6*d*. each, and of these 1245 paid 1*s*. extra.

Then, by the question, $(15568-x)\times 6+1245\times 12=101610$.

Solving the equation we have, $x=1123$.

$\therefore$ The number of men admitted without payment $=1123$. *Ans.*

Example 7. Three railway tickets, a 1st, a 2nd, and half a third class ticket were purchased for 16*s*. $10\frac{1}{2}$*d*. The 1st class ticket cost $1\frac{2}{3}$ times as much as the 2nd, and the 2nd class $1\frac{1}{2}$ times as much as a whole 3rd class ticket. The distance travelled was 45 miles. Find the cost of each ticket and the rate per mile for each class.

Suppose a 3rd class ticket costs x pence. Then a 2nd class ticket costs $1\frac{1}{2}x$ pence, and a 1st class ticket costs $1\frac{2}{3}\times 1\frac{1}{2}x$ pence.

$\therefore$ By the question, $\frac{1}{2}x+1\frac{1}{2}x+1\frac{2}{3}\times 1\frac{1}{2}x=202\frac{1}{2}$. [16*s*. $10\frac{1}{2}$*d*. $=202\frac{1}{2}$*d*.]

$$\therefore \quad x+3x+5x=\frac{405}{2}\times 2.$$

$$\therefore \quad 9x=405.$$

$$\therefore \quad x=45.$$

$\therefore$ A 3rd class ticket costs 45*d*. = 3*s*. 9*d*.,

$\therefore$ A 2nd ... ... 45*d*. $\times 1\frac{1}{2}$ = 5*s*. $7\frac{1}{2}$*d*. ; etc.

And the rates per mile are $\frac{45}{45}$*d*. or 1*d*. (3rd class) ; $1\frac{1}{2}$*d*. (2nd class) ; etc.

Example 8. A certain number is divided into two parts such that 5 times the first part added to 18 times the second equals 7 times the whole. Find the ratio of the parts.

Let x and y be the parts of the number.

Then, by the question, $5x+18y=7(x+y)$,

$$\therefore \quad 5x+18y=7x+7y,$$

$$\therefore \quad 18y-7y=7x-5x,$$

$$\therefore \quad 11y=2x;$$

$$\therefore \quad \frac{x}{y}=\frac{11}{2}.$$

$\therefore$ The 1st part is to the 2nd part as 11 : 2.

Example 9. The price of 5 horses and 6 oxen is R680, that of 4 horses and 7 oxen is R610. Find the price of an ox. [See *Ex.* 1, page 276.]

Let Rx and Ry denote the price of a horse and of an ox, respectively. Then

$$5x+6y=680, \text{..................(i)}$$

and $$4x+7y=610 \text{..................(ii)}$$

Multiplying the line (i) by 4 and the line (ii) by 5, we have

$$20x+24y=2720,$$

and $$20x+35y=3050;$$

$\therefore$ By subtraction, $-11y=-330$,

or, $y=30$.

$\therefore$ The price of 1 ox = R30. *Ans.*

Example 10. A man rows down a river 18 miles in 4 hours with the stream, and returns in 12 hours; find the rate at which he rows and the rate at which the stream flows. [See *Ex.* 5, page 264]

Let x m. p. h. be the man's rate in still water and y m. p. h. the rate of the stream. Then

$x+y$ = the man's rate with the stream ....(i)

and $x-y$ = ... against ... ...(ii)

Then, by the question, $\left.\begin{array}{l}4(x+y)=18, \\ \text{and } 12(x-y)=18\end{array}\right\}$

$$\therefore \left.\begin{array}{l}x+y=\frac{9}{2}, \\ \text{and } x-y=\frac{3}{2};\end{array}\right\}$$

whence $x=3$, and $y=1\frac{1}{2}$.

$\therefore$ The man's rate in still water = 3 m. p. h., and the rate of the stream = $1\frac{1}{2}$ m. p. h.

Note. From (i) and (ii) we have

$x=\frac{1}{2}$ (man's rate with stream + man's rate against stream) ;

$y=\frac{1}{2}$ (man's rate with stream − man's rate against stream).

Hence we have, *generally*, that

(i) ***a man's rate in still water is half the sum of his rates with and against the stream ;***

(ii) ***the rate of the stream is half the difference between the man's rates with and against the stream.***

***Example* 11.** A man standing on a railway platform noticed that a train took 21 seconds to completely pass through the station, which was 88 yd. long, and that it was 9 seconds in passing him. Find the length of the train, and the rate of the train in miles p.h.

Let x yd. be the length of the train. Then the train travels x yd. in 9 sec. and $(88+x)$ yd. in 21 sec.

$\therefore \frac{x}{9}=\frac{88+x}{21}$ [∵ both sides represent the no. of yd. run by the train in 1 sec.]

Multiplying both sides by 63, we have, $7x=264+3x$,

$\therefore \quad 4x=264,$

$\therefore \quad x=66.$

$\therefore$ The length of the train $=66$ yd.

Hence the train travels 66 yd. in 9 sec. Therefore it travels $\frac{66\times60\times60}{9\times1760}$ or 15 miles in an hour.

***Example* 12.** In running from a station P to a station Q a train travels at $\frac{5}{7}$ths of its usual speed. Shew that it will take $\frac{7}{5}$ths of its usual time.

Let x m. p. h. be the usual speed of the train, and let y miles be the distance from P to Q.

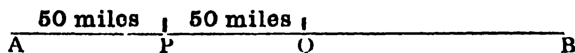
Then the usual time $=\frac{y}{x}$ hr.

and the actual time taken $=\frac{y}{\frac{5}{7}x}$ hr. $=\frac{7}{5}\times\frac{y}{x}$ hr.

$=\frac{7}{5}$ths of the usual time.

And, *generally*, if a body moves at $\frac{n}{m}$ of its usual velocity, it will take $\frac{m}{n}$ of its usual time to cover the same distance.

Example 13. A train running from A to B meets with an accident 50 miles from A, after which it moves with $\frac{3}{5}$ths of its original speed, and arrives at B 3 hours late. Had the accident happened 50 miles further on, it would have been only 2 hours late. Find the original speed of the train and the distance from A to B.



Let P be the place where the accident happenes and let Q be a place 50 miles from P. Then it is obvious that owing to the reduction in speed the train takes $(3-2)$ hr. or 1 hr. more than its usual time in running from P to Q.

Now, suppose that x m. p. h. is the original speed of the train. Then after the accident at P its speed is $\frac{3}{5}x$ m. p. h. Therefore the usual time taken by the train in running from P to $Q=\frac{50}{x}$ hr. and the actual time $=\frac{50}{\frac{3}{5}x}$ hr.

$\therefore$ By the question, $\frac{50}{\frac{3}{5}x}-\frac{50}{x}=1$,

$$\therefore \quad \frac{250}{3}-50=x,$$

$$\therefore \quad x=83\tfrac{1}{3}-50=33\tfrac{1}{3}.$$

Hence the original speed of the train $=33\frac{1}{3}$ m. p. h.

Again, let y miles be the distance from Q to B.

Then, by the question, $\frac{y}{\frac{3}{5}\times 33\frac{1}{3}}-\frac{y}{33\frac{1}{3}}=2$,

$$\therefore \quad \frac{y}{20}-\frac{3y}{100}=2,$$

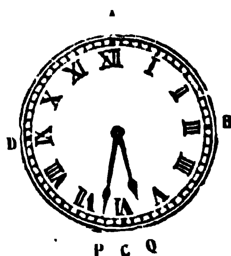
$$\therefore \quad 5y-3y=200,$$

$$\therefore \quad y=100.$$

Hence the distance from A to $B=(50+50+100)$ miles $=200$ miles. *Ans.*

Example 14. A man who went out between 5 and 6 and returned between 6 and 7, found that the hands of his watch had exactly changed places. When did he go out ?

Suppose that the man went out at x minutes past 5; and let, in the annexed diagram, the minute-hand and hour-hand be then at P and Q respectively. Then the arc ABP contains x minute-spaces; and $\therefore$ the arc ABQ contains $(25+\frac{x}{12})$ minute-spaces. Now, when the man returned, the minute-hand had moved through the arc $PDA+ABQ$, *i.e.*, through $\{(60-x)+(25+\frac{x}{12})\}$ minute-spaces; while the hour-hand had moved through the arc $QCP(=ABP-ABQ)$, *i.e.*, through $\{x-(25+\frac{x}{12})\}$ minute spaces. But the minute-hand moves through 12 times the arc moved through by the hour hand in the same time.



$$\therefore \quad (60-x)+\left(25+\frac{x}{12}\right)=12\left\{x-\left(25+\frac{x}{12}\right)\right\};$$

$$\text{or} \qquad 60-x+25+\frac{x}{12}=12x-300-x;$$

$$\therefore \quad 720-12x+300+x=144x-3600-12x;$$

$$\therefore \quad -143x=-4620;$$

$$\therefore \quad x=\frac{4620}{143}=32\tfrac{4}{13}.$$

Hence the man went out at $32\frac{4}{13}$ minutes past 5.

[The arithmetical solutions of the above examples are left to the student as exercises.]

Example 15. A challenged B to ride a bicycle race of 1040 yards. He first gave B 120 yards' start and lost by 5 seconds, he then gave B 5 seconds' start and won by 122 feet. How long does each take to ride the distance?

Suppose A takes x seconds, and B, y seconds to ride 1040 yards.

From the first race we have, $x-5=\frac{920}{1040}y=\frac{23}{26}y$............(i)

... ... second... ... ... $x+5=\frac{1000}{1040}y=\frac{25}{26}y$............(ii)

Subtracting (i) from (ii), $10=\frac{2}{26}y$; $\therefore$ $y=130$.

Substituting this value of y in (i), $x-5=115$; $\therefore$ $x=120$.

Hence A takes 120 sec. and B takes 130 sec. to ride 1040 yards.

Example 16. A starts from a place P to go to a place Q; at the same time B starts from Q for P. After meeting they arrive at their destinations in 2 and 3 hours respectively. Shew that the ratio of their speeds is as $\sqrt{3}:\sqrt{2}$.

Suppose that A and B meet at R and let A's rate of motion be x m. p. h. and B's y m. p. h.

P R Q

Then $RQ=2x$ miles and $PR=3y$ miles. Now the time taken by A in going from P to $R=\frac{3y}{x}$, and the time taken by B in going from Q to $R=\frac{2x}{y}$. It is obvious that these two times are equal.

$$\therefore \frac{3y}{x}=\frac{2x}{y},$$

$$\therefore 3y^2=2x^2,$$

$$\therefore \frac{x^2}{y^2}=\frac{3}{2},$$

$$\therefore \frac{x}{y}=\frac{\sqrt{3}}{\sqrt{2}},$$

$$\therefore x:y=\sqrt{3}:\sqrt{2}.$$

And, generally, if A and B arrive at their destinations in T and T' hours after meeting the ratio of their speeds is as $\sqrt{T'}:\sqrt{T}$.

EXAMPLES. 153

(Most of the following examples admit of solution without the use of algebraical symbols. For the sake of comparison such examples should be worked out both arithmetically and algebraically.)

1. The sum of two numbers is 53 and their difference is 11 ; find the numbers.

2. Divide ₨112 among A, B and C, so that B may have ₨12 more than A, and C may have ₨13 more than B.

3. Divide ₨101. 4a among A, B and C, so that B may have twice as much as A, and C thrice as much as B.

4. Divide ₨100 among 3 men, 5 women and 10 boys, so that each man may receive 4 times as much as a boy, and each woman twice as much as a boy.

5. A purse contains £8. 0s. 6d. in shillings and sixpences. The number of coins is 201 ; how many are there of each kind ?

6. I have a certain number of rupees to be distributed among a certain number of boys. I find that if I give each boy ₨3, I shall have left ₨7 ; but if I give each boy ₨5, I shall require ₨3 more. Find the number of boys and the number of rupees I have.

7. Find the number whose half exceeds its fifth part by 6.

8. A and B have R40 between them. If A gives $\frac{1}{3}$ of his share to B, then B will have as much as A had to start with Find their original shares.

9. Find the number whose square is equal to the difference between the squares of 6467 and 4683.

$$\text{Now, } \sqrt{6467^2 - 4683^2} = \sqrt{(6467 + 4683)(6467 - 4683)}$$
$$= \sqrt{11150 \times 1784}$$
$$= \sqrt{(2 \times 5 \times 5 \times 223) \times (2 \times 2 \times 2 \times 223)}$$
$$= \sqrt{2^2 \times 2^2 \times 5^2 \times 223^2} = \text{etc.}$$

10. A journey of 132 miles can be performed either by steamer or by train. The charge by steamer is at the rate of 12a for 20 miles, and by train at the rate of 1a. per mile. I cannot afford to pay more than R6 for the journey. What is the least distance which I must travel by steamer ?

11. A goods train starts from a station P to a station Q; at the same time a passenger train starts from Q to P. If they arrive at Q and P 6 hours and $1\frac{1}{2}$ hours after they passed each other, show that the passenger train travels twice as fast as the goods train.

12. A man who went out between 4 and 5 and returned between 5 and 6 found that the hands of his watch had exactly changed places. When did he go out ?

13. A person, standing on a railway platform 264 yards long, noticed that a train, which passed him in 8 seconds, passed completely through the station in 20 seconds. Find the length of the train, and the rate of the train in miles per hour.

14. A certain number is divided into two parts such that 5 times the first part added to 11 times the second part makes 7 times the whole. Find the ratio of the parts.

15. I buy 8 cows and 10 sheep for R270 ; for the same sum I can have 5 cows and 40 sheep. Find the price of a cow and a sheep.

16. A motor car after travelling 20 miles develops engine trouble and then proceeds at $\frac{3}{4}$ths of its former rate and arrives at its destination 1 hr 20 min. late. Had the trouble arisen 30 miles further on, it would have been only 50 min late. Find the original speed of the motor car and the distance travelled.

17. A man walks to a railway station at $3\frac{1}{4}$ m. p h. to catch the 10 o'clock train but misses it by 3 minutes ; the next day he walks at $3\frac{1}{2}$ m. p. h. to catch the same train but misses it again by 1 minute. Find the distance to the station.

18. A man swimming in a stream which flows $1\frac{1}{2}$ m. p. h. finds that in a given time he can swim twice as far with the stream as he can against it : at what rate does he swim ?

19. *A* and *B* run a mile race. First *A* gives *B* a start of 40 yards and beats him by 10 seconds. During the second heat *A* gives *B* a start of 15 seconds and beats him by 18 yards. Find the speed of each ?

MISCELLANEOUS EXAMPLES. 154.

1. Find the least number which being added to 1409 will make the result divisible by 23.

2. A boy receiving ₨2. 4*a.* a week has 8*a.* stopped every fourth week ; if there are 48 weeks in the school-year, how much does he get in 2 years ?

3. What are the prime factors in 45090045, and what is the smallest whole number by which it must be multiplied in order to become a perfect square ?

4. Find the least fraction which, being added to $\frac{1}{8}+\frac{1}{6}\div\frac{1}{7}-\frac{1}{7}\times\frac{1}{8}-\frac{1}{3}$, shall make the result an integer.

5. Find, by Practice, the value of $37\frac{1}{2}$ md. of sugar at ₨9. 13*a.* 6*p.* per md.

6. If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in $\frac{3}{8}$ of the time ?

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7. Find the greatest and least numbers of four digits exactly divisible by 34.

8. I distribute a sum of money among 32 men, giving ₨50. 7*a.* 6*p.* to the first, ₨51. 7*a.* 6*p.* to the next, ₨52. 7*a.* 6*p.* to the next, and so on, increasing the sum by ₨1 each time ; how much would each get if I divided the money equally ?

9. Determine the least number, by which 378 must be multiplied to produce a number exactly divisible by 336.

10. A screw advances ·392 of an inch at each turn ; how many turns must be taken for it to advance 9·8 inches ?

11. If 12 iron bars, each 4 ft. long, 3 in. broad and 2 in. thick, weigh 576 lb., how much will 11 weigh, each 6 ft. long, 4 in. broad and 3 in. thick ?

12. Read off as decimals of £1 :—(i) 19*s.*, (ii) 12*s.* 6*d.*

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13. The population of a town is 5720, and there are 320 more men than women ; how many are there of each sex ?

14. A labourer, who works on week-days only, earns *7a. 9p.* a day ; supposing that the 1st of January 1885 was on a Sunday, find the amount of his earnings during the year.

15. Four bells ring at intervals of 3, $3\frac{1}{4}$, $3\frac{1}{2}$ and $3\frac{3}{4}$ seconds respectively, beginning together ; how often during 24 hours will the four bells ring together again ?

16 By what number must $\frac{1}{2}+\frac{1}{3}$ of $\frac{1}{6}-\frac{1}{4}$ be multiplied in order to produce the least possible integer ?

17. A certain number of men subscribed £63. 0*s.* 9*d.*, each subscribing as many pence as there were men ; how many men were there ?

18. Read off £7631. 19*s.* $5\frac{1}{2}$*d.* as the decimal of £1 to three places. Hence find the number of farthings in the sum.

19. To the fourth part of a certain number I add 79, and obtain 100 as the sum ; what is the number ?

20. Divide R101. 15*a.* 3*p.* among 20 men, giving to each of 5 of them twice as much as to each of the others.

21. 720 gallons of cocoanut oil and 450 gallons of castor oil are to be put into an exact number of barrels, all of the same size, without mixing the two oils together ; find the least number of barrels required.

22. The palace of the King of Babylon contained a thousand rectangular court yards, each 60 metres long and 54 metres broad. The court yards were all paved with marble slabs, 18 inches long by 18 inches broad. Required the total number of slabs. (A metre = 39·37 inch.)

23. The perimeter of a rectangle is 110 ft. ; the difference of two sides is 11 ft. ; find its area as the decimal of an acre.

24. If a man can perform a journey of 170 miles in $4\frac{1}{4}$ days of 11 hours each, in how many days of $8\frac{1}{4}$ hours each, will he perform a journey of 470 miles ?

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25. To a certain number I add 3, and multiply the sum by 4, then divide the product by 5, and get 7 as quotient and 1 as remainder ; what is the number ?

26. A man bought 40 pieces of ribbon, all equally long, for R137. 8*a.* at 2*a.* 9*p.* a yard ; how many inches were there in each piece ?

27. Two taps can fill a cistern in 4 and 6 hours respectively. When the waste-pipe is left open along with the taps, the cistern is filled in 24 hours. In what time will the waste-pipe empty the cistern when full ?

28. What is the capacity of a vessel, out of which, when it is half full, $4\frac{1}{2}$ gallons being drawn, there remain $\frac{1}{6}$ of the whole content ?

29. A square space, containing 113 sq. yd. 7 sq. ft., is to be lengthened by 3 ft. in one of its dimensions, and to be shortened by 3 ft. in the other ; what will then its area be ?

30. If a person walks 7 miles in $2\frac{1}{2}$ hours, how long will a second person take to walk 10 miles, supposing that the first walks $2\frac{1}{4}$ miles while the second walks $2\frac{1}{4}$?

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31. Fourteen years ago a man was six times as old as his son whose present age is 20 years ; what is the present age of the father ?

32. A man buys 20 seers of milk at 3*a.* 6*p.* per seer ; how much water must he add to it that he may gain ₨1. 4*a.* by selling the mixture at 3*a.* per seer ?

33. I had coins of one kind weighing 2295 grains ; and of this I spent coins weighing 1035 grains ; show that a single coin cannot weigh more than 45 grams.

34. Two clocks begin to strike 12 together ; one strikes at an interval of 2·916 seconds, the other, of 2·083 seconds ; what decimal of a minute is there between their seventh strokes ?

35. Find the cost of painting the walls of a square room, 10 ft. high and 16 ft. long, with one door 8 ft. by 4 ft., and 2 windows, each 5 ft. by 2, the amount saved by each window being ₨1. 14*a.* What additional height would increase the cost by ₨12 ?

36. A merchant of Calcutta indented from London goods worth £226, and paid £34 for freight and packing. He sold half the goods at a gain of 2 annas per rupee ; at what gain per rupee must he sell the remainder that he may clear ₨500 on the whole outlay ? [₨1 = 1*s.* $7\frac{1}{2}$*d.*]

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37. Find the greatest fraction, the numerator of which is composed of 3, 5, 1, 0 and the denominator of 3, 2, 8, 0.

38. In the number 13579 insert a zero, somewhere between 1 and 9 so as (i) to make the greatest possible difference in the number, (ii) to make the least possible difference ; and find the difference of these two differences.

39. Two persons buy 600 oranges each at 24 for a half-rupee : one sells them at 5*a.* 6*p.* a dozen, and the other at 8*a.* 3*p.* a score ; who gains more, and by how much ?

39a. A number is exactly divisible by 7 and by 13, and it is known that the number is between 400 and 500 ; what is the number ?

40. How long will it take to walk round a square field 13 ac. 3 ro. 9 sq. po in area at the rate of 3 miles per hour ?

41. Find the length of the inner edge of a cubical cistern which will hold 256 lb. of water, supposing that a cu. ft. of water weighs 1000 oz.

42. A person after paying an income-tax of 1 anna in the R, devotes $\frac{1}{18}$ of the remainder of his income to purposes of charity, and finds that he has R5175 left ; what is his income ?

43. A person has a number of oranges to dispose of : he sells half of what he has and one more to *A*, half of the remainder and one more to *B*, half of the remainder and one more to *C* ; by which time he has disposed of all he had : how many had he at first ?

44. A certain number of men, twice as many women and three times as many children earned R16. 2*a.* in 3 days ; each man earned 12*a.*, each woman 8*a.* and each child 5*a.* a day : how many women were there ?

45. Find the greatest weight that will measure (*i.e.*, divide exactly) a lb. Avoir., and a lb. Troy.

46. A boy running up a staircase finds that when he goes up two steps at a time there is one step over ; when he goes up three at a time there are two over ; and when he goes up four at a time there are three over. Find the number of stairs, which is somewhere between 40 and 50.

47. How many bricks, 6 in. by 3 in. by 3 in., will be required for a wall, 16 ft. by 10 ft. by 2 ft. allowing $\frac{1}{18}$ of the space for mortar ?

48. A creditor received on a debt of R3600 a dividend of 9*a.* 10*p.* in the R ; and a further dividend of 6*a.* 8*p.* upon the remainder. What did he receive altogether, and what fraction was it of the entire debt ?

49. *A* has R150, *B* has R120 ; if *C* had R16 more than what he has, then *B* and *C* together would have as much as *A* : how much has *C* ?

50. Divide £30 10*s.* 8*d.* into two sums of money, one of which contains as many shillings as the other contains fourpences.

51. 378 oranges and 462 mangoes are to be distributed among boys so that each boy gets as many oranges and as many mangoes as any other boy ; find the largest possible number of boys, and the least possible number of fruits each boy may get.

52. What number is greater than its fifth part by $\frac{1}{5}$?

53. Find how much card-board is required to make a cubical box and its cover, the edge of the box is 9 in., and the rim of the cover extends 3 in. deep down each side.

54. A work can be completed in 36 days by 30 men working 6 hours a day; in what time would 18 men and 60 women, working 9 hours a day, complete it, supposing that 3 men can do as much work as 5 women?

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55. A gentleman's monthly expenses are R150 less than his income; if his income be increased by R100 a month and expenses decreased by R50, how much will he be able to save in a year?

56. A man with a fixed yearly income and a fixed daily expenditure saves in ordinary years $\frac{1}{27}$ of his income. In leap years he saves R63 9*a*. What is his income?

57. Three persons *A*, *B*, *C* start on a tour, each with £20 in his pocket, and agree to divide their expenses equally. When they return, *A* has £3. 11*s*. 9*d*, *B* has £2. 5*s*. and *C* has 17*s*. 3*d*. What ought *A* and *B* to pay to *C* to settle their accounts?

58. A man walks at the rate of 128 yards per minute; find the least whole number of minutes he will take to walk over an exact number of miles.

59. The external dimensions of an open box are 5 ft., $4\frac{1}{2}$ ft. and 3 ft.; find the cost of painting the outside at 3 annas per sq. yd. What will be the cost of painting the inside at the same rate, if the box is made of $\frac{1}{2}$ inch plank?

60. Three men can do as much work as 5 boys; the wages of three boys are equal to those of two men. A work, on which 40 boys and 15 men are employed, takes 8 weeks and costs £350; how long would it take if 20 boys and 20 men were employed, and how much would it cost?

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61. What quantity of water must an inn-keeper add to a barrel of beer, which cost him £50, to reduce the price of £1. 5*s* a gallon?

62. A certain number of men mow 4 acres in 3 hours, and a certain number of others mow 8 acres in 5 hours: how long will they be mowing 11 acres, if they all work together?

63. At 10 minutes to 2 in the afternoon a clock is 55 seconds slow, and at 6 in the evening it is 30 seconds slow: at what hour will it show true time?

64. A train leaves Calcutta at 7 A. M. for Goalundo, 153 miles distant, and travels at the rate of 20 miles an hour: another train leaves Goalundo for Calcutta at 11-30 A. M. and travels at the rate of 22 miles an hour; when, and where, will the trains pass each other.

65. A cistern, 6 ft. long, 5 ft. wide and 4 ft. deep, contains pulp for making paper. If $\frac{3}{5}$ of the volume of the pulp be lost

in the process of drying, how many sheets of paper, 16 in. by 10 in., will be obtained, if 400 sheets in thickness go to an inch ?

66. If 7 men and 5 boys can reap 168 acres in 18 days, how many days will 15 men and 5 boys take to reap 700 acres, one man being able to do three times as much work as a boy ?

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67. A gramophone with 5 rolls of music costs £7 ; the same gramophone with 20 rolls of music costs £9. How much should be paid for the gramophone with 50 rolls of music ?

68. Two pipes, *A* and *B*, fill a cistern in 25 and 30 minutes respectively. Both pipes being opened, find when the first must be closed that the cistern may be just filled in 15 minutes.

69. If $\frac{1}{5}$ of a sheep be worth $\frac{2}{3}$ of a rupee, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of a cow, how much must be given for 106 cows ?

70. The cubic content of an open cistern, 6 ft. long and 4 ft. broad, is 20 cu. ft. ; what will be the cost of lining the inside of it at 1*s.* per sq. ft. ?

71. Two persons walking at the rate of $3\frac{1}{2}$ and 4 miles per hour respectively, set off from the same place in opposite directions to walk round a park, and meet in 20 minutes. Find the length of the path round the park.

72. If it takes 120 men to supply, in 5 days' work, a fortress with provisions for 5 months, when the garrison is 650 strong, how many will be required to supply it in 3 days for 4 months, after the garrison had been reduced by 130 men ?

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73. A bag contains a certain number of shillings, twice as many sixpenny pieces and 3 times as many fourpenny pieces ; the whole sum amounts to 2 guineas : find the number of each.

74. A room, whose height is 9 ft., and length twice its breadth, takes 189 yards of paper, 2 ft. wide, for its four walls ; find its length.

75. *A* can do a piece of work in 20 days ; *A* and *B* together can do it in $11\frac{1}{9}$ days. *A* works alone for 8 days, *A* and *C* together for 6 days, and *B* finishes it in 3 days. Find in what time *B* and *C* together could do it.

76. One clock gains 8 min., and another loses 4 min., in 24 hours. They are set at right at noon on Sunday. Determine the time indicated by each clock when the one appears to have gained 12 minutes on the other.

77. The whole time occupied by a train 110 yards long, travelling at the rate of 30 miles an hour, in crossing a bridge is 12 seconds ; find the length of the bridge.

78. If a family of 9 persons spend R480 in 8 months, how much will serve a family (living upon the same scale) of 24 persons for 16 months ?

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79. Simplify $\dfrac{\pounds 7.\ 6s.\ 8d.}{\pounds 3.\ 4s.} \times \dfrac{\frac{1}{2} - \frac{1}{3} \text{ of } \frac{1}{4} - \frac{1}{6}}{(\frac{1}{2} - \frac{1}{3}) \text{ of } (\frac{1}{4} - \frac{1}{6})}$.

80. A room twice as long as it is broad is carpeted at 9*s.* a sq. yd., and the walls are painted at 1*s.* 6*d.* a sq. yd., the respective costs being £44. 2*s.* and £8. 8*s.* Find the dimensions of the room.

81. A cistern would be filled by a tap, *A*, in $3\frac{1}{2}$ hours, or emptied by a tap, *B*, in 3 hours. The cistern being half full, *A* is turned on at 8 o'clock, and *B* at 15 min. to 9 ; find when the cistern will again be half full.

82. If 2 guineas make 3 napoleons, and 15 rix-dollars make 4 napoleons, and 6 ducats make 7 rix-dollars, how many ducats are there in £490 ?

83. A person rows a distance of 3 miles down a stream in 40 minutes, but without the aid of the stream it would have taken him an hour ; what is the rate of the stream per hour ? And how long would it take him to return against it ?

84. A boat propelled by 6 oars which take 25 strokes per minute travels at the rate of $7\frac{1}{2}$ miles an hour ; find the rate of a boat propelled by 4 oars which take 32 strokes per minute ; the work done by each oar during one stroke in the latter case being a quarter as much again as in the former case.

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85. A wagon, loaded with 1246 equal packages, weighs 26 tons 14 cwt. ; if the wagon itself weighs twice as much as the packages, find the weight of each package.

86. *A* did $\frac{3}{5}$ of a piece of work in 6 hours, *B* did $\frac{3}{4}$ of what remained in 2 hours and *C* finished it in half an hour. How long would they have been doing the whole if they had worked together ?

87. A clock loses 5 minutes a day. It shows correct time at noon on a Monday. After how many days will it again show correct time on a Monday ?

88. A privateer, running at the rate of 10 miles an hour, discovers a ship, 18 miles off, making way at the rate of 8 miles an hour ; how many miles can the ship run before she is overtaken ?

89. If the wages of 25 men amount to R766. 10*a.* 8*p.* in 16 days, how many men must work 24 days to receive R1035, the daily wages of the latter being one-half those of the former ?

90. 55 gallons of a mixture of wine and water contain 5 gallons more wine than water; find the ratio of wine to water in the mixture.

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91. Bring $\left\{\left(\frac{5\frac{1}{4}-\frac{1}{3}\text{ of }2\frac{5}{6}}{\frac{3}{5}\times 4\frac{1}{3}+1\frac{1}{2}}+\frac{2\frac{3}{7}}{4\frac{3}{7}}\right)\div 21\frac{28}{29}\times 3\frac{19}{206}\right\}$ cwt. to the fraction of $4\frac{1}{7}$ tons.

92. *A* can do half a piece of work in 3 hours, being twice as much as *B* can do; *A*, *B* and *C* can together do the whole in $2\frac{1}{2}$ hours; in how many hours will *C* do a piece of work which *B* can do in 9 hours?

93. How many seconds will a train, 184 feet in length, travelling at the rate of 21 miles an hour, take in passing another train, 223 feet long, proceeding in the same direction at the rate of 16 miles an hour?

94. *A* can give *B* 20 yards' start in a mile race and can give *C* 40 yards' start; how much start can *B* give *C* in a mile race?

95. A piece of work must be finished in 36 days, and 15 men are set to do it, working 9 hours a day; but after 24 days it is found that only $\frac{3}{5}$ of the work is done. If 3 additional men be then put on, how many hours a day will they all have to labour, in order to finish the work in time?

96. *Two equal wine glasses are filled with mixtures of wine and water in the ratios of 2 of wine to 3 of water and 3 of wine to 4 of water; when the contents are mixed in a tumbler, find the strength of the mixture.

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97. Divide R47 between *A*, *B* and *C* in such a manner that *B* may receive R2 more than 3 times, and *C* R3 more than 4 times the amount to be received by *A*.

98. At what times between 2 and 3 are the hands of a clock $5\frac{1}{2}$ minute-divisions apart?

99. Three boys agree to start together and run, until all come together again, round a circular court 15 yards in circumference. One runs at the rate of 6, the second, 7, and the third, 8 miles an hour. In how many seconds will the race end?

100. In a game of skill *A* can give *B*, and *B* can give *C*, 10 points out of a game of 50; how many should *A* give *C*?

101. If 7 cows and 20 sheep be worth £12, and 3 cows and 16 sheep be worth £7, find the price of a cow and of a sheep.

102. Two equal wine glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of wine; they are then filled up with water, and the contents mixed in a tumbler: find the ratio of wine to water in the tumbler.

103. Express $\cdot\dot{6}$ of R17. 8*a*. + $\cdot\dot{5}$ of £1. 14*s*. 6*d*. as the fraction of R170, a rupee being worth 2 shillings.

104. *A* can do a piece of work in 8 days, which *B* can destroy in 3. *A* has worked 6 days, during the last 2 of which *B* has been destroying; how many days must *A* now work alone in order to complete his task?

105. A train 110 yd. long overtook a person walking along the line at the rate of 3 miles an hour, and passed him completely in 9 seconds; afterwards it overtook another person and passed him in $9\frac{3}{8}$ seconds. At what rate was this second person walking?

106. In a hundred yards' race *A* can give *B* four and *C* five yards' start; if *B* were to race *C*, giving him 1 yard in a hundred, which would win?

107. If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days, how long will it take 2 men and 2 boys to reap 10 acres?

108. Gold and silver are mixed together in a mass of 30 oz., so that for every 6 parts of gold there are 4 parts of silver. How much gold must be added to the mass, so that for every 5 parts of gold there may be 3 parts of silver?

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109. A publican bought 10 gallons of wine at £1 . 7 . 6 per gallon; he mixed some water and filled quart bottles with it; how much water must have been added, supposing that the cost price of the contents of each bottle was thereby reduced to 5*s*. $8\frac{3}{4}$*d*.?

110. If 12 oxen be worth 29 sheep, 15 sheep worth 25 hogs, 17 hogs worth 3 loads of wheat, and 8 loads of wheat worth 13 loads of barley; how many loads of barley must be given for 340 oxen?

111. *A* and *B* are two spouts attached to a cistern. *A* can fill it in 10 min., and *B* can empty it in 15 min. If *A* and *B* be opened alternately for 1 minute each, in what time will the cistern be filled?

112. A race course is one mile long; *A* and *B* run a race and *A* wins by 80 yards; *A* and *C* run over the same course and *A* wins by 20 seconds; *B* and *C* run and *B* wins by 5 seconds. In what time can *A* run a mile?

113. If I can walk a certain distance in 112 days when I rest 5 hours each day, how long will it take me to walk twice as far, if I walk twice as fast and rest twice as long each day?

114. A cask contains 12 gallons of a mixture of wine and water in the ratio of 3 to 1; how much of the mixture must be drawn off, and water substituted for the mixture in the cask to become half and half?

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115. A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides, of 6 ft. in breadth, and also has within it a path of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s.* 8*d.* per sq. ft. and of the grass 3*s.* per sq. yd., find the whole cost of laying out the court.

116. To complete a piece of work, *A* would take twice as long as *B* and *C* together, and *B* 3 times as long as *A* and *C* together; *A*, *B*, *C* together can do it in 12 days. In what time could each do it by himself?

117. A down-train usually travels at the rate of 30 miles an hour and meets an up-train 50 miles from the terminus. On one occasion, on account of an accident, it only went at the rate of 20 miles an hour and met the up-train $41\frac{2}{3}$ miles from the terminus. Find the speed of the up-train.

118. *A* can walk 5 miles an hour, and the rates at which *A* and *B* walk are in the ratio of 7 to 6; how many seconds' start must *A* give *B* that he may just beat him in a 3 mile race?

119. If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine, how many pumps with a length of stroke of $2\frac{1}{2}$ feet, working 10 hours a day for 12 days, will be required to empty the same mine; the strokes of the former set of pumps being performed four times as fast as the other?

120. If 7 horses and 12 cows cost as much as 10 horses and 6 cows, compare the prices of a horse and a cow.

XLI. DIVISION INTO PROPORTIONAL PARTS.

265. To divide a given quantity into *proportional parts* is to divide it into parts which shall be proportional to certain given numbers.

Example 1. Divide R873 among *A*, *B*, *C*, so that their shares may be in the proportion of 2, 3 and 4.

If we divide R873 into 9 (*i.e.*, 2+3+4) equal parts, then *A* will have 2, *B* will have 3 and *C* will have 4 of these parts.

Hence *A*'s share = R$\frac{873}{9}\times 2$ = R194.

B's share = R$\frac{873}{9}\times 3$ = R291.

C's share = R$\frac{873}{9}\times 4$ = R388.

Example 2. Divide £287 into parts proportional to $1\frac{1}{2}$, 2 and $3\frac{1}{3}$.

$1\frac{1}{2} : 2 : 3\frac{1}{3} = \frac{3}{2} : 2 : \frac{10}{3} = \frac{9}{6} : \frac{12}{6} : \frac{20}{6} = 9 : 12 : 20.$

Now proceed as in the preceding example.

Example 3. A certain sum of money was divided between *A*, *B*, *C* in the proportion of 5, 6 and 9; *A* received £45; what was the sum divided?

Since $5+6+9=20$, if the whole sum were divided into 20 equal parts, *A*'s share would contain 5 of these parts. Hence the value of one part $=£\frac{45}{5}$; $\therefore$ the whole sum $=£\frac{45}{5}\times 20=£180$.

Example 4. Divide ₨50 among *A*, *B*, *C*, so that *B*'s share may be half as much again as *A*'s, and *C*'s share $\frac{2}{3}$ of *A*'s and *B*'s together.

B's share $= 1\frac{1}{2}$ of *A*'s share;

$\therefore$ *A*'s share + *B*'s share = *A*'s share + $1\frac{1}{2}$ of *A*'s share

$= (1+1\frac{1}{2})$ of *A*'s share $= 2\frac{1}{2}$ of *A*'s share;

$\therefore$ *C*'s share $= \frac{2}{3}$ of $2\frac{1}{2}$ of *A*'s share $= \frac{5}{3}$ of *A*'s share;

$\therefore$ *A*'s share : *B*'s : *C*'s $= 1 : 1\frac{1}{2} : \frac{5}{3}$; etc.

Example 5. Divide 52 into 3 parts such that $\frac{1}{3}$ of the first part $=\frac{1}{2}$ of the second part $=5$ times the third part.

$\frac{1}{2}$ of the 2nd part $= \frac{1}{3}$ of the 1st part,

$\therefore$ the 2nd part $= \frac{2}{3}$ of the 1st part.

Again, 5 times the 3rd part $= \frac{1}{3}$ of the 1st part,

$\therefore$ the 3rd part $= \frac{1}{15}$ of the 1st part.

$\therefore$ 1st part : 2nd part : 3rd part

= 1st part : $\frac{2}{3}$ of the 1st part : $\frac{1}{15}$ of the 1st part

$= 1 : \frac{2}{3} : \frac{1}{15}$; etc.

Example 6. ₨82 is given to 5 men, 8 women and 10 boys, in such a way that a woman is to receive twice as much as a boy, and a man as much as a woman and a boy together; what do the women receive?

8 women receive as much as 16 boys;

and 5 men receive as much as 5 women and 5 boys,

or as 10 boys and 5 boys,

or as 15 boys;

$\therefore$ men's share : women's : boys' $= 15 : 16 : 10$; etc.

Example 7. How many rupees, half-rupees and quarter-rupees, of which the numbers are proportional to 3, 4 and 5, are together worth R50 ?

Values of three groups of coins are

as 3 rupees : 4 half-rupees : 5 quarter-rupees,

or as 12 quarter-rupees : 8 quarter-rupees : 5 quarter-rupees,

or as 12 : 8 : 5 ;

∴ the amount in rupees $= R\frac{50}{25} \times 12 = R24$;

the amount in half-rupees $= R\frac{50}{25} \times 8 = R16$;

and the amount in qr.-rupees $= R\frac{50}{25} \times 5 = R10$.

Therefore there are 24 rupees, 32 half-rupees, and 40 qr.-rupees.

Example 8. Divide £100 between *A*, *B*, *C*, *D*, so that

A's share : *B*'s = 2 : 3, *B*'s : *C*'s = 4 : 5, and *C*'s : *D*'s = 7 : 8.

We find as in *Ex.* 4, Art. 255, that the shares of *A*, *B*, *C*, *D* are proportional to 56, 84, 105 and 120 ; etc.

Note. Use of algebraical symbols often simplifies the solution of problems like those above. We shall solve *Examples* 4 and 7 algebraically.

Example 4. Suppose *A*'s share is Rx, then *B*'s share is R$\frac{3}{2}x$ and *C*'s share is R$\{\frac{2}{3} \times (x + \frac{3}{2}x)\} = R\frac{5}{3}x$.

Then, by the question, $x + \frac{3}{2}x + \frac{5}{3}x = 50$,

$\therefore \frac{25}{6}x = 50$,

$\therefore x = 12$.

Hence *A*'s share R12 ; *B*'s share $(R\frac{3}{2} \times 12) = R18$; etc.

Example 7. Suppose the no. of rupees is $3x$; then the no. of half-rupees will be $4x$ and of quarter-rupees $5x$. Now $4x$ half-rupees = R$2x$ and $5x$ quarter-rupees = R$\frac{5}{4}x$.

Then, by the question, $3x + 2x + \frac{5}{4}x = 50$,

$\therefore \frac{25}{4}x = 50$,

$\therefore x = 8$.

Hence the no. of rupees is $3x$ or 24, of half-rupees $4x$ or 32 ; etc.

EXAMPLES. 153.

(Oral.)

Divide

1. R50 into two parts in the ratio of 2 : 3.
2. £220 " " " " " " 4 : 7.

Divide

3.	72 in.	into	three	parts	proportional	to	1, 2, 3.
4.	₹5. 10*a*.	„	„	„	„	„	1, 3, 5.
5.	1600 md.	„	„	„	„	„	2, 5, 9.
6.	24 seers	„	„	„	„	„	3, 4, 5.
7.	₹36	„	four	„	„	„	3, 7, 11, 15.
8.	£13	„	three	„	„	„	$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

EXAMPLES. 156.

1. Divide ₹15. 10*a*. into parts proportional to 1, 2, 3, 4.

2. Divide £18. 9*s*. into parts proportional to 3, $2\frac{1}{2}$, 1, $\frac{1}{3}$.

3. Divide 26 tons in the proportion of 3·5, 2·25, $3\frac{1}{3}$, $3\frac{1}{12}$.

4. Divide $532\frac{1}{2}$ into parts which shall have the same ratio to one another as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

5. Divide £4. 17*s*. 6*d*. into two parts one of which is $\frac{5}{8}$ of the other.

6. A sum of money was divided into parts proportional to $3\frac{3}{4}$, 4, 5·5 ; the smallest part was ₹30 ; what was the sum divided ?

7. A sum of money was divided between *A*, *B*, *C*, in proportion to their ages which were 10, 12, 13 years respectively ; *A*'s share was £55 ; find the other shares.

8. Gunpowder is composed of saltpetre, sulphur and charcoal, in parts proportional to 75, 10 and 15 ; how many pounds of charcoal are there in 6 cwt. of gunpowder ?

9. How much of the above gunpowder can be made with 25 lb. of sulphur ?

10. In a certain battle an army lost 4 men wounded and 2 killed out of every 25, and it mustered 38,000 men unhurt ; what was the number of men in the army at first ?

11. Divide ₹90, between three persons, so that for every rupee given to the first, the second may get 12 annas and the third may get 8 annas.

12. Divide ₹36 between *A*, *B* and *C*, so that *A* gets $\frac{3}{4}$ of *B*'s share, and *C* gets $\frac{2}{3}$ of *A*'s share.

13. Divide ₹360 among *A*, *B*, *C*, so that *A* may get 3 times as much as *B*, and *B* and *C* together $\frac{1}{2}$ as much as *A*.

14. Divide ₹32 between *A*, *B*, *C*, so that *A* may receive 3 times as much as *B*, and *C* $\frac{1}{3}$ of what *A* and *B* together receive.

15. Divide £14 between *A* and *B*, so that $\frac{1}{2}$ of *A*'s money may be equal to $\frac{2}{3}$ of *B*'s.

16. Divide 30 into 3 parts such that $\frac{1}{3}$ of the first part $=\frac{2}{5}$ of the second $=\frac{1}{2}$ of the third.

17. ₨21 is divided between A, B, C; A's share is $\frac{3}{5}$ of B's; it is also $\frac{2}{5}$ of B's and C's together; find each one's share.

18. Divide £1. 13*s.* $4\frac{1}{2}$*d.* between A, B, C, D, so that A's share may be $\frac{3}{10}$ of D's, C's share $\frac{3}{10}$ of A's, and B's share the sum of A's and C's.

19. Divide £3. 6*s.* between 5 men, 7 women and 10 boys, so that each woman may have $\frac{3}{5}$ of each man's share, and each boy $\frac{2}{3}$ of each woman's share.

20. ₨110 is to be divided among 10 men, 16 women and 20 children; if each man's share is to be equal to the shares of 2 women and the 16 women are to have twice as much as the 20 children, how much will each woman receive?

21. A number of men, women and children are in the proportion of 3, 4, 5; divide £3. 5*s.* 3*d.* among them, so that the shares of a man, a woman and a child may be proportional to 4, 3, 1.

22. Divide £39 among A, B, C, so that A's share : B's share $=3:2$, B's share : C's share $=4:3$.

23. A certain kind of brass is composed of copper, zinc, lead and tin; the ratio of copper to zinc is $1:2$, of zinc to lead $3:5$ and of lead to tin $7:8$; find the quantity of zinc in 1 cwt. of the brass.

24. Four towns are to provide according to their population a contingent of 140 men. The populations of the towns are 1058, 1587, 2116 and 2645 respectively; find the number of men to be provided by each town.

25. 700 coins consist of rupees, half-rupees and quarter-rupees; the values of the rupees, the half-rupees and the quarter-rupees are as $2:3:5$; find the number of the rupees.

26. How many rupees, eight-anna pieces and four-anna pieces, of which the numbers are proportional to $2\frac{1}{2}$, 3 and 4, are together worth ₨80?

27. If 2 men do as much work as 5 women, and 6 women as much as 10 children, divide a week's wages of ₨38 among 8 men, 9 women and 15 children.

28. The sum of three fractions is $\frac{122}{135}$; 14 times the first $=15$ times the second $=18$ times the third; find the fractions.

29. Divide ₨142 among A, B, C, so that for every ₨5 given to A, B may get ₨3, and for every ₨7 given to B, C may get ₨5.

30. Areas of circles are to one another as the squares of their radii. Divide a circle of 1 ft. radius into three equal parts by concentric circles.

31. If the weight of pure silver and of alloy in a rupee be in the ratio of 11 to 1, and the price of pure silver be ₨2 . 10 . $5\frac{1}{11}$ per oz. Avoir., find the weight of a rupee (in grains) supposing its value to be that of pure silver it contains.

32. An estate is divided amongst 3 persons in the ratio of 7, 8 and 10. Find the value of the estate when ₨2500 added to the largest share would make it equal to half of the whole.

33. A number of mangoes is to be divided amongst 4 persons in shares which are as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$; what must the number *at least* be that this may be done without cutting any of the mangoes?

XLII. FELLOWSHIP OR PARTNERSHIP.

266. Suppose that three persons *A*, *B*, *C* are partners in trading, and that *A* has a capital of ₨3000 in the business, *B* has ₨5000 and *C* has ₨6000; and they gain ₨1400: how should the profit be divided?

It is obvious that the profit must be divided into parts proportional to 3000, 5000 and 6000; which may be done by the method explained in the preceding section.

The above is an example of what is called **Simple Fellowship**, the capitals contributed by the several partners being supposed to continue in the business for the same period of time.

267. Suppose, again, that *A*, *B*, *C* are partners in trading, and that *A* has a capital of ₨3000 in the business for 3 months, *B* has ₨5000 for 6 months, and *C* has ₨6000 for 7 months, at the end of which time the gain is ₨720: how should this be divided?

Now, a capital of ₨3000 employed for 3 months may be taken to be equivalent to a capital of ₨9000 (*i.e.*, ₨3000 × 3) employed for 1 month; ₨5000 for 6 months may be taken to be equivalent to ₨30000 (*i.e.*, ₨5000 × 6) for 1 month; and ₨6000 for 7 months, equivalent to ₨42000 (*i.e.*, ₨6000 × 7) for 1 month. Hence the profit must be divided in the proportion of 9000, 30000 and 42000; which may be done in the usual way.

Consequently, if the capitals of partners be employed for different periods of time, the period of time must be made the same for all, by multiplying each capital by the measure of the corresponding period of time.

Note. In working, the several sums of money must be expressed in terms of the same unit, as also the several periods of time.

The above is an example of what is called **Compound Fellowship**, the capitals contributed by the several partners being employed in the business for different periods of time.

EXAMPLES. 157.

1. *A*, *B*, *C* enter into partnership; *A* furnishes ₨350, *B* ₨500 and *C* ₨750; what should be the share of each in ₨320 profit?

2. A bankrupt owes ₨2000 to two creditors, namely, ₨1200 to one, and ₨800 to the other; his assets are ₨700; what does each creditor lose?

3. *A*, *B*, *C*, *D* engage in business with a joint capital of £7550; at the end of a year *A* receives £200, *B* £235, *C* £120 and *D* £200; how much capital did *C* put in?

4. *A*, *B*, *C* are partners, *A* receiving $\frac{3}{5}$ of the profits, and *B* and *C* sharing the remainder equally. *A*'s income is increased by ₨75 when the profits rise from $\frac{1}{12}$ of the capital to $\frac{1}{10}$. Find the respective capitals invested.

5. *A* and *B* are partners in a business in which *A* has $7\frac{1}{2}$ annas share and *B* $8\frac{1}{2}$ annas; *B*, being the working partner, receives $\frac{1}{25}$ of all the profit; the rest is divided in proportion to the capital; what does *B* receive out of ₨6080?

6. *A*, *B*, *C* engage in business with a joint capital of £18000; *A* gives £2000 more than *B*, and *B* £2000 more than *C*; divide a profit of £1080 between them.

7. *A*, *B*, *C* enter into partnership; *A* puts in £70 for 5 months, *B* £50 for 6 months and *C* £30 for 8 months. They gain £44. 10*s*. How should the profit be divided?

8. *A*, *B*, *C* pasture in the same field. *A* has in it 10 oxen for 7 months, *B* has 12 oxen for 5 months, and *C* has 15 oxen for 3 months. The rent is ₨17. 8*a*. How much of the rent should each pay?

9. *A* starts business with a capital of £2200 on the 16th of April, and on the 3rd of July admits a partner *B* with a capital of £1800. The profits amount to £449. 16*s*. by the 31st of December. What is each person's share?

10. *D* and *E* become partners, *D* bringing ₨5400 and *E* ₨4500. At the end of 3 months *D* doubles his capital and a new partner *F* is admitted who brings ₨5700; and at the end of 5 months *E* trebles his capital. The year's profits amount to ₨1200; how ought this to be divided?

11. *A* and *B* start a business with capitals as 5 : 7. They withdraw respectively $\frac{2}{3}$ and $\frac{3}{4}$ of their capitals at the end of 4 months. At the end of the year a profit of £226 is divided; find *A*'s share.

12. *A* and *B* entered into partnership with £700 and £600 respectively. After 3 months *A* withdrew $\frac{2}{7}$ of his stock but after

3 months more he put back $\frac{3}{8}$ of what he had withdrawn. The profits at the end of the year are £725 : how much of this should *A* receive ?

13. *A* and *B* start a business, *A* puts in double of what *B* puts. *A* withdraws $\frac{1}{3}$ of his stock at the end of 3 months but at the end of 7 months puts back $\frac{1}{3}$ of what he has taken out, when *B* takes out $\frac{1}{4}$ of his stock. *A* receives ₨300 profits at the end of the year ; what does *B* receive ?

14. *A* and *B* hire a meadow for 6 months. *A* puts in 21 cows for 4 months ; how many can *B* put in for the remaining 2 months, if he pays $\frac{5}{7}$ of what *A* pays ?

XLIII. ALLIGATION.

268. The following are examples of **Alligation** or the mixing of things of the same kind but of different qualities.

Example 1. How must a grocer mix teas at 2*s.* 6*d.* a lb. and 3*s.* 9*d.* a lb. so that the mixture may be worth 3*s.* a lb. ?

When the mixture is made and sold at 3*s.* a lb., each lb. of the cheaper tea in it brings a gain of 6*d.*, and each lb. of the dearer tea brings a loss of 9*d.* Therefore 9 lb. of the cheaper tea brings a gain of 54*d.* and 6 lb. of the dearer brings a loss of 54*d.* Hence, in order that there may be neither any gain nor any loss, for every 9 lb. of the cheaper tea we must take 6 lb. of the dearer ; therefore the proportion is 9 parts to 6, that is, *the teas must be mixed in the inverse ratio of the differences of the two prices and the mean price.*

Or, *algebraically* thus :

Let x lb. of the first kind of tea be mixed with y lb. of the second.

Then the price of the mixture $=(\frac{5}{2}x+\frac{15}{4}y)$ shillings.

But it is also equal to $3(x+y)$ shillings.

$$\therefore \quad \tfrac{5}{2}x+\tfrac{15}{4}y=3x+3y,$$

$$\therefore \quad 10x+15y=12x+12y,$$

$$\therefore \quad 3y=2x,$$

$$\therefore \quad \frac{x}{y}=\frac{3}{2}.$$

$\therefore$ The proportion of the mixture is 3 parts to 2.

Example 2. In what proportion should teas at 2*s*. 6*d*., 3*s*., 4*s*. 3*d*. and 4*s*. 9*d*., a lb. be mixed to make a mixture worth 4*s*. a lb. ?

The first two prices are under, and the last two above, the mean price. We take equal quantities of the teas at the first two prices, and the mixture is worth 2*s*. 9*d*. a lb. ; we also take equal quantities of the teas at the last two prices, and the mixture is worth 4*s*. 6*d*. a lb. Now we mix these two mixtures as in *Ex*. 1, and we find that these must be taken in the proportion of 6 to 15 or 2 to 5. Consequently the teas are mixed in the proportion of 1, 1, $\frac{5}{2}$, $\frac{5}{2}$.

Note. Instead of taking equal quantities we might take the teas in any proportion to make the first two mixtures ; and consequently an example of this kind (in which the number of ingredients is more than two) may have an unlimited number of solutions.

Example 3. In what ratio must a grocer mix sugar at 6*a*. per seer with sugar at 4*a*. per seer so that by selling the mixture at 5*a*. 3*p*. per seer he may gain $\frac{1}{6}$ of his outlay ?

$1\frac{1}{6}$ of the cost price of a seer of the mixture = 5*a*. 3*p*. ; ∴ cost price of a seer of the mixture = 5*a*. 3*p*. ÷ $1\frac{1}{6}$ = 4*a*. 6*p*. Now proceeding as in *Ex*. 1, we find that sugar at 6*a*. per seer must be mixed with sugar at 4*a*. per seer in the ratio of (4*a*. 6*p*. − 4*a*.) to (6*a*. − 4*a*. 6*p*.) *i.e.*, of 1 to 3.

Example 4. There are two vessels of equal capacity, one full of milk, and the second one-third full of water. The second vessel is then filled up out of the first ; the contents of the second are then poured back into the first till it is full and then again the contents of the first are poured back into the second till it is full. What is the proportion of milk in the second vessel ?

Let M be the vessel containing milk and W the vessel containing water.

	First vessel.	Second vessel.
1st operation	1 M.	$\frac{1}{3}$ W.
2nd „	$\frac{1}{3}$ M.	$\frac{1}{3}$ W + $\frac{2}{3}$ M.
3rd „	$\frac{1}{3}$ M + $\frac{2}{3}$($\frac{1}{3}$ W + $\frac{2}{3}$ M).	$\frac{1}{3}$ ($\frac{1}{3}$ W + $\frac{2}{3}$ M).
4th „	$\frac{1}{3}$ {$\frac{1}{3}$ M + $\frac{2}{3}$($\frac{1}{3}$ W + $\frac{2}{3}$ M)}.	[$\frac{1}{3}$($\frac{1}{3}$ W + $\frac{2}{3}$ M) + $\frac{2}{3}${$\frac{1}{3}$ M + $\frac{2}{3}$($\frac{1}{3}$ W + $\frac{2}{3}$ M)}].

Simplifying the quantity on the right hand side, we get the proportions of water and milk in the second vessel.

$$[\tfrac{1}{9}\text{W}+\tfrac{2}{9}\text{M}+\tfrac{2}{3}\{\tfrac{1}{3}\text{M}+\tfrac{2}{9}\text{W}+\tfrac{4}{9}\text{M}\}]=\tfrac{1}{9}\text{W}+\tfrac{2}{9}\text{M}+\tfrac{2}{9}\text{M}+\tfrac{4}{27}\text{W}+\tfrac{8}{27}\text{M}$$

∴ proportion of milk $=\frac{2}{9}\text{M}+\frac{2}{9}\text{M}+\frac{8}{27}\text{M}=\frac{20}{27}\text{M}$.

Ans. $\frac{20}{27}$ of the second vessel is milk.

EXAMPLES. 158.

1. How must sugar at 4*a*. per seer be mixed with sugar at 5*a*. per seer to make a mixture worth 4*a*. 3*p*. per seer?

2. In what ratio must tea worth 2*s*. 7*d*. per lb. be mixed with tea worth 3*s*. 8*d*. per lb. to make a mixture worth 3*s*. per lb.?

3. Tea at 2*s*. 6*d*. per lb. is mixed with tea at 4*s*. 2*d*. per lb., and the mixture is sold for 3*s*. 5*d*. a lb.; how were they mixed?

4. In what ratio must a grocer mix coffee at 3*s*. per lb. with chicory at 7*d*. so that by selling the mixture at 2*s*. per lb. he may gain $\frac{1}{12}$ of his outlay?

5. A grocer buys black tea at 2*s*. 6*d*. per lb. and green tea at 3*s*. 9*d*. per lb.; how must he mix them so that by selling the mixture at 3*s*. per lb. he may gain $\frac{1}{6}$ of his outlay?

6. In what proportion should water and wine at 12*s*. 6*d*. a gallon be mixed to reduce the price to 10*s*. a gallon?

7. Currants at 5*d*. per lb. are mixed with currants at 9*d*. per lb. to make a mixture of 17 lb. worth 7*d*. per lb.; how many pounds of each are taken?

8. A person bought 60 md. of rice of two different sorts for R153. 12*a*. The better sort cost R3 per md. and the worse R2. 4*a*. per md. How many maunds were there of each sort?

9. A liquid *P* is $1\frac{3}{7}$ times as heavy as water, and water is $1\frac{2}{5}$ times as heavy as another liquid *Q*; how much of the liquid *P* must be added to 7 gallons of the liquid *Q* so that the mixture may weigh as much as an equal volume of water?

10. A mass of gold and silver weighing 9 lb. is worth £318. 13*s*. 6*d*.; if the proportions of gold and silver in it were interchanged, it would be worth £129. 10*s*. 6*d*.; supposing that the price of gold is £3. 17*s*. $10\frac{1}{2}$*d*. per oz., find the proportion of gold and silver in the mass, and the price of silver per oz.

11. A merchant has wines worth 7*s*., 9*s*., 11*s*. and 15*s*. a gallon respectively: how must he mix them to obtain a mixture worth 10*s*. a gallon, using equal parts of the first two kinds, and also equal parts of the last two kinds?

12. In what proportion must a grocer mix teas at 2*s*. 6*d*., 3*s*. and 4*s*. 6*d*. per lb. to make a mixture worth 4*s*. per lb., using equal parts of the first two kinds?

13. A man has whisky worth 22*s*. a gallon, and another lot worth 18*s*. a gallon; equal quantities of these are mixed with water to obtain a mixture of 50 gallons worth 16*s*. a gallon; find how much water the mixture contains.

14. A grocer buys teas at 2*s*. 6*d*., 3*s*. and 3*s*. 9*d*. per lb. respectively: how must he mix them so as to obtain a mixture worth 3*s*. 3*d*. per lb., using the first two kinds in the proportion of 2 to 3?

15. A grocer wishes to mix teas at 2*s.*, 3*s.*, 3*s.* 6*d.* and 4*s.* per lb. respectively ; how must he mix them (using the first two kinds in the proportion of 2 : 3, and the last two in the proportion of 3 : 4) so that by selling the mixture at 3*s.* 4*d.* per lb., $\frac{1}{25}$ of the receipts may be clear profit ?

16. A person fills a glass with medicine and drinks $\frac{1}{4}$ of it. He then fills up the glass with water and drinks $\frac{1}{3}$ of it. Again he fills up with water and drinks $\frac{1}{2}$. How much of the medicine does he drink altogether, and how much each time ?

17. A vessel is filled with spirits ; $\frac{1}{5}$ of the contents is drawn off into another equal vessel, which is then filled with water. The first vessel is then filled up again out of the second one, and then the second one is filled up again out of the first. Find what proportion of spirits the second vessel finally contains.

XLIV. AVERAGE VALUE.

269. Suppose a man spends R4 on a certain day and R6 on the next day and R5 on the day after. In three days he spends altogether R4+R6+R5 or R15. If he had spent $\left(\frac{R4+R6+R5}{3}=\frac{R15}{3}\right)$ or R5 each day, his total expenditure would have been the same. R5 is said to be the **average** or **mean** daily expenditure of the man for the three days. The **average** or **mean value** of any number of quantities of the same kind is their sum divided by the number of them.

Example 1. Find the average age of four boys who are 10, 11, 13 and 14 years old respectively.

Average age $=\frac{10+11+13+14}{4}$ years $=12$ years.

And, *generally*, the average of n quantities of the same kind, represented by $a, b, c, d, e, \ldots = \frac{a+b+c+d+e+\ldots}{n}$.

Conversely, the sum of n quantities of the same kind $= n \times$ their average.

Example 2. The average age of 5 children is 7 years, which is increased by 6 when the age of the father is included ; find the age of the father.

The average age of 5 children = 7 yr. ;

∴ the sum of the ages of 5 children = 7 × 5 or 35 yr.

Again, the average age of 5 children and father = 7 + 6 or 13 yr.

∴ the sum of the ages of the 5 children and their father = 13 × 6 or 78 yr.

Hence the age of the father = 78 − 35 or 43 yr.

Example 3. A man walks from P to Q at the rate of 5 miles an hour and returns from Q to P at the rate of 3 miles an hour. What is the average rate for the whole distance ?

Let x miles be the distance from P to Q.

Then in going from P to Q the man takes $\frac{x}{5}$ hr.,

and in returning from Q to P ... ... $\frac{x}{3}$ hr. ;

$\therefore$ in going and returning ... ... $\left(\frac{x}{5}+\frac{x}{3}\right)$ hr.

$\therefore$ The man's average rate of walking $=2x \div \left(\frac{x}{5}+\frac{x}{3}\right)$ mi.

$= 3\frac{3}{4}$ miles. *Ans.*

EXAMPLES. 159.

Find the average of the numbers,

1. 1, 2, 3, 4, 5.

2. 8, 10, 13, 15, 17, 20.

3. $3\frac{1}{2}$, $7\frac{3}{4}$, $8\frac{1}{4}$, $9\frac{1}{2}$, 10.

4. 1·3, 7·6, 8·9, 3·1, ·8.

5. Find the average age of five boys who are 15, 13, 11, 9 and 8 years old respectively ?

6. What was the average daily expenditure of a man in 1880, who spent ₨765 . 10 . 9 in the first half-year and ₨881 . 5 . 3 in the last ?

7. The population of a town was 28750 in 1870 and 30000 in 1880 ; find the average annual increase between the two dates.

8. Of 20 men 12 gain £3. 7*s.* each and 8 men gain £2. 8*s.* each ; what is the average gain per man ?

9. Five men weighed respectively 8 st. 8 lb., 9 st. 4 lb., 10 st., 10 st. 10 lb. and 11 st. 6 lb. ; what is the average weight per man ?

10. If 20 chairs are bought at ₨5 each, and 15 at ₨4. 8*a.* each, and 15 more at ₨4 each, what is the average price of a chair ?

11. A train travels 1 mile in the first 10 min., $1\frac{1}{2}$ miles in the next 10 min., 2 miles in the next, $1\frac{1}{2}$ miles in the next, and 1 mile in the next : what is the average speed of the train per hour ?

***12.** The average weight of 6 men is 10 st. ; two of them weigh 9 st. 7 lb. each ; find the average weight of the others.

13. The average age of 8 men, 7 women and 1 boy is 45 years, that of the 8 men being 48 years and of the 7 women being 46 ; determine the age of the boy.

14. The average weight of 7 men is diminished by 3 lb. when one of them who weighs 10 stones is replaced by a fresh man ; find the weight of the new man.

15. The average age of a class of 20 boys is 12 years ; what will be the average age if 5 new boys receive admission in the class, whose average age is 7 years ?

16. If the chairs in Question 10, are sold so as to gain $\frac{1}{4}$ of the cost price, what is the average selling price of a chair.

17. The average price of a chair, a table, and a cot is R19 ; the average price of the table, the cot and a book-shelf is R22 : if the price of the book-shelf be R16, find the price of the chair.

18. The average temperature of Monday, Tuesday, Wednesday and Thursday is 60° ; the average for Tuesday, Wednesday, Thursday and Friday is 63° ; if the ratio of the temperatures for Monday and Friday be 21 : 25, find these temperatures.

19. Find the average of all proper fractions having integral numerators and 20 for the denominator.

20. A train travels from Calcutta to Burdwan at the rate of 50 miles per hour and returns from Burdwan to Calcutta at the rate of 40 miles per hour. Find the average rate for the whole distance.

XLV. PERCENTAGE.

270. The term **per centum** or **per cent.** means *for a hundred.*

We have already seen that when two or more fractions are compared they are reduced to a common denominator. In some cases it is found convenient to express fractions with 100 as a common denominator. For example, suppose that in an examination a boy has got 18 marks out of 25 in English, 13 out of 20 in Mathematics and 7 out of 10 in History. The measures of his success in the three papers may be expressed by the fractions $\frac{18}{25}$, $\frac{13}{20}$ and $\frac{7}{10}$. Expressing the fractions with 100 as a common denominator, the measures of his success may be expressed by the fractions $\frac{72}{100}$, $\frac{65}{100}$ and $\frac{70}{100}$. That is, if the maximum in each paper had been 100 marks he would have got 72 marks in English, 65 in Mathematics and 70 in History. This is usually expressed by saying that the *percentage* of marks obtained by the boy in the three papers is 72, 65 and 70 respectively. Therefore a fraction expressed with

100 as its denominator is called a **percentage**, and the numerator which expresses the number of hundredths is called the **rate per cent.**

We thus see that percentages give us a third method of measuring parts of a whole, vulgar and decimal fractions giving us the other two methods. A percentage may be regarded as a fraction with a new notation. We may write the same quantity as R$\frac{37}{100}$ or R·37 or 37% of R1.

Suppose that a trader who has a capital of R4000 gains R200; he gains R5 for every hundred of his capital. This is expressed by saying that *the trader's gain is* 5 *per cent.*

Note. The symbol % or the letters *p. c.* are used as an abbreviation for the words *per cent.*

Example 1. What fraction of a number does 5 p. c. of it denote?

5 p. c. of a number $=\frac{5}{100}$ of the number $=\frac{1}{20}$ of the number.

Example 2. How much is $6\frac{1}{4}$ p. c. of R320?

The percentage $=\frac{6\frac{1}{4}}{100}$ of R320 $=\frac{1}{16}$ of R320 = R20.

EXAMPLES. 160.

(*Oral.*)

What fractions are denoted by the following rates per cent.?

1. 25. **2.** 40. **3.** 50. **4.** 75. **5.** 5.

6. 4. **7.** 5. **8.** 1. **9.** 100. **10.** 150.

What percentage is

11. 3 of 5? **12.** 13 of 20? **13.** 25 of 75?

14. 5 of 1? **15.** ·01 of 1?

16. What is cent per cent. of 100?

17. How much is 4 p. c. of 200?

18. How much is 20 p. c. of 80?

19. How much is $1\frac{1}{2}$ p. c. of 200?

EXAMPLES. 161.

What fractions are denoted by the following rates per cent.?

1. $12\frac{1}{2}$. **2.** $33\frac{1}{3}$. **3.** $\frac{1}{4}$. **4.** $\frac{3}{8}$. **5.** 125.

Find the value of

6. 5 p. c. of R700. **7.** $7\frac{1}{2}$ p. c. of £140. **8.** $\frac{3}{4}$ p. c. of £20.

9. 35% of 3480 men. **10.** $\frac{1}{8}$% of a sq. ft. **11.** 8·5% of 50 cwt.

12. A man's income is ₨3000 a year ; if he spends $6\frac{1}{4}$ p. c. of it each month, how much does he save in a year ?

13. Five per cent. of the total population of a town are Englishmen ; the rest are Hindus : if the population of the town be 37820, what is the number of Hindus ?

14. A man's income in 1928 was £500 ; in 1929 it was increased by 20 p. c. ; what was his income in 1929 ?

15. Find the difference between $\frac{3}{4}$ of ₨70 and $\frac{3}{4}$ p. c. of ₨70.

16. A testator bequeathed by will $\frac{2}{5}$ of his estate to his son, 60 p. c. of the remainder to his daughter, and the remainder to his widow ; the son got £75 more than the daughter. How much did the widow receive ?

Example 3. What *rate per cent.* does the fraction $\frac{3}{8}$ denote ?

The fraction, $\frac{3}{8}=\frac{3\times 100}{8\times 100}=\frac{\frac{300}{8}}{100}=\frac{37\frac{1}{2}}{100}$;

$\therefore$ rate per cent. $=37\frac{1}{2}$.

Example 4. What per cent. of ₨40 is ₨3 ?

The fraction $=\frac{3}{40}=\frac{300}{40\times 100}=\frac{\frac{300}{40}}{100}=\frac{7\frac{1}{2}}{100}$;

$\therefore$ rate per cent. $=7\frac{1}{2}$.

Note. The following equivalents are useful and should be committed to memory :

$2\frac{1}{2}\%=\frac{1}{40}$; $5\%=\frac{1}{20}$; $10\%=\frac{1}{10}$; $12\frac{1}{2}\%=\frac{1}{8}$; $20\%=\frac{1}{5}$.
$25\%=\frac{1}{4}$; $33\frac{1}{3}\%=\frac{1}{3}$; $50\%=\frac{1}{2}$; $66\frac{2}{3}\%=\frac{2}{3}$; $75\%=\frac{3}{4}$.

EXAMPLES. 162.

What rates per cent. do the following fractions denote ?

1. $\frac{1}{6}$. **2.** $\frac{1}{30}$. **3.** $\frac{3}{7}$. **4.** $\frac{7}{20}$.
5. $\frac{8}{9}$. **6.** $\frac{19}{50}$. **7.** $\frac{75}{10}$. **8.** $\frac{69}{80}$.

What per cent. of

9. ₨26 is ₨13 ? **10.** ₨40 is ₨8 ? **11.** £3 is 12*s*.
12. ·25 is $\frac{1}{7}$? **13.** $\frac{1}{3}$ is ·7 ? **14.** $\dot{6}$ is ·$\dot{3}$?

15. Of 3420 men in a town, 420 died ; what per cent. survived ?

16. Out of a debt of ₨2500, ₨1900 is paid ; what per cent. of the debt still remains unpaid ?

17. The number of boys in a school in January was 320 ; in February it increased to 360. Find the increased per cent.

18. A mass of gunpowder is made with 2 lb. $5\frac{1}{2}$ oz. of nitre, 5 oz. of sulphur and $7\frac{1}{2}$ oz. of charcoal ; find the percentage composition of the powder.

19. Standard gold contains 11 parts pure gold out of 12 ; what per cent. is dross ?

Example 5. Of what sum of money is R30, 5 p. c. ?

5 p. c. of the sum = R30,

or $\frac{5}{100}$ of the sum = R30 ;

$\therefore$ the sum = R30 $\times \frac{100}{5}$ = R600.

EXAMPLES. 163.

Of what number is

1. 22, 10 p. c. ? **2.** 57, $4\frac{3}{4}$ p. c. ? **3.** 30, 120 p. c. ?

4. 81, $\frac{3}{4}$ p. c. ? **5.** $2\frac{1}{2}$, $2\frac{1}{2}$ p. c. ? **6.** $3\frac{1}{2}$, ·27 p. c. ?

7. A man spends R3250 a year, which is $66\frac{2}{3}$ p. c. of his yearly income ; find his income ?

8. A man spends 60 p. c. of his income and saves R2000 ; what is his income ?

9. If a tax of 10 p. c. on the income of a man yields R300, how much will an income-tax of 5 pies in the R produce ?

271. Miscellaneous questions on percentage.

Example 1. A reduction of $12\frac{1}{2}$ per cent. in the price of mangoes enables a purchaser to obtain 4 more for a rupee. What is the reduced price ? How many could he get for 8 annas before the reduction in price.

Owing to the reduction in price the purchaser saves $12\frac{1}{2}$ p. c. of $\frac{1}{4}$ of R1, that is, $2a.$ With this sum he gets 4 mangoes at the reduced price. $\therefore$ The reduced price of a mango $=\frac{2}{4}a.=6p.$

Again, $(100-12\frac{1}{2})$ p. c. or $(1-\frac{1}{8})$ or $\frac{7}{8}$ of the original price of a mango $=6p.$ $\therefore$ The original price of a mango $=\frac{8}{7}\times\frac{1}{2}a.=\frac{4}{7}a.$

$\therefore$ He could get $(8\div\frac{4}{7})$ or 14 mangoes before the reduction in price.

Example 2. If the duty on imported sugar be increased by 25 p. c., by how much per cent. must a man reduce his consumption of that article so as not to increase his expenditure.

Present price of 1 seer = 125 p. c. of the former price of 1 seer

$=\frac{5}{4}$ of the former price of 1 seer

= former price of $\frac{5}{4}$ seer ;

$\therefore$ former price of 1 seer = present price of $\frac{4}{5}$ seer.

Therefore, in order that the expenditure may remain the same as before, for 1 seer consumed formerly, $\frac{4}{5}$ seer must be consumed now ; that is, the consumption must be reduced by $\frac{1}{5}$ or by 20 p. c.

Example 3. If the import duty on motor cars be reduced by 40 p. c. of its present amount, by how much per cent. must the import of cars increase in order that, (i) the revenue may be unaltered ; (ii) the revenue may be increased by 10 p. c.

(i) The new duty is 60 p. c. or $\frac{3}{5}$ of the former duty.

Therefore, in order that the revenue may remain unaltered, the number of cars imported must be $\frac{5}{3}$ of the present number. Therefore the import must increase by $(\frac{5}{3}-1)$ or $\frac{2}{3}$ or by $66\frac{2}{3}$ p. c.

(ii) The new revenue is to be $\frac{110}{100}$ or $\frac{11}{10}$ of the former revenue.

Therefore the number of cars imported must be $(\frac{11}{10}\div\frac{3}{5})$ or $\frac{11}{6}$ of the present number. Therefore the import must increase by $(\frac{11}{6}-1)$ or $\frac{5}{6}$ or by $83\frac{1}{3}$ p. c.

Note. The use of algebraical symbols will make the solution of the last two examples clearer.

Example 2. Let x be the number of seers consumed originally and let y annas be the price per seer. Then the original expense on sugar was $x \times y$ annas. The present price of sugar per seer $=\frac{125}{100}y$ annas.

Therefore to keep the expense unaltered the number of seers now to be consumed

$$=xy\div\frac{125}{100}y=\frac{100}{125}x=\frac{400}{500}x$$

$$=\frac{80}{100}x=\frac{80}{100}\text{ of the original no. of seers.}$$

$\therefore$ The consumption must be decreased by 20 p. c.

Example 3. Let x be the number of cars imported originally, and let £y be the duty on each car. The present revenue derived from the import is £xy.

The new duty on each car $=\frac{60}{100}$ of £y = £$\frac{3}{5}y$.

(i) The new revenue is the same as before, namely, £xy ;

$\therefore$ the number of cars to be imported

$$=\frac{xy}{\frac{3}{5}y}=\frac{5}{3}x=\frac{500}{300}x=\frac{166\frac{2}{3}}{100}x=\frac{166\frac{2}{3}}{100}\text{ of the present number.}$$

$\therefore$ The import must increase by $66\frac{2}{3}$ p. c.

(ii) The new revenue = £$\frac{110}{100}xy$;

$\therefore$ the number of cars to be imported

$$=\frac{110}{100}xy\div\frac{3}{5}y=\frac{550}{300}x=\frac{183\frac{1}{3}}{100}x=\frac{183\frac{1}{3}}{100}\text{ of the present number.}$$

$\therefore$ The import must increase by $83\frac{1}{3}$ p. c.

Example 4. A parliamentary grant is made at the rate of 5*s.* per head for all the children at elementary schools. If this grant be distributed at the rate of 5*s.* 9*d.* per child in town and 3*s.* 3*d.* per child in country schools, what percentage of the total number of children are in each class of school ?

Let x be the number of children in town schools out of 100 children in both town and country schools. Then $(100-x)$ is the number of children in country schools.

Then the cost for town schools $=(x\times 69)d.$,

and " " " country " $=(100-x)\times 39d.$

$\therefore$ The total cost $=\{69x+39(100-x)\}$ pence.

But the grant $=(100\times 5\times 12)d.=6000$ pence.

$\therefore$ By the question, $69x+39(100-x)=6000$,

$\therefore\ 30x=2100$,

$\therefore\ x=70$.

Thus the required percentages are 70 and 30. *Ans.*

[The arithmetical solution of the above example is left to the student as an exercise.]

Example 5. If the annual increase in the population of a town is 5 per cent., and the present number of inhabitants is 8000, what will it be in three years time ? What was it three years ago ?

For each 100 of population at the beginning of any year there are 105 at the end of the year.

$\therefore\ \dfrac{\text{Population at the end of any year}}{\text{Population at the beginning of that year}}=\dfrac{105}{100}$;

$\therefore$ population at the end of any year = pppulation at the beginning of any year $\times\frac{105}{100}$,

$\therefore$ population at the end of the first year $=\frac{105}{100}\times 8000$.

Similarly, population at the end of the second year

$=\frac{105}{100}\times$ population at the beginning of the second year

$=\frac{105}{100}\times\frac{105}{100}\times 8000=(\frac{105}{100})^2\times 8000$.

Finally, population at the end of the third year

$=\frac{105}{100}\times$ population at the beginning of the third year

$=\frac{105}{100}\times(\frac{105}{100})^2\times 8000=(\frac{105}{100})^3\times 8000=9261$.

Again, the present population $=(\frac{105}{100})^3\times$ population three years ago ;

$\therefore\ 8000=(\frac{105}{100})^3\times$ population three years ago.

Hence the population three years ago

$=8000\div(\frac{105}{100})^3=$ etc.

Note. Generally, if P be the population of a town at the commencement of any year and the annual increase be r per cent., then the population at the end of n years $=P\left(1+\frac{r}{100}\right)^n$.

And if the annual decrease be r per cent., then the population at the end of n years $=P\left(1-\frac{r}{100}\right)^n$.

Example 6. In a boarding house the number of boarders increases at a certain rate per cent. per annum. Four years ago the number of boarders was 81; now it is 144. What will the number of boarders be two years hence?

Suppose the number of boarders increases at r p. c. per annum.

$$\text{Then} \quad 81\left(1+\frac{r}{100}\right)^4=144,$$

$$\therefore \quad \left(1+\frac{r}{100}\right)^4=\frac{144}{81},$$

$$\therefore \quad \left(1+\frac{r}{100}\right)^2=\sqrt{\frac{144}{81}}=\frac{12}{9}.$$

$$\text{The no. of boarders 2 years hence}=144\left(1+\frac{r}{100}\right)^2$$

$$=144\times\tfrac{12}{9}$$

$$=192. \quad \textit{Ans.}$$

MISCELLANEOUS EXAMPLES. 164.

1. The price of a bottle of red ink is 20 p. c. more than that of a bottle of black ink. If a bottle of red ink cost 12 annas, how much will a bottle of black ink cost?

2. A trader in his first year gains 8 p. c. of his capital, but in the second year loses 10 p. c. of what he had at the end of the first year, and his capital is Rs224 less than at first; find his original capital.

3. A trader's capital increased 10 p. c. every year; at the end of 3 years it was Rs6050; what was his capital at first?

4. In a mixed school 25 per cent. of the scholars are infants under 7, and the number of girls above 7 is $\frac{4}{5}$ of the boys above 7, and amounts to 36; find the number of children in the school.

5. A man spends 5 p. c. of his income in insuring life, and this part is exempted from income-tax; his income-tax which is laid at 4 pies in the rupee, amounts to Rs30. 5*a*., find his gross income.

6. Three casks contain equal quantities of wine ; a mixture is formed by taking 25 p. c. of the first cask, 35 p. c. of the second and 45 p. c. of the third ; what per cent. of the whole quantity is taken ?

7. Two mixed schools have 90 and 120 children respectively ; in the first 60 p. c. and in the second 50 p. c. of the children are boys ; what per cent. of the children in the two schools are boys ?

8. In a town the numbers of male and female inhabitants are 3450 and 3020 respectively ; the decrease in the former is 10 p. c., while the increase in the latter is 5 p. c. Find the increase or decrease per cent. of the total population.

9. In a mixture of coffee and chicory the coffee is 40 per cent. ; to 500 lb. of the mixture a quantity of chicory is added, and then the coffee is $36\frac{4}{11}$ p. c. How many pounds of chicory are added ?

10. If *A*'s income be 10 per cent. more than *B*'s, how much per cent. is *B*'s income less than *A*'s ?

11. *A* sells his goods 10 per cent. cheaper than *B*, and 10 per cent. dearer than *C* ; how much per cent. are *C*'s rates lower than *B*'s ?

12. The price of sugar being raised 10 p. c., by how much per cent. must a man reduce his consumption of that article so as not to increase his expenditure.

13. A reduction of 20 p. c. in the price of tea would enable a purchaser to obtain 3 lb. more for a sovereign ; what is the reduced price ?

14. A reduction of $33\frac{1}{3}$ p. c. in the price of oranges would enable a purchaser to obtain 8 more for a rupee ; what was the price before the reduction ?

15. Ten per cent. of the inhabitants of a town having died of plague, a panic set in, during which 25 p. c. of the remaining inhabitants left the town. If the population was then reduced to 3375, what was it originally ?

16. In an election to the Legislative Assembly 4 p. c. of the constituency refused to vote and of the two candidates the one who was supported by 52 p. c. of the whole constituency was returned by a majority of 500. How many actually voted ?

17. The Calcutta Corporation makes a grant at the rate of ₨3. 12*a*. per head for all the children reading in primary schools. If this grant is distributed at the rate of ₨4. 5*a*. per child in schools for boys and ₨2. 7*a*. per child in schools for girls, what percentage of the total number of children are in each class of school ?

18. If the annual increase in the population of a town is 4 per cent., and the present number of inhabitants be 5 lacs; what will it be in three years time?

19. The population of a town increased 7 p. c. from 1920 to 1923, and its population in the latter year was 13910; what was its population in 1920?

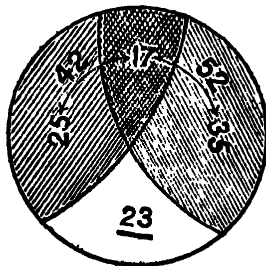
20. Show that if rats in a certain village increase in number by 50 per cent. per annum, they will, after an interval of 8 years, be more than 25 times as numerous as they were.

21. In a boarding house the number of boarders increases at a certain rate per cent. per annum. Four years ago the number of boarders was 49; now it is 196. What will it be two years hence?

22. If the tax on tea be reduced by 10 p. c., by how much per cent. must the consumption increase so that there may be an increase of 8 p. c. in the revenue from this source?

23. In an examination 52 per cent. of the candidates fail in English and 42 per cent. fail in Mathematics. If 17 per cent. fail both in English and Mathematics, find the percentage of those who pass in both subjects.

[17 p. c. fail in both English and Mathematics. ∴ (52 − 17) or 35 p. c. fail in Egnlish only; and (42 − 17) or 25 p. c. fail in Mathematics only. Hence out of 100 candidates, (17 + 35 + 25) or 77 candidates fail in one or both subjects.]



XLVI. COMMISSION, BROKERAGE, PREMIUM.

272. An **agent** is a man who acts for another.

Commission is the sum of money paid to an agent for buying or selling goods or property of any kind. It is usually *a percentage* upon the value of goods bought or sold.

The agent is sometimes called a **broker**, especially when he buys or sells Government Promissory Notes, Shares of Companies, etc., and the commission, **brokerage**.

Premium is the sum of money paid to an *Insurance Company* which, in consideration thereof, undertakes to make good a loss incurred through fire or shipwreck, or to pay a certain sum of money after a man's death to his relatives. The instrument containing the contract is called the **Policy of Insurance**; and the stamp duty on the policy is called the **Policy duty**. Premium is usually *a percentage* upon the sum of money which the insurer or his relatives are to receive.

Commission, Brokerage and Premium are therefore names given to a percentage in particular cases.

Example 1. An agent buys goods worth R750, and receives a commission of $2\frac{1}{2}$ per cent.; how much does he get?

Commission $= \frac{2\frac{1}{2}}{100}$ of R750 $= \frac{75}{4} =$ R18. 12*a*.

Example 2. A cargo, valued at £760, is to be insured at 5 p. c. premium; what sum must be insured that, in case of loss, the value of cargo and the premium paid may be recovered?

If every £95(£100 − £5) be insured for £100, then in case of loss both the value of goods and premium paid will be recovered.

Now since £95 must be insured for £100,

£1 .... £$\frac{100}{95}$,

∴ £760........................... £$\frac{760 \times 100}{95}$,

or £800. *Ans.*

EXAMPLES. 165.

1. A broker purchases goods worth R5000; what is his commission at $3\frac{1}{2}$ per cent.?

2. What is the cost of insuring cargo valued at £7000, the premium being $3\frac{1}{2}$ per cent.?

3. A commission agent sells 720 bales of jute at R7 per bale; what commission does he receive at $1\frac{1}{2}$ per cent.?

4. An agent buys a house for R6750, and receives commission at R3. 12*a*. per cent.; what has his employer to pay altogether?

5. A broker received $\frac{1}{8}$ p. c. for buying Government Promissory Notes. His brokerage amounted to R35; what was the value of the Promissory Notes bought?

6. A ship is insured for $\frac{5}{4}$ of its value at $1\frac{3}{4}$ p. c., and the premium is £20; what is the ship worth?

7. The premium on a policy of insurance at 4 p. c. is R120; find the amount of the policy.

8. How much must be paid to insure a cargo worth £5720, the premium being 25*s*., policy duty 1*s*. 6*d*., and brokerage 9*s*. per £100 respectively.

9. For what sum must a merchant insure a cargo worth ₨9760 at $2\frac{3}{8}$ p. c. so that in case of loss both the cargo and premium may be recovered?

10. Goods worth £7740 are insured at $3\frac{1}{4}$ per cent., so that in case of loss both the value of goods and premium may be recovered; find the amount of premium paid.

11. Cargo worth £5000 is to be insured, so that in case of loss its value and all the expenses connected with its insurance may be recovered. The premium is $2\frac{1}{10}$ per cent., policy duty $\frac{13}{20}$ per cent. and brokerage $\frac{1}{4}$ per cent.; for what sum must the cargo be insured and what is the amount of the whole expense paid on insurance?

XLVII. PROFIT AND LOSS.

273. The terms "Profit" and "Loss" are used in business transactions. If a merchant buys goods at one price and sells them at a different price, and if his *selling price* is greater than his *buying price or cost price*, the difference between the two prices is called his **Profit** or **Gain**. But if his *cost price* is greater than his *selling price*, the difference between them is called his **Loss**.

Suppose a merchant buys an article for ₨15 and sells it for ₨20; his profit is ₨5. Again, suppose the same merchant buys an article for ₨20 and sells it for ₨25; his profit is ₨5. In each case his *actual* profit is ₨5, but the *relative* profit in the first case is greater than in the second case; for, in the first case the profit is $\frac{1}{3}$ of the cost price while in the second case it is only $\frac{1}{4}$ of the cost price, *i.e.*, in the first case he gains $33\frac{1}{3}$% while in the second case he gains only 25%. Hence in order to compare profits, it is not enough merely to compare actual profits, but we should also take into account what it has cost a merchant to get these profits. Under this head, therefore, we estimate a profit or loss, not absolutely, but in relation to the cost price, that is, as so much per cent. on the *cost price*.

274. To find the gain or loss per cent.

Example 1. If chairs are bought at ₨5 each, and sold at ₨5. 9*a*. each, what is the gain per cent.?

The gain is 9*a*. on ₨5 or 80*a*.; and we have to find what per cent. of 80*a*. is 9*a*.

Now, the fraction $\frac{9}{80}=\frac{900}{80\times100}=\frac{\frac{900}{80}}{100}=\frac{11\frac{1}{4}}{100}$;

$\therefore$ the gain is $11\frac{1}{4}$ per cent.

Example 2. If chairs are bought at R5 each, and sold at R4. 8*a.* each, what is the loss per cent. ?

The loss is 8*a.* on R5 or 80*a.* ; and we have to find what per cent. of 80*a.* is 8*a.*

Now, the fraction $= \frac{8}{80} = \frac{1}{10} = \frac{10}{100}$;

∴ the loss is 10 per cent.

Example 3. I sell 15 articles for the same money as I paid for 20 ; what do I gain per cent. on my outlay ?

Suppose the cost price of 20 articles is R100. Then since 15 articles are sold for R100, 20 articles are sold for R100 × $\frac{20}{15}$ = R$\frac{400}{3}$ = R133$\frac{1}{3}$. ∴ The gain on R100 is R33$\frac{1}{3}$, that is, the gain is 33$\frac{1}{3}$ p. c.

EXAMPLES. 166.

(*Examples* 1—10 *should be taken orally.*)

Find the actual gain, and also the gain per cent., if

1.	an article costing	R100	is sold for	R120.
2.	,, ,, ,,	R500	,, ,, ,,	R550.
3.	,, ,, ,,	R1000	,, ,, ,,	R2000.
4.	,, ,, ,,	R2. 8*a.*	,, ,, ,,	R3.
5.	,, ,, ,,	5*s.* 6*d.*	,, ,, ,,	6*s.*

Find the actual loss, and also the loss per cent., if

6.	an article costing	R100	is sold for	R80.
7.	,, ,, ,,	R500	,, ,, ,,	R450.
8.	,, ,, ,,	R2000	,, ,, ,,	R1000.
9.	,, ,, ,,	R2. 8*a.*	,, ,, ,,	R2.
10.	,, ,, ,,	5*s.* 6*d.*	,, ,, ,,	5*s.*

11. I sell for R20 that for which I gave R16 ; what is my gain per cent. ?

12. At what rate per cent. is the loss on selling for £11 . 9 . 8$\frac{1}{4}$ what cost £15 . 6 . 3 ?

13. I sell 20 articles for the same money as I paid for 25 ; what do I gain per cent. on my outlay ?

14. If the selling price of $\frac{3}{4}$ of a number of toys be equal to the cost price of the whole, find the profit per cent.

15. 70 gallons of wine are bought for £50, and 9 gallons are lost by leakage ; the remainder is sold at 1*s.* 10$\frac{1}{2}$*d.* a pint ; find the gain or loss per cent. on the outlay.

16. Centain articles are bought at £12. 15*s*. for 100, and are sold at $2\frac{1}{2}$ guineas for a dozen ; find the gain or loss per cent.

17. A person by selling 48 yards of cloth gained the cost of 16 yards ; find the gain per cent.

275. To find the selling price, *when the cost price and the gain or loss per cent. are given.*

Example 1. A horse is bought for R80, and is sold at a profit of 25 p. c. ; what does the profit amount to, and for how much is the horse sold ?

First Method : Profit=25 p. c. of R80=$\frac{25}{100}$ of R80=R20.

∴ The horse is sold for R80+R20, or R100.

Second Method :

Selling price of horse=$\frac{125}{100}$ of R80=$\frac{5}{4}$ of R80=R100.

Example 2. A horse is bought at R80, and is sold at a loss of 20 p. c. ; what does the loss amount to, and for how much is the horse sold.

First Method : Loss=20 p. c. of R80=$\frac{20}{100}$ of R80=R16.

∴ The horse is sold for R80−R16, or R64.

Second Method :

Selling price of horse=$\frac{80}{100}$ of R80=$\frac{4}{5}$ of R80=R64.

Example 3. Some goods are bought for R90 ; for how much must they be sold so as to gain 10 per cent. ?

First Method :

When the cost price is R100, the selling price is R110,

∴ ....R1, ...R$\frac{110}{100}$,

∴ ....R90, ...R$\frac{110\times 90}{100}$

=R99.

Second Method :

The selling price=110 p. c. of cost price

=$\frac{110}{100}$ of R90=R99.

EXAMPLES. 167.

(*Examples* 1—5 *should be taken orally.*)

Find the selling price when

1. cost price is £200, and profit is 5 p. c.

2. „ „ „ £400, „ „ „ $2\frac{1}{2}$ p. c.

3. „ „ „ R12. 8*a*. „ „ „ 10 p. c.

4. „ „ „ R500, „ loss „ $7\frac{1}{2}$ p. c.

5. „ „ „ £8. 12*s*. „ „ „ 25 p. c.

6. 320 maunds of rice were bought at R5 per maund, and sold at a loss of 5 p. c.; find the total loss and the selling price per seer.

7. A merchant buys certain goods at £6 . 19 . 3 per cwt. and pays 15*s*. per ton for expenses; at what price per lb. must he sell them so as to gain 15 p. c. on his total outlay?

8. If oranges are bought at the rate of 15 for a rupee, how many must be sold for a rupee so as to gain 25 p. c.?

9. The cost price of a book is 7*s*. 6*d*.; if the expenses of sale be 5 p. c. upon this, and the profit 20 p. c., what would be the retail price?

10. 24 gallons of ale are bought at 2*s*. a gallon and 30 gallons of porter at 1*s*. a gallon, and they are mixed together. If 13 gallons of the mixture be lost by leakage, and 20 gallons sold at 2*s*. 3*d*. a gallon, at what price per gallon must the remainder be sold to gain 20 p. c. on the whole cutlay?

276. To find the cost price, ***when the selling price and the gain or loss per cent. are given.***

Example 1. By selling sugar at R12 per md. I gain 20 p. c.; what price per md. did I buy it?

First Method:

When the selling price is R120, the cost price is R100,

∴ ....R1, ...R$\frac{100}{120}$,

∴ ....R12, ...R$\frac{100\times12}{120}$

=R10.

Second Method:

120 p. c. of the cost price=selling price,

or $\frac{120}{100}$ of the cost price=R12;

∴ the cost price=R12×$\frac{100}{120}$=R10.

Example 2. By selling sugar at R12 per md. I lose 25 p. c.; at what price per md. did I buy it?

(100−25) p. c. or 75 p. c. of the cost price=selling price,

or $\frac{75}{100}$ of the cost price=R12;

∴ the cost price=R12×$\frac{100}{75}$=R16.

EXAMPLES. 168.

1. An article is sold at R4. 4*a*. at a profit of $33\frac{1}{3}$ p. c.; find the cost price.

2. A horse is sold for R440, at a loss of 12 p. c.; how much did it cost?

3. A quantity of sugar is sold at 6*a*. 9*p*. per seer ; the gain is $12\frac{1}{2}$ p. c. and the total gain is R15. What is the quantity of sugar sold ?

4. If oranges are sold at the rate of 11 for the rupee, and the gain is $8\frac{1}{3}$ p. c., at what rate were they purchased ?

5. A dealer sells two gramophones at R100 each, gaining 25 p. c. on one gramophone and losing 25 p. c. on the other ; what is his gain or loss on the whole transaction ?

277. To find the selling price, *corresponding to one of two given rates of gain or loss per cent.*

Example. If 10 p. c. be lost by selling an article for R72, for how much should it have been sold so as to gain 5 per cent. ?

90 p. c. of the cost price = R72,

∴ 15 .... = R12,

∴ 105 .... = R84. *Ans.*

Here we have passed from 90 *to* 105 *by way of* 15, *the G. C. M. of the two numbers.*

EXAMPLES. 169.

1. A horse was sold for R240 at a loss of $5\frac{1}{2}$ p. c. ; for what should it have been sold to gain 26 p. c. ?

2. By selling tea at 3*s.* per lb. a grocer gains only 5 p. c. ; by how much must he raise the price so as to gain 15 p. c. ?

3. If by selling 7 mangoes for R1 . 2 . $4\frac{1}{2}$, there be a profit of $16\frac{2}{3}$ per cent., at what price per dozen must they be sold to gain 20 per ent. ?

4. If a man lose 4 p. c. by selling oranges at the rate of 12 a rupee, how many a rupee mus he sell them so as to gain 44 p. c. ?

5. A bankrupt's stock was sold for R5205 at a loss of 17 per cent. on the cost price ; had the stock been sold in the ordinary course of trade it would have realized a profit of 20 per cent. How much was it sold under the trade price ?

278. To find the gain or loss per cent., *corresponding given selling prices.*

Example. By selling a house for £69 there is a loss of 8 p. c. ; what would be the loss or gain per cent for £78 ?

£69 = 92 p. c. of the cost price,

∴ $£1 = \frac{92}{69}$ ....,

∴ $£78 = \frac{92 \times 78}{69}$ ....,

= 104 ....

∴ There would be a gain of 4 per cent.

EXAMPLES. 170.

1. If by selling goods for R141 there be a loss of 6 p. c., what will be the loss or gain per cent. by selling them for R159 ?

2. Goods were sold for R37. 8*a.* with a gain of $12\frac{1}{2}$ p. c. ; what would have been gained or lost by selling them for R33. 8*a.* ?

3. If 8 p. c. is lost by selling an article for R38. 4*a*, find the gain per cent. on selling it for R57.

4. By selling a bicycle for R85 a man lost $7\frac{1}{2}$ p. c. Find what would have been his gain or loss per cent. if he had sold it for R96. 8*a.*

5. By selling a watch for R240 a dealer gains 25 p. c. How much would he have gained per cent. by it for R204 ?

6. If a profit of 5 p. c. is made by selling tea at £12. 5*s.* per cwt., what per cent. profit would be made by selling it at 2*s.* 4*d.* per lb. ?

279. Miscellaneous questions on Profit and Loss.

Example 1. Goods pass successively through the hands of *A*, *B* and *C*, each of whom in selling adds as profit 10 per cent. of the price he paid ; if *C* sells the goods for R73. 3*a.*, how much did *A* pay ?

What cost *A* R100, he sells for R110.

$\therefore$ *A*'s selling price, *i.e.*, *B*'s buying price $=\frac{110}{100}\times$ *A*'s cost price ;

$\therefore$ *B*'s selling price, *i.e.*, *C*'s buying price

$=\frac{110}{100}\times$ *B*'s buying price $=\frac{110}{100}\times\frac{110}{100}\times$ *A*'s cost price ;

$\therefore$ R73. 3*a.* or 1331*a.* = *C*'s selling price $=\frac{110}{100}\times$ *C*'s buying price

$=\frac{110}{100}\times\frac{110}{100}\times\frac{110}{100}\times$ *A*'s cost price $=\frac{11\times11\times11}{10\times100}\times$ *A*'s cost price.

$\therefore$ *A*'s cost price $=\frac{10\times100\times1331}{11\times11\times11}a. = 1000a.$ = R62. 8*a.*

Example 2. A man buys a certain number of oranges at 20 for 6*a.* and an equal number at 30 for 6*a.* He mixes them and sells them at 25 for 6*a.* What is his gain or loss per cent. ?

Let us suppose that he bought 60 oranges of each kind. (We could have taken any number but 60 is the most convenient as it is the L. C. M. of 20 and 30.)

Then the cost of 60 oranges of the first kind = 18*a.*,

and .... second... = 12*a.* ;

$\therefore$ the total cost of 120 oranges = 30*a.*

He sells these 120 oranges for $\frac{120}{25}\times6a. = \frac{24\times6}{5}a. = \frac{144}{5}a.$

$\therefore$ The man's loss $= 30a. - \frac{144}{5}a. = \frac{6}{5}a.$

This loss is on the total cost of 120 oranges, that is, on 30*a.* Hence we have the proportion :

$$30a. : 100a. :: \tfrac{6}{5}a. : \text{loss on } 100a.;$$

∴ loss on $100a. = \frac{6 \times 100}{5 \times 30}a. = 4a.$

∴ The man loses 4 p. c. on his outlay.

Example 3. How must a grocer mix teas at 2*s.* 6*d.* a lb. and 3*s.* 9*d.* a lb. so that by selling the mixture at 3*s.* 4½*d.* a lb. he may gain 12½ p. c.?

By selling the mixture at 3*s.* 4½*d.* per lb. the gain is 12½ p. c.

∴ The cost price of the mixture per lb.

$$=(3s.\ 4\tfrac{1}{2}d.) \times \frac{100}{112\frac{1}{2}} = \frac{40\frac{1}{2} \times 100}{112\frac{1}{2}}d. = \frac{81 \times 100}{225}d. = 36d. = 3s.$$

Now proceed as in *Example* 1, Art. 268.

Example 4. A tradesman's prices are 20 p. c. above cost price. If he reduces the marked prices by 10 p. c. for cash payment, what profit does he make?

Suppose the cost price = ₨100.

Then the marked price = ₨120.

Reduced price for cash payment = ₨120 × $\frac{90}{100}$ = ₨108.

∴ The profit = 108 − 100 or 8 p. c.

Example 5. A man sells an article at a loss of 20 p. c. If he had sold for ₨50 more he would have gained 5 p. c. Find the cost price of the article.

The difference between 105 p. c. of the cost price and 80 p. c. of the cost price = ₨50.

∴ 25 p. c. of the cost price = ₨50;

∴ $\frac{25}{100}$ or $\frac{1}{4}$..................... = ₨50.

∴ The cost price = ₨50 × 4 = ₨200.

Example 6. A man having bought a quantity of sugar for ₨80, sells ¼ of it at a loss of 5 p. c.; by what rate must he raise that selling price, in order that by selling the rest at the increased rate he may gain 5 p. c. on his outlay?

The man must altogether get 105 p. c. of ₨80, that is, $\frac{105}{100}$ of ₨80 or ₨84.

Now, ¼ of the quantity of sugar bought is sold for 95 p. c. of ₨20, that is, for $\frac{95}{100}$ of ₨20 or ₨19. Therefore the remaining ¾ of the quantity of sugar must be sold for ₨84 − ₨19 or ₨65. ∴ Each ¼ of the remaining quantity must be sold for ₨$\frac{65}{3}$ or ₨21⅔; ∴ the former selling price of ¼ of the quantity must be raised by ₨21⅔ − ₨19 or ₨2⅔.

Hence, because R$2\frac{5}{7}$ is $14\frac{2}{7}$ p. c. of R19, the former selling price must be raised by $14\frac{2}{7}$ p. c.

Example 7. A dishonest tradesman marks his goods at an advance of 5 p. c. on the cost price, but uses a fraudulent balance, whose beam is horizontal when the weight in one scale is one-fifteenth more than the weight in the other. What is his actual gain per cent. ?

$$\text{Marked price} = \text{nominal cost price} \times \frac{105}{100}.$$

But because of the fraudulent balance,

$$\text{nominal cost price} = \text{actual cost price} \times \frac{16}{15},$$

$$\therefore \quad \text{marked price} = \text{actual cost price} \times \frac{16}{15} \times \frac{105}{100}$$

$$= \text{actual cost price} \times \frac{112}{100}.$$

Hence the actual gain = 12 p. c.

MISCELLANEOUS EXAMPLES. 171.

1. Tea which cost R60 per md. is retailed at R2. 8*a.* per seer, and there is a waste of 10 p. c. ; what is the rate of profit per cent. ?

2. Sulphuric acid worth 3*d.* per lb. absorbs moisture and becomes $2\frac{1}{2}$ p. c. heavier ; what is it then worth per lb. ?

3. A merchant sells tea to a tradesman at a profit of 40 p. c., but the latter becoming bankrupt pays only 12*s.* in the £ ; how much per cent. does the merchant gain or lose on his outlay ?

4. A tradesman's prices are 30 p. c. above the cost price ; if he allows his customers 10 p. c. on his bill, what profit does he make ?

5. How much per cent. must a tradesman add on to the cost price of his goods, that he may make 20 p. c. profit after allowing his customers a reduction of 5 p. c. on his bill ?

6. The price of flour being raised 20 per cent., by how much per cent. must a man reduce his consumption of that article so as not to increase his expenditure ?

7. An article when sold at a gain of 5 p. c. yields R15 more than when sold at a loss of 5 p c. ; what was its prime cost ?

8. A man sells an article at a loss of 10 p. c. ; if he had received R5 more, he would have gained $12\frac{1}{2}$ p. c. What did the article cost him ?

9. A piece of cloth is sold for R40. 10*a.* at a profit of 30 p. c. If it had been sold at R1. 12*a.* per yard, the profit would have been R12. 8*a.* ; how many yards are there in the piece ?

10. A man embarks his capital in three successive ventures. In the first he clears 80 p. c., and in each of the others he loses 15 p. c.; what per cent. does he gain or lose on his original outlay?

11. A boy buys a number of apples at 6 for 4*a.* and a third of the number at 4 for 2*a.*; at what rate must he sell them to gain 20 p. c. on his outlay? Supposing his total profit to be R4, how many did he buy?

12. A man having bought a quantity of tea for R75, sells $\frac{1}{3}$ of it at a loss of 4 p. c.; by what rate per cent. must he raise that selling price, in order that by selling the rest at the increased price he may gain 4 p. c. on his outlay?

13. I bought note paper at the rate of 8 annas for 5 quires, and sold it so as to gain as much on the cost of 32 quires as 8 quires were sold for; at what price did I sell the paper per quire?

14. How must a grocer mix teas at 3*s.* a lb. and 3*s.* 6*d.* a lb., so that by selling the mixture at 3*s.* 8*d.* a lb. he may gain 10 p. c.?

15. A man bought a certain number of oranges at 3 a penny and four-fifths of that number at 2 a penny; by selling them at 9 for 4*d.* he gained 3*d.*; how many of each kind did he buy?

16. By buying milk at 4 seers a rupee and selling it at 5 seers a rupee after adding 9 seers of water to it, a milk-woman gains 25 p. c. How many seers of milk did she buy?

17. A merchant mixes 35 lb. of one kind of tea at 5*s.* per lb. with a few pounds of another kind of a tea at 7*s.* per lb.; if by selling the mixture at 6*s.* 6*d.* per lb. he gains $14\frac{2}{7}$ p. c., with how many pounds of the second kind of tea did he mix the first?

18. What quantity of corn at R5 per maund must a tradesman mix with 60 maunds at R6 per maund, in order to gain 20 per cent. by selling the whole at 2 annas 6 pies per seer?

19. A man bought a French watch bearing a duty of 25 per cent., and sold it at a loss of 5 per cent.; had he sold it for £3 more, he would have cleared 1 per cent. in his bargain. What had the French maker for the watch?

20. I must sell my stock of sugar at 2*a.* 6*p.* per lb. to gain $33\frac{1}{3}$ per cent.; by mixing it with an inferior sugar in the proportion of 4 to 1, I gain $33\frac{1}{3}$ p. c. by selling at R1. 9*a.* 6*p.* for $7\frac{1}{2}$ lb. Find the cost of the inferior sugar per lb.

21. A grocer proposes to sell his tea at 10 per cent. profit, but adulterates it by adding $\frac{1}{3}$ of its weight of an inferior tea which costs him $\frac{3}{4}$ of the price of the better; what profit per cent. does he make? Also in what proportion must he mix the two kinds so as to gain 20 per cent.?

22. A merchant buys 1575 cubits of cloth. He sells $\frac{1}{3}$ of it at a gain of 6 p. c., $\frac{1}{5}$ at a gain of 8 p. c., $\frac{1}{7}$ at a gain of 12 p. c., and the rest at a loss of 3 p. c. If he had sold the whole at a gain of 5 p. c. he would have received R120. 12*a*. more than he did. What was the prime cost of a yard ?

23. How must wine at 20*s*. a gallon and brandy at 45*s*. a gallon be mixed, so that by selling the mixture at 35*s*. a gallon there may be a gain of 15 p. c. on the price of the wine and 20 p. c. on the price of the brandy ?

24. A mixture of two kinds of wine, at 20*s*. and 25*s*. a gallon, is sold at a gain of 10 p. c. If the two kinds had been sold separately at a gain of 15 p. c. and 8 p. c. respectively, the total profit would have been the same. In what proportion were the two kinds of wine mixed together ?

25. A manufacturer sells an article to a wholesale dealer at a profit of 20 p. c., the wholesale dealer sells it to a retail dealer at a profit of 25 p. c. and the retail dealer sells it to a customer for R2. 3*a*. making a profit of 40 p. c. Find the actual cost of manufacture.

26. A man buys a certain number of mangoes at 20 for a rupee and an equal number at 25 a rupee. He mixes them together and sells them at 21 a rupee. What does he gain or lose per cent. ?

27. A man buys mangoes at 6*a*. a dozen and an equal number at 9*a*. a score ; he sells them at 9*a*. a dozen and thus makes a profit of R4. 13*a*. How many mangoes did he buy ?

28. A tradesman marks his goods at 25 p. c. above cost price. If he reduces the marked price by $12\frac{1}{2}$ p. c. for cash payment, what profit does he make ?

29. The cost of an article is 60 p. c. of the price at which it is marked for sale ; and the article is sold subject to a reduction of 10 p.c. for cash payment. How much per cent. does the seller gain ?

30. A tradesman by means of a false balance, defrauds to the extent of 10 p. c. in buying goods, and also defrauds in selling. What per cent. does he gain on his outlay by his dishonesty ?

31. A man sells a house, at a loss, for R400 ; had he sold it for R500 his gain would have been $\frac{2}{3}$ of his former loss ; find the cost price of the house.

32. A merchant has goods worth £300 ; he sells one-third of them so as to lose 10 p. c. By how much per cent. should he raise that selling price in order to gain 10 p. c. on the whole ?

33. A dishonest tradesman marks his goods at an advance of 10 p. c. on the cost price, but uses a fraudulent balance, whose beam is horizontal when the weight in one scale is

one-fifth more than the weight in the other. What is his actual gain per cent. ?

34. In the above example, if the tradesman, by mistake, weighs out the goods from the wrong scale, find his actual gain or loss per cent.

XLVIII. SIMPLE INTEREST.

280. **Interest** is money paid for the use of money lent. The money lent is called the **Principal**. The **Amount** is the sum of the principal and interest at the end of any time. The **rate of interest** is the money paid for the use of a certain sum for a certain time. Thus, if I borrow a sum of money on the condition that for the use of every rupee in the loan for a month I shall pay an interest of $\frac{1}{2}$ anna ; I am said to borrow *at the rate of* $\frac{1}{2}$ anna per rupee per month. Again, if I borrow on the condition that for the use of every R100 in the loan for one year I shall pay an interest of R5, I am said to borrow *at the rate of* 5 per cent per annum.

Note. *Per annum* means *for a year*.

281. When the interest is calculated simply on the original principal it is called **Simple** Interest.

Note 1. The term interest is generally used in the sense of *simple* interest.

Example 1. Find the simple interest on R24 for 5 months at $\frac{1}{2}$ anna per rupee per month.

Interest on R1 for 1 month $= \frac{1}{2}a. = R\frac{1}{32}$,

$\therefore$ ....R24 for 1 month $= R\frac{1}{32} \times 24$,

$\therefore$ ....R24 for 5 months $= R\frac{1}{32} \times 24 \times 5$

$= R3.\ 12a.$

Hence, to find the interest we multiply the principal by 5 and by $\frac{1}{32}$, that is, we multiply it by $\frac{5}{32}$. The work in practice should stand thus :

```
          R
         24
          5
32 ) 120 ( R3. 12a.   Ans.
      96
      24
      16
     384 ( 12
     32
      64
      64
```

EXAMPLES. 172.

Find the simple interest on

1. R58 for 4 months at 6*p*. per rupee per month.
2. R76 for 9 months at 2 pice per rupee per month.
3. R240 for 1 year at 3*p*. per rupee per month.
4. R375 for 15 months at $\frac{3}{4}$ anna per rupee per month.
5. R29 for 3 years 3 months at 2*p*. per rupee per month.
6. R720 for 18 months at 4*p*. per rupee per month.

Example 2. Find the simple interest on R728 for 5 years at 4 per cent. per annum.

Interest on R100 for 1 year = R4,
∴ .... R1 for 1 year = R$\frac{4}{100}$,
∴ .... R728 for 1 year = R$\frac{728\times4}{100}$,
∴ .... R728 for 5 years = $\frac{728\times4\times5}{100}$ [= R$\frac{728}{5}$],
= R145. 9*a*. $7\frac{1}{5}$*p*.

Hence we deduce the following rule :

Multiply the principal by the rate per cent. and by the number of years, and divide the product by 100.

The work should stand thus :

We divide R14560 by 100 by cutting off the two figures on the right ; thus the quotient is R145 , and R60 is the remainder ; this remainder is equal to 960*a*. ; this divided by 100 gives 9*a*. as quotient and 60*a*. as remainder ; this remainder is equal to 720*p*. ; this divided by 100 gives 7·2*p*. as quotient.

```
            R
           728
             4
          2912
             5
100 ) R145,60
           16
      a.  9,60
            12
      p.  7,20
```

∴ Interest = R145. 9*a*. 7·2*p*.
= R145. 9*a*. $7\frac{1}{5}$*p*.

Note 2. The amount may be obtained by adding the interest to the principal. Thus the amount in the above example

= R728 + R145. 9*a*. $7\frac{1}{5}$*p*.
= R873. 9*a*. $7\frac{1}{5}$*p*.

If the amount only is wanted we may also proceed thus :

Interst on R100 for 5 years at 4 p. c. = R20.
∴ The amount of R100 in 5 years = R120,
∴ .... R1 .... = R$\frac{120}{100}$,
∴ .... R728 .... = R$\frac{728\times120}{100}$
= R873. 9*a*. $7\frac{1}{5}$*p*.

Note 3. If P=principal, I=interest, t=time in years, and r=rate per cent., then the rule for finding the simple interest may be expressed by the formula—

$$I=\frac{P\times r\times t}{100}.$$

EXAMPLES. 173.

N. B. The rate per cent. is understood to be *per annum* unless otherwise stated.

Find the simple interest on

1. R200 for 3 yr. at 4 p. c.
2. £300 for 4 yr. at 5 p. c.
3. R750 for 7 yr. at 6 p. c.
4. £128 for 15 yr. at 3 p. c.
5. R450 for 11 yr. at $4\frac{1}{2}$ p. c.
6. £800 for $3\frac{1}{2}$ yr. at 4 p. c.

Find the simple interest and the amount of

7. R495. 4*a.* for $2\frac{3}{4}$ yr. at 3%
8. £325. 5*s.* for 4 yr. at $2\frac{1}{2}$%
9. R225. 11*a.* 9*p.* for 4 years at 1 per cent. per month.

Find the amount only of

10. R250 for 2 yr. at 7 p. c.
11. £304 for 5 yr. at $4\frac{1}{2}$ p. c.
12. R335 for $3\frac{1}{2}$ years at $\frac{3}{4}$ per cent. per month.
13. £720. 8*s.* 6*d.* for $2\frac{1}{4}$ years at $2\frac{2}{3}$ per cent.
14. £329. 9*s.* $4\frac{1}{2}$*d.* for $7\frac{1}{6}$ years at $3\frac{1}{4}$ per cent.
15. £220 for 7 months at $4\frac{3}{4}$ per cent.

282. When the rate per cent. and the number of years (or either of them) are fractional numbers, it is convenient first to multiply these two, and then multiply the principal by the product.

Example. Find the simple interest on R345. 10*a.* 3*p.* for 2 years 6 months at $5\frac{1}{4}$ per cent.

Now, 2 years 6 months = $2\frac{1}{2}$ years ;

and, $2\frac{1}{2}\times 5\frac{1}{4}=\frac{5}{2}\times\frac{21}{4}=\frac{5\times 7\times 3}{8}$.

R.	a.	p.
345	10	3
		5
1728	3	3
		7
12097	6	9
		3
8) 36292	4	3
R45 36	8	$6\frac{3}{8}$
16		
a. 5.84		
12		
p. 10.14$\frac{3}{8}$		

See Example 2, Art. 281.

The interest = R45. 5*a.* 10·14$\frac{3}{8}$*p.*

= R45. 5*a.* $10\frac{23}{160}$*p.*

283. As a general rule, it will be found more convenient to work in decimals by first *decimalizing* the principal, that is, by expressing it as the decimal of R1 (or £1). As the interest is generally less than the principal, and, therefore, the product of the rate per cent. and the number of years less than 100, it will, in most cases, be sufficient to decimalize the money *corrected* to three places only. The final result should also be taken *correct* to three places. This process will then give an answer correct to the *nearest* pie (or farthing).

Example 1. Find the simple interest on R400. 14*a.* 9*p.* for $5\frac{1}{2}$ yr. at 4 p. c.

R400. 14*a.* 9*p.* = R400·922.

Since $\frac{4\times 5\frac{1}{2}}{100}=\frac{22}{100}$, we have to multiply the principal by 22 and to divide the product by 100.

```
              R
           400·922
                22
           -------
           801·844
          8018·44
   100 )  8820·284
           88·20284
```

∴ Interest = R88·203
= R88. 3*a.* 3*p.*

Example 2. Find the simple interest on £325. 12*s.* $4\frac{1}{2}$*d.* for $3\frac{1}{2}$ yr. at $4\frac{1}{2}$ p. c.

£325. 12*s.* $4\frac{1}{2}$*d.* = £325·619.

Since $\frac{3\frac{1}{2}\times 4\frac{1}{2}}{100}=\frac{\frac{63}{4}}{100}=\frac{15\frac{3}{4}}{100}=\frac{16-\frac{1}{4}}{100}$, we have to multiply by $(16-\frac{1}{4})$ and to divide the product by 100.

```
                       £
                    325·619
                         16
                    -------
                   5209·904
¼ × 325·619 =        81·40475
          100 )    5128·49925
                     51·2849...
```

∴ Interest = £51·285
= £51. 5*s.* $8\frac{1}{2}$*d.*
(See Art. 186.)

We might have multiplied the decimalized principal by 63 and then divided the product by 400.

EXAMPLES. 174.

N. B. When the time is given in *months* and *days*, 12 months are reckoned to the year, and 30 days to the month.

Find the simple interest on

1. R375 for $3\frac{1}{3}$ years at $2\frac{2}{3}$ per cent.

2. £450 for $6\frac{2}{3}$ years at $3\frac{1}{3}$ per cent.

3. £875 for 3 years 4 months 15 days at $5\frac{1}{3}$ per cent.

Find, to the nearest pie, the simple interest on

4. R309. 10*a.* 3*p.* for 5 months 10 days at $4\frac{1}{3}$ per cent.

5. R21. 15*a.* 9*p.* for 2 years 9 months at $3\frac{1}{3}$ per cent.

6. R101. 13*a.* for 1 year 7 months 6 days at $\frac{1}{2}$ per cent. per month.

284. When interest has to be calculated from one day of the year to another, it is customary to include *one only* of the days named.

Example. Find the interest on £320 from January 4th to May 30th, at 3 per cent.

Number of days $= 27 + 28 + 31 + 30 + 30 = 146$;

145 days $= \frac{146}{365}$ of a year $= \frac{2}{5}$ yr. ; and $3 \times \frac{2}{5} = \frac{6}{5}$.

£
320
6
5) 1920
£3.84
20
s. 16.80
12
d. 9.60   ∴ the interest = £3. 16*s.* $9\frac{3}{5}$*d.*

Note 2. It should be noted that factors of 365 are 5 and 73.

285. When the number of days is not 73 or a multiple of 73, the fraction $\frac{\text{rate per cent.} \times \text{time}}{100}$ can always be so arranged as to have 73 in the denominator.

For example, suppose we have to find the simple interest on £1320. 11*s.* 3*d.* from January 2nd to May 31st at 3 per cent.

Here the number of days = 149 ; 149 days $= \frac{149}{365}$ of a year.

$$\therefore \frac{\text{rate per cent.} \times \text{time}}{100} = \frac{3 \times 149}{100 \times 365} = \frac{3 \times 149 \times 2}{100 \times 73 \times 5 \times 2}.$$

$$= \frac{894}{1000 \times 73} = \frac{\cdot 894}{73}.$$

The principal = £1320·563.

$$\therefore \text{ Interest} = \frac{£1320\text{·}563 \times \text{·}894}{73} = \frac{£1180\text{·}583}{73}.$$

Division by 73 may be effected in the ordinary way, or, more conveniently, by the following rule, known as the **third-tenth-and-tenth rule** :

To the number to be divided add successively its third part, then a tenth of this quotient ; and again a tenth of this second quotient ; then divide the sum by 100, *and subtract* $\frac{1}{10000}$ *of the resulting number.*

Applying this rule to the above example, we have

$$\begin{array}{rr} & \pounds \\ & 1180{\cdot}583 \\ \tfrac{1}{3}= & 393{\cdot}527 \\ \tfrac{1}{10}\text{ of }\tfrac{1}{3}= & 39{\cdot}352 \\ \tfrac{1}{10}\text{ of }\tfrac{1}{10}\text{ of }\tfrac{1}{3}= & 3{\cdot}935 \\ \hline & 1617{\cdot}397 \end{array}$$

Dividing this sum by 100, we get £16·17397.

Subtract $\frac{1}{10000}$ of this result ·00161

16·17236.

∴ The interest required = £16·172

= £16. 3*s.* $5\frac{1}{4}$*d.* correct to the nearest farthing.

To prove the rule. By actual division, we have

$\frac{1}{73}$ = ·01369863......(i)

Also, $1+\frac{1}{3}+(\frac{1}{10}\text{ of }\frac{1}{3})+\frac{1}{10}\text{ of }(\frac{1}{10}\text{ of }\frac{1}{3}) = 1+{\cdot}33\frac{1}{3}+{\cdot}03\frac{1}{3}+{\cdot}00\frac{1}{3}$

$= 1+{\cdot}37 = 1{\cdot}37.$

Dividing this sum by 100, we get ·0137.

Subtract $\frac{1}{10000}$ of this sum ·00000137

·01369863......(ii).

The results (i) and (ii) are the same, which proves the rule.

EXAMPLES. 175.

N. B. When the time is given in *days* or *years and days*, the year is taken to consist of 365 days.

Find the simple interest on

1. £400 from April 4th to June 16th at 3 p. c.
2. Rs.750 from Feb. 23rd to Sep. 30th at $4\frac{1}{2}$ p. c.
3. Rs.321. 8*a.* from Dec. 10th, 1927, to May 4th, 1928, at $3\frac{3}{4}$ p. c.
4. £847. 15*s.* from Jan. 1st to April 1st at $2\frac{1}{2}$ p. c.
5. Rs.349. 8*a.* 9*p.* from June 1st to Oct. 4th at $5\frac{3}{4}$ p. c.
6. Rs.309. 12*a.* for 1 year 73 days at $2\frac{2}{3}$ p. c.

286. Inverse questions on Simple Interest.

It is evident from the formula

$$I = \frac{P \times r \times t}{100}.........(A),$$

that when out of the four quantities, I, P, r and t, any three are given, the fourth can always be found.

I. To find the rate per cent.

Example 1. At *what rate* per cent. will R425 amount to R476 in 3 years ?

Interest on R425 for 3 years = R51, (*i.e.*, R476 − R425).

∴ .... R1 for 3 years = $R\frac{51}{425}$,

∴ .... R1 for 1 year = $R\frac{51}{425 \times 3}$,

∴ .... R100 for 1 year = $R\frac{51 \times 100}{425 \times 3}$ = R4 ;

∴ the rate per cent. = 4.

Or thus : From formula (A), $P \times r \times t = I \times 100$; ∴ $r = \frac{I \times 100}{P \times t}$ = etc.

EXAMPLES. 176.

At what rate per cent. will

1. R300 amount to R337. 8*a.* in 5 years ?
2. R825 amount to R905. 7*a.* in 3 years ?
3. £142. 10*s.* amount to £163. 13*s.* 11¼*d.* in 4½ years ?
4. The interest on R22214. 4*a.* amount to R462. 12*a.* 9*p.* in 7 months 10 days ?
5. A given sum of money double itself in 20 years ?
6. The interest on any sum of money be ⅖ths of the amount 20 years ?
7. The interest on £1368. 15*s.* become £14. 4*s.* 7½*d.* from July 5th to Nov. 20th ?
8. At what rate per rupee per month will R250 amount to R312. 8*a.* in 8 months ?

II. To find the time.

Example 2. In *how many years* will £300 amount to £405 at 5 per cent. ?

Interest on £300 for 1 year = $£\frac{300 \times 5}{100}$ = £15 ; and interest on £300 for the required number of years = £405 − £300 = £105.

∴ The required number of years = $\frac{£105}{£15}$ = 7.

Or thus : From formula (A), $P \times r \times t = I \times 100$; ∴ $t = \frac{I \times 100}{P \times r}$ = etc.

EXAMPLES. 177.

In what time will

1. R475 amount to R532 at 4 p. c. ?
2. R266 . 10 . 8 amount to R293 . 5 . 4 at 3 p. c. ?
3. £1451 . 6 . 8 amount to £1667 . 4 . 4⅘ at 4½ p. c. ?

4. In how many years and months will the interest on £3125 amount to £556 . 12 . $9\frac{3}{4}$ at $3\frac{3}{4}$ p. c. ?

5. In how many years, months and days will ₹425 amount to ₹474 . 3 . 8 at 5 p. c. ?

6. In how many days will the interest on £121 . 13 . 4 amount to £2 . 0 . 5 at $6\frac{1}{4}$ p. c. ?

7. In how many years will a sum of money treble itself at $3\frac{1}{3}$ p. c. ?

8. In what time will the interest on any sum of money at $6\frac{1}{4}$ p. c. be ·1875 of the principal ?

9. In what time will the interest on any sum of money at 5 p. c. be $\frac{1}{8}$ of the amount ?

10. On Feb. 1st, 1929, a person borrowed £400 at $6\frac{1}{4}$ p. c., promising to return it as soon as the interest amounted to £5 ; on what date did the loan expire ?

11. In how many months will ₹3200 amount to ₹4000 at 3 pies per rupee per month ?

III. To find the principal.

Example 3. *What principal* will amount to ₹1000 in 10 years at $2\frac{1}{2}$ per cent. ?

Interest on ₹100 for 10 years at $2\frac{1}{2}$ p. c. = ₹25 ;

∴ ₹100 amounts to ₹125 in 10 yr. at $2\frac{1}{2}$ p. c.

Of the amount ₹125 the principal = ₹100,

∴ .... ₹1 .... = ₹$\frac{100}{125}$,

∴ .... ₹1000 .... = ₹$\frac{100 \times 1000}{125}$

= ₹800. *Ans.*

Or thus : Since amount $= P + I = P + \frac{P \times r \times t}{100} = P\left(1 + \frac{r \times t}{100}\right)$

therefore in this case

$$P\left(1 + \frac{2\frac{1}{2} \times 10}{100}\right) = 1000,$$

$$\therefore \quad P(1 + \cdot 25) = 1000,$$

$$P = \frac{1000}{1 \cdot 25} = 800.$$

∴ The required principal = ₹800.

Example 4. *What principal* will produce ₹33. 5*a.* 4*p.* as simple interest at $2\frac{2}{3}$ p. c. in $3\frac{1}{3}$ years ?

Interest on ₹100 for $3\frac{1}{3}$ yr. at $2\frac{2}{3}$ p. c. = ₹$\frac{80}{9}$.

When interest is R$\frac{80}{9}$, the principal = R100,

$\therefore$ .... R1, ... = R$100 \times \frac{9}{80}$;

$\therefore$ .... R$33\frac{1}{3}$, ... = R$100 \times \frac{9}{80} \times 33\frac{1}{3}$

$$= \text{R}\frac{100 \times 9 \times 100}{80 \times 3} = \text{R}375.$$

Or thus : From the formula $\frac{P \times r \times t}{100} = I$, we have

$$\frac{P \times 2\frac{2}{3} \times 3\frac{1}{3}}{100} = 33\frac{1}{3} = \frac{100}{3},$$

$$\therefore \quad P = \frac{100 \times 100}{3 \times 2\frac{2}{3} \times 3\frac{1}{3}} = \frac{100 \times 100 \times 9}{3 \times 80} = \text{etc.}$$

EXAMPLES. 178.

What principal will amount to

1. R900 in 5 years at 4 per cent. ?
2. R4546 . 10 . 8 in 1½ years at 5¼ per cent. ?
3. £190. 15*s*. in 3 years at 4 per cent. ?
4. £1153 . 9 . 4½ in 3 years 7 months at 2¾ per cent. ?
5. R459 . 2 . 8 in 2 years 4 months and 12 days at 6¼ per cent.?
6. R737. 8*a*. in 100 days at 3¾ per cent. ?
7. R809 at 5⅝ per cent. from April 20th to July 2nd ?
8. R255. 7*a*. 6*p*. in 1¼ years at 3 pice per rupee per month ?

What principal will produce

9. R37. 8*a*. 8*p*. interest in 4 years 3 months at 3⅓ per cent. ?
10. £23 . 7 . 1½ interest in 15 years at 4⅝ per cent. ?
11. Find, to the nearest pie, the sum that must be invested at 3¾ per cent. for 13 years to amount to R1000.
12. Find, to the nearest penny, the principal whose interest amounts to £100 in 2 years 5 months and 10 days at 4 per cent.

MISCELLANEOUS EXAMPLES. 179.

1. The interest on a sum of money at the end of 6 years is ⅜ths of the sum itself ; what rate per cent. was charged ?

2. A money-lender lent a sum of money for 3 years 7 months at 1½ pice per rupee per month. At the end of the time he received R1003 . 14 . 6 ; what was the sum lent ?

3. A sum of money increases by $\frac{1}{12}$ of itself every year, and in 7 years it amounts to R902. 8*a*. ; find the sum.

4. £275 increases by $\frac{1}{10}$ of itself per year: how long will it take to amount to £357. 10*s.*?

5. A sum of money amounts in 6 years at 5 per cent. simple interest to R442; in how many years will it amount to R510?

6. R500 is borrowed at the beginning of the year at a certain rate of interest, and after 7 months R350 more is borrowed at half the previous rate. At the end of the year the interest on both loans is R34. 6*a.* What is the rate of interest at which the first sum was borrowed?

7. What sum of money laid out at $3\frac{3}{4}$ per cent. will give R1 interest a day?

8. The principal and interest for 5 years are together R550, and the interest is $\frac{3}{8}$ of the principal; find the principal and the rate per cent. per annum.

9. The principal and interest for a certain time at $3\frac{1}{2}$ per cent. are together £450, and the interest is $\frac{2}{7}$ of the principal; find the time.

10. What sum lent out at 5 per cent. will produce in $4\frac{1}{2}$ years the same amount of interest as R500, lent out at 6 per cent, will produce in 4 years?

11. If an investment of £75 becomes £78. 15*s.* in 8 months, what sum invested at the same rate of interest will become £201. 17*s.* 6*d.* in 10 months?

12. *A* bequeaths to *B* a certain sum of money, which after paying a legacy duty of 10 per cent. yields an income of £810 when placed at interest of 3 per cent. Find the amount bequeathed.

13. A person who pays 4*p.* in the R income-tax, finds that a fall of interest from 4 to $3\frac{3}{4}$ per cent. diminishes his net yearly income by R47. What is his capital?

14. A sum of money doubles itself in 20 years; in how many years would it treble itself?

XLIX. COMPOUND INTEREST.

287. When money is lent at interest, and the interest is not paid at the end of the period when it becomes due, but is retained by the borrower (who makes use of it), it may, if so stipulated, be added on to the principal and then the interest for the next period is charged *not* on the *original* principal but on the *new* principal which is the amount at the end of the period. In such a case the principal increases from period to period and the interest for each period is the interest on the amount at the end of each preceding period. Such interest is called **Compound** Interest.

For example, suppose R2000 is lent at 5 p. c. compound interest, the interest being payable yearly. The interest at the end of 1 yr. is R100. Instead of paying R100, suppose the borrower retains it and makes use of it, (or we may consider the sum as having been paid and borrowed again), then at the end of one year. *i.e.*, at the beginning of the second year he has in his possession R2100 of the lender's money. The interest for the second year would be charged on R2100 and not on R2000, *i.e.*, the principal for the second year is R2100, which is the amount at the end of the first year. The interest on R2100 is R105, and if the borrower retains this interest also, the amount due at the end of the second year which is R2205 will be the principal at the beginning of the third year. The interest on R2205 for one year is R110. 4*a.* The amount at the end of the third year or the principal for the fourth year is R2315. 4*a.* ; and so on. The compound interest may be obtained by adding together the interests for the several periods. Thus in this case the compound interest for the first three years is (R100+R105+R110. 4*a.*) or R315. 4*a.* But it is usually obtained by subtracting the *original principal* from the *final amount*. Thus, R2315. 4*a.* – R2000=R315. 4*a.* (=Compound interest for 3 yr.).

Example 1. Find the compound interest on R200 for 4 yr. at 5 p. c. per annum.

R	
200	Prin. for the 1st yr.
10	Int. for the 1st yr.
210	Amt. at the end of 1st yr. Or Prin. for the 2nd yr.
10·5	Int. for the 2nd yr.
220·5	Amt. at the end of 2nd yr. Or Prin. for the 3rd yr.
11·025	Int. for the 3rd yr.
231·525	Amt. at the end of 3rd yr. Or Prin. for the 4th yr.
11·57625	Int. for the 4th yr.
243·10125	Amt. at the end of 4th yr.
200	Original principal.
43·10125	Interest required.

∴ The required interest is R43. 1*a.* 7*p.*, to the nearest pie.

Here to find each year's interest we multiply the principal by $\frac{5}{100}$ *i.e.*, we multiply by 5 (mentally) and divide the product

by 100; the division by 100 being effected by setting down each figure two places to the right, the decimal points remaining vertically below one another throughout.

The compound interest is obtained by subtracting the original principal from the final amount.

Example 2. Find the compound interest on R321. 8*a*. for 3 years at $2\frac{1}{2}$ per cent. per annum.

Now, R321. 8*a*. = R321·5, and $2\frac{1}{2}$ p. c. = 2 p. c. + $\frac{1}{2}$ p. c.
= 2 p. c. + $\frac{1}{4}$ of 2 p. c.

R	
321·5	Prin. for 1st yr.
6·43 1·6075	Int. for 1st yr.
329·5375	Prin. for 2nd yr.
6·5907 1·6477	Int. for 2nd yr.
337·7759	Prin. for 3rd yr.
6·7555 1·6889	Int. for 3rd. yr.
346·2203	Amt. at the end of 3 yr.
321·5	Original principal.
24·7203	Interest required.

∴ The required interest is R24. 11*a*. 6*p*., to the nearest pie.

Since it is unnecessary to calculate interest more accurately, at the most, than to the nearest pie (or farthing), we only want the answer correct to three places of decimals. We need, therefore, retain in the working only four decimal places, or five, at the most, if there be several years' interest to be calculated.

Note. 1. The compound interest might also, as has already been explained, be obtained by adding together the interest for the 1st year, interest for the 2nd year and interest for the 3rd year. If the interest for $2\frac{3}{4}$ years were required, it would be obtained by adding together the interest for the 1st year, interest for the 2nd year and $\frac{3}{4}$ of the interest for the 3rd year.

Note 2. If the interest is payable *half-yearly*, the result may be obtained by finding the interest for double the number of years at half the rate per cent.

EXAMPLES. 180.

N. B. The interest is understood to be payable *yearly* unless otherwise stated.

Find, to the nearest pie, the compound interest on

1. R400 for 2 yr. at 5 p. c.
2. R520 for 2 yr. at 4 p. c.
3. R500 for $2\frac{1}{2}$ yr. at 3 p. c.
4. R1000 for 3 yr. at $4\frac{1}{2}$ p. c.

Find, to the nearest penny, the amount, at compound interest, of

5. £650 in 3 yr. at 4 p. c. **6.** £320. 8*s.* in 2 yr. at $3\frac{1}{2}$ p. c.

7. £600 in $2\frac{1}{4}$ yr. at 3 p. c. **8.** £250 in $2\frac{3}{4}$ yr. at $1\frac{1}{2}$ p. c.

9. Find the compound interest on ₨350 for 1 yr. at 4 p. c. per annum, the interest being payable half-yearly.

10. Find the compound interest on £200 for $1\frac{1}{2}$ yr. at 10 p. c. per annum, the interest being payable quarterly.

288. The following method of finding the amount at compound interest is often useful.

Example 1. Find the amount, at compound interest, of ₨5000 in 3 years at 4 p. c.

Amount of ₨100 at the end of 1 yr. = ₨104;

∴ .... ₨1 .... = ₨$\frac{104}{100}$;

∴ .... any sum..................... = ₨$\frac{104}{100}$ of the sum.

Also, amount of any sum at the end of 2 yr. = $\frac{104}{100}$ of the amount at the end of 1st yr.

= $\frac{104}{100}$ of $\frac{104}{100}$ of that sum

= $(\frac{104}{100})^2$ of that sum.

Similarly, amount in 3 years = $(\frac{104}{100})^3$ of that sum; and so on.

[And, **generally**, if P be the principal, r the rate per cent. and n the number of years, and A the amount, then

$$A = P\left(1 + \frac{r}{100}\right)^n.$$ [Cf. *Example* 5, page 326.]

Hence, to find the amount of ₨5000 in 3 years, we have to multiply ₨5000 by $(104)^3$, and divide the product by $(100)^3$.

Process:

```
 ₨     5000
        104
     ------
     520000
      104
     ------
      208
      52
     --------
     54080000
      104
     --------
    21632
    5408
   -------------
₨5624·320000 = amt. in 3 years, which
              = ₨5624. 5a. 1·44p.  Ans.
```

Division by $(100)^3$ is effected by marking off 6 decimal places in the final product.

Example 2. Find the amount of R400 for $2\frac{1}{2}$ years at 6 per cent. compound interest.

$$\text{Amount} = R400 \times \frac{106}{100} \times \frac{106}{100} \times \frac{103}{100} = \text{etc.}$$

EXAMPLES. 181.

Find, (by the method of Art. 288) to the nearest pie, the amount, at compound interest, of

1. R1000 in 2 yr. at 5 p. c.
2. R300 in 3 yr. at 3 p. c.
3. R700 in $2\frac{1}{2}$ yr. at 4 p. c.
4. R750 in 3 yr. at $4\frac{1}{2}$ p. c.
5. R2000 in $2\frac{1}{4}$ yr. at 4 p. c.
6. R4000 in $2\frac{2}{3}$ yr. at 3 p. c.
7. R1 in $1\frac{1}{2}$ yr. at $3\frac{1}{2}$ p. c.
8. R10 in $3\frac{1}{4}$ yr. at $3\frac{1}{4}$ p. c.
9. R3000 in $1\frac{1}{2}$ yr. at 4 p. c. per annum, interest being due half-yearly.
10. R350 in $1\frac{3}{4}$ yr. at 4 p. c. per annum, interest being due quarterly.

289. Inverse questions on Compound Interest.

I. To find the principal.

Example. What principal will amount to R551. 4*a.* in 2 years at 5 per cent. compound interest?

$$\text{Principal} \times \left(\frac{105}{100}\right)^2 = R551{\cdot}25.$$

$$\therefore \text{Principal} = R551{\cdot}25 \times \left(\frac{100}{105}\right)^2$$

$$= R500.$$

290. To find the rate per cent.

Except in very simple and special cases the rate per cent. cannot be found without the aid of logarithms.

Example. Find the rate per cent., compound interest, at which R400 will amount to R441 in two years.

We have $400\left(1 + \frac{r}{100}\right)^2 = 441,$

$$\therefore \left(1 + \frac{r}{100}\right)^2 = \frac{441}{400},$$

$$\therefore 1 + \frac{r}{100} = \sqrt{\frac{441}{400}} = \frac{21}{20} = 1 + \frac{1}{20},$$

$$\therefore \frac{r}{100} = \frac{1}{20},$$

$$\therefore r = 5.$$

Hence the required answer = 5 p. c.

EXAMPLES. 182.

What sum lent at compound interest will amount to

1. £100 in 2 yr. at 5 p. c.? **2.** £132. 6s. in 2 yr. at 5 p. c.?

3. £270. 8s. in 2 yr. at 4 p. c.? **4.** £3413. 16s. in $2\frac{1}{4}$ yr. at 4 p.c.?

5. £1000 in $3\frac{1}{2}$ yr. at 6 p. c.? **6.** £1 in $3\frac{1}{4}$ yr. at 8 p. c.?

7. At what rate per cent., compound interest, will R576 amount to R625 in 2 years?

8. Find the rate per cent., compound interest, at which R64 will amount to R125 in 3 years.

291. To find the time.

In order to find the *time* when the amount, the principal and the rate per cent. are given, we find the amounts for successive years at the given rate per cent. until an amount approximately equal to the given amount is obtained.

Example. In how many years will R400 amount to R457. 8*a.* $7\frac{1}{5}$*p.* at 5 per cent. per annum compound interest?

R457. 8*a.* $7\frac{1}{5}$*p.* = R457·5375.

R	
400	Principal for 1st yr.
20	Int. for 1st yr.
420	Principal for 2nd yr.
21	Int. for 2nd yr.
441	Principal for 3rd yr.
22·05	Int. for 3rd yr.
463·05	Amt. in 3 yr.

When the interest for the third year is added, the amount is larger than R457·5375. Hence the required number of years lies between 2 and 3. The interest for the fractional part of the year is R457·5375 − R441 = R16·5375. But the interest for the 3rd yr. is R22·05.

$$\therefore \text{ the fraction of the year} = \frac{16{\cdot}5375}{22{\cdot}05} = {\cdot}75 = \frac{3}{4}.$$

$\therefore$ the number of years is $2\frac{3}{4}$.

EXAMPLES. 183.

(*Compound interest.*)

In what time will

1. £120 amount to £132. 6*s.* at 5 per cent. per annum?

2. R250 „ „ R270. 6*a.* $4\frac{4}{5}$*p.* at 4 p. c. per annum?

3. R700 amount to R772. 4*a*. 2*p*. at 4 p. c. per annum ?

4. R3125 „ „ R3413. 12*a*. $9\frac{3}{5}$*p*. at 4 p. c. per annum ?

5. R500 produce an interest of R38. 6*a*. 6*p*. at 3 p. c. per annum ?

6. £800 „ „ „ „ £172. 8*s*. $0\frac{3}{4}$*d*. at 5 p. c. per annum ?

7. R578 „ „ „ „ R35. 3*a*. $2\frac{2}{5}$*p*. at 3 p. c. per annum ?

8. £250 „ „ „ „ £10. 9*s*. 1*d*. at $1\frac{1}{2}$ p. c. per annum ?

292. Miscellaneous questions on Simple and Compound Interest.

Example 1. The defference between the simple and compound interest on a given sum for 3 yr. at 5 p. c. per annum is R15. 4*a*. Find the sum.

Let Rx be the required sum.

Then the amt. of Rx at 5 p. c. at the end of 3 yr. = R$(\frac{105}{100})^3 \times x$.

∴ The compound interest on Rx = (R$\frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} - 1)x$

= R·157625x.

The simple interest on Rx for 3 yr. at 5 p. c. = R·15x.

∴ The difference between the simple and compound interest = R(·157625 − ·15)x = R·007625x.

But the difference given is R15. 4*a*. = R15·25.

$$\therefore \quad ·007625x = 15·25,$$

$$\therefore \quad x = \frac{15·25}{·007625} = 2000,$$

i.e., the required sum is R2000.

Example 2. The interest on a given sum of money for one year is £5. 8*s*. 4*d*., the compound interest for two years is £11. 1*s*. Find the rate per cent.

Simple interest for 2 years = £10. 16*s*. 8*d*. ;

∴ (£11. 1*s*. − £10. 16*s*. 8*d*.) or 4*s*. 4*d*. is the interest on £5. 8*s*. 4*d*. for one year.

Now, 4*s*. 4*d*. = £$\frac{13}{60}$, and £5. 8*s*. 4*d*. = £$\frac{65}{12}$.

∴ The interest on £$\frac{65}{12}$ for one year = £$\frac{13}{60}$,

∴ .... £1 .... = £$\frac{13}{60} \times \frac{12}{65}$,

∴ ....£100...................... = £$\frac{13}{60} \times \frac{12}{65} \times 100$

= £4.

∴ The rate of interest is 4 p. c. per annum.

MISCELLANEOUS EXAMPLES. 184.

1. Find the difference between the simple and compound interest on R500 for 3 years, at 4 p. c.

2. Prove that the amount at compound interest for 2 years at 2 per cent. is 1·0404 times the principal.

3. Prove that the difference between the simple and compound interest for 3 years at 5 per cent. is ·007625 times the principal.

4. The difference between the simple and compound interest on a certain sum of money for 2 years at 4 p.c. is R1; find the sum.

5. A person at the beginning of each year lays aside R1000, and employs the money at 5 p. c. compound interest; how much will he be worth at the end of 3 years?

6. Show that the amount of £700 at 10 p. c. per annum compound interest will in 8 years exceed the amount of £1000 at $6\frac{1}{4}$ p. c. per annum simple interest.

7. What sum will amount to £1591. 13*s*. 2·16*d*. in 3 yr. at compound interest, the interest for the first, second, and third years being 3, 2 and 1 p. c. respectively?

8. Find what is the least number of years in which a sum of money will become more than double in amount if put out at compound interest at the rate of 10 p. c. per annum.

9. The population of a town is 64000 and its annual increase is 10 per cent.; what will be the number of its inhabitants at the end of 3 years?

10. A merchant commenced with a certain capital, and gained annually at the rate of 30 per cent. At the end of 3 years he is worth R21970. What was his original capital?

11. Interest being due yearly, the difference between the total simple interest and the total compound interest on a certain principal lent in each case for 2 yr. at $3\frac{1}{2}$ p. c. is £12. 5*s*. Find this principal.

12. A money-lender borrows money at 4 per cent. per annum, and pays the interest at the end of the year; he lends it at 6 per cent. per annum payable half-yearly, and receives the interest at the end of the year; by this means he gains R104. 8*a* a year; how much money does he borrow?

L. PRESENT WORTH AND DISCOUNT.

293. Suppose a customer purchases goods and promises to pay to the merchant R104 as the price of the goods at the end of 1 year, but by chance he gets money and wants to pay off the price by a cash payment immediately. Should he

pay ₨104? Since the customer wants to pay off the debt 1 year before it is due, he should be allowed some reduction. What should the reduction be? Suppose the current rate of interest is 4 p. c. If ₨100 be put out to interest, it will amount to ₨104 at the end of 1 year. It will, therefore, be no loss to the merchant if he accepts ₨100 as immediate cash payment. ₨100, in this case, is called the **Present Worth** or **Present Value** of ₨104; and ₨4, the reduction allowed, is called the **True** or **Mathematical Discount**, or simply **Discount** of ₨104.

Similarly, if the customer proposes to pay off his debt by cash payment after six months have elapsed, he will pay such a sum of money as would, if put out to interest by the merchant for the remaining 6 months at 4 p. c. per annum, amount to ₨104.

Thus the **Present Worth** or **Present Value** of an amount due at the end of a given time is that sum which with its interest for the given time will be equal to the amount.

Discount is the allowance made for the payment of a sum of money before it is due.

From the definition of present worth, it follows that a debt which is due at some future period equitably discharged by paying the present worth at once. Hence

discount is equal to the interest on the present worth.

And *Amount = Present Worth + Discount.*

Note. It will be observed that the relation between the amount and the present worth and the discount is the same as that between the amount and the principal and the interest which we have already considered in the section on Interest. Hence a large number of questions that we shall discuss here will be but repetitions, *under new names*, of those already dealt with, the terms *principal* and *interest* being replaced by *present worth* and *discount* respectively.

Example 1. Find the present worth of ₨825, due $2\frac{1}{2}$ years hence, reckoning interest at 4 per cent.

[*N. B.* This corresponds to *Ex.* 3, Art. 285.]

₨100 amounts to ₨110 in $2\frac{1}{2}$ years at 4 p. c.

∴ Present worth of ₨110 = ₨100,

∴ ....₨1 $= ₨\frac{100}{110}$,

∴ ....₨825 $= ₨\frac{100 \times 825}{110}$

$= ₨750$. *Ans.*

[Discount = ₨825 − ₨750 = ₨75.]

EXAMPLES. 185.

Find the present worth of

1. R204, due 4 years hence, interest at 5 per cent.
2. R1518. 12*a.*, due in 4 years, at $5\frac{3}{8}$ per cent.
3. R3776. 4*a.*, due 18 months hence, at 4 per cent.
4. £1522. 1*s.* 6*d.*, due 3 years hence, at $4\frac{3}{8}$ per cent.
5. £1607. 18*s.* 4*d.*, due $4\frac{1}{2}$ years hence, at 3 per cent.
6. £1156. 2*s.* 8*d.*, due $3\frac{1}{3}$ years hence, at $4\frac{1}{2}$ per cent.
7. R1626, due 4 months 10 days hence, at $4\frac{1}{2}$ per cent.
8. R183, due 25 days hence, at 4 per cent.
9. R24845. 15*a.*, due 3 years hence, at $7\frac{1}{2}$ per cent. compound interest.
10. £1050. 12*s.* 6*d.*, due 2 years hence, at $2\frac{1}{2}$ per cent. compound interest.

Example 2. Find the discount on R600, due 4 years hence, interest being reckoned at 5 per cent.

Interest on R100 for 4 years at 5 p. c. = R20.

∴ Discount on R120 = R20,

∴ ....R1 $= R\frac{20}{120}$,

∴ ....R600 $= R\frac{20 \times 600}{120}$

$= R100$. *Ans.*

[Present worth = R600 − R100 = R500.]

EXAMPLES. 186.

Find the discount on

1. R355. 4*a.*, due 4 months hence, at $4\frac{1}{2}$ per cent. interest.
2. R2830. 3*a.* 4*p.*, due 7 months hence, at 5 per cent.
3. R6901. 14*a.*, due 9 months hence, at 3 per cent.
4. R2980. 6*a.* 8*p.*, due 11 months hence, at 4 per cent.
5. £370. 4*s.* $8\frac{1}{4}$*d.*, due 15 months hence, at $4\frac{3}{8}$ per cent.
6. £275. 6*s.* 8*d.*, due $1\frac{1}{2}$ years hence, at $4\frac{1}{2}$ per cent.
7. £241. 12*s.* 4*d.*, due 146 days hence, at $4\frac{1}{3}$ per cent.
8. £121. 15*s.*, due 5 months hence, at $3\frac{1}{2}$ per cent.
9. R5208. 12*a.*, due $3\frac{1}{2}$ years hence, at $4\frac{1}{2}$ per cent.
10. R2516. 4*a.*, due 4 yr. 9 mo. 18 da. hence, at $6\frac{1}{4}$ per cent.
11. R6077. 8*a.* 6*p.*, due 4 years hence, at 5 p. c. compound interest
12. £413. 8*s.* 9*d.*, due 2 years hence, at 5 p. c. compound interes

294. Inverse Questions.

Example 1. If the discount on R282. 8*a*., is R32. 8*a*. reckoning interest at 4 per cent., when is the amount due ?

[*N. B.* This corresponds to *Ex.* 2, Art 285.]

Amount = R282. 8*a*. ; discount = R32. 8*a*. ; ∴ present worth = R250.

∴ Interest on R250 for the required number of years = R32. 8*a*.

and interest on R250 for 1 year at 4 per cent. = R10 ;

$$\therefore \text{ the required number of years} = \frac{R32.\ 8a.}{R10} = 3\tfrac{1}{4}.$$

∴ The amount is due $3\frac{1}{4}$ years hence.

EXAMPLES. 187.

When is the sum due, if the

1. discount on R1010. 10*a*. at 5 per cent. interest is R91. 14*a*. ?
2. discount on R1518. 12*a*. at $5\frac{5}{8}$ p. c. is R268. 12*a*. ?
3. discount on £520 . 17 . 6 at $4\frac{1}{2}$ p. c. is £70 . 17 . 6 ?
4. discount on £5747 at $3\frac{1}{2}$ p. c. is £147 ?
5. present worth of R3850 at 4 p. c. is R3500 ?
6. P. W. of R15941. 6*a*. 6*p*. at $3\frac{7}{8}$ p. c. is R13750 ?
7. P. W. of £8776. 6*s*. $10\frac{1}{2}$*d*. at $2\frac{1}{2}$ p. c. is £8721. 16*s*. 8*d*. ?

Example 2. If the discount on R528. 12*a*., due $3\frac{1}{2}$ years hence, be R78. 12*a*., at what rate per cent. is the interest calculated ?

[*N. B.* This corresponds to *Ex.* 1, Art. 286.]

Amount = R528. 12*a*. ; discount = R78. 12*a*. ; ∴ present worth = R450.

Interest on R450 for $3\frac{1}{2}$ years = R78. 12*a*. ;

$$\therefore \ldots\ldots\ldots\ldots\ldots R1 \text{ for } 3\tfrac{1}{2} \text{ years} = R\frac{78\frac{3}{4}}{450};$$

$$\therefore \ldots\ldots\ldots\ldots\ldots R1 \text{ for 1 year} = R\frac{78\frac{3}{4}}{450 \times 3\frac{1}{2}};$$

$$\therefore \ldots\ldots\ldots\ldots\ldots R100 \text{ for 1 year} = R\frac{78\frac{3}{4} \times 100}{450 \times 3\frac{1}{2}} = R5.$$

∴ Rate per cent. = 5.

EXAMPLES. 188.

What is the rate of interest, if the

1. discount on R350, due 2 years hence, is R100 ?

2. discount on ₹7480, due 4 years hence, is ₹680 ?

3. discount on £397. 2. $2\frac{6}{7}$, due 4 years hence, is £71. 12. $2\frac{6}{7}$?

4. discount on £538 . 10 . $7\frac{1}{16}$, due $2\frac{3}{4}$ years hence, is £37 . 17 . $3\frac{1}{16}$?

5. present worth of ₹1260, due 4 years hence, is ₹1125 ?

6. P. W. of ₹2673. 2*a*., due $3\frac{1}{2}$ years hence, is ₹2275 ?

7. P. W. of £2857. 10*s*., due $12\frac{1}{2}$ years hence, is £2000 ?

295. Miscellaneous questions on P. W. and Discount

Example 1. On what sum of money, due at the end of 2 years, does the discount, at 4 per cent., amount to ₹20 ?

Here, interest on P. W. for 2 years = ₹20.

Now, ₹8 is the interest for 2 years on ₹100,

∴ ₹4..₹50,

∴ ₹20..₹250 ;

∴ the P. W. = ₹250 ; and ∴ amount = ₹270. *Ans.*

Example 2. If the interest on ₹500 at 5 per cent. be equal to the discount on ₹575, when is the latter sum due ?

Here, ₹500 = P. W. of ₹575 ; ∴ ₹75 = interest on ₹500.

Now, the interest on ₹500 for the required number of years = ₹75, but the interest on ₹500 for 1 year at 5 per cent. = ₹25 ;

∴ the required number of years $= \frac{₹75}{₹25} = 3$.

∴ The sum is due 3 years hence.

Example 3. The interest on a certain sum of money is ₹22, and the discount on the same sum for the same time and at the same rate is ₹20 ; find the sum.

Int. on the sum = Int. on P. W. + Int. on Disc.

= Disc. on the sum + Int. on Disc.

∴ Int. on the sum − Disc. on the sum = Int. on Disc.

Hence ₹2 = Int. on ₹20,

∴ ₹22 = ... ₹220. *Ans.*

Note. It should be carefully noted that ***the difference between the interest and discount on a sum of money for a certain time and at a certain rate is equal to the interest on that discount for that time and at that rate.***

EXAMPLES. 189.

1. On what sum of money, due at the end of 16 months, does the discount, at $4\frac{3}{4}$ per cent., amount to ₨484. 8*a.* ?

2. If the discount on a certain sum of money, due 8 months hence, at $2\frac{1}{2}$ per cent., be ₨883 . 10 . 8, what is the sum ?

3. The discount on a certain sum of money, due at the end of $2\frac{1}{4}$ years, at $2\frac{1}{3}$ per cent., is £32. 10*s.* : find the sum.

4. If the interest on ₨2275 at $3\frac{1}{2}$ per cent. be equal to the discount on ₨2593 8*a.* for the same time and at the same rate, when is the latter sum due ?

5. If the interest on £800 at 3 per cent. be equal to the discount on £838, when is the latter sum due ?

6. If the interest on £148 for 5 years is equal to the discount at the same rate on £173. 18*s.*, due 5 years hence, what is the rate of interest ?

7. The interest on a certain sum of money is ₨120, and the discount on the same sum for the same time and at the same rate is ₨100 ; find the sum.

8. The interest on a certain sum of money is ₨336, and the discount for the same time and at the same rate is ₨300 ; find the sum.

9. The discount on a certain sum, due 2 years hence, is ₨50, and the interest on the same sum for 2 years is ₨56. 4*a.* : find the sum, and the rate per cent. per annum.

10. The interest on a certain sum, at 5 per cent., for a certain time is ₨50, and the discount for the same time at the same rate is £40 : find the sum, and the time.

11. If the difference between the interest and discount on a sum for 3 years at 3 per cent. be ₨1, what is the sum ?

12. If the difference between the interest and discount on a certain sum of money for 9 months at 4 per cent. be 15*s.*, find the sum.

13. *A* offers for a house ₨800, and *B* offers ₨815 to be paid at the end of 4 months. Which is now the better offer, if the rate of interest is 5 per cent. per annum ?

14. A man buys 250 md. of sugar for ₨2500 payable at the end of 6 months, and the same day sells them at ₨10 per md. ready money : what does he gain by the transaction, reckoning interest at 5 per cent. per annum ?

15. A tradesman marks his goods with two prices, one for ready money and the other for 6 months' credit : what ratio should

the two prices bear to each other, allowing interest at 4 per cent. ? If the credit price of an article be ₨50, what is the cash price ?

16. Five copies of a book can be bought for a certain sum payable at the end of a year and six copies of the same book can be bought for the same sum in ready money ; what is the *rate of interest* ?

17. The discount on ₨550 for a certain time is ₨50 ; what is the discount on the same sum for twice that time ?

18. The interest on £720 for a certain time is £18 ; find the discount on the same sum for the same time.

19. If the discount on a sum of money, due 6 months hence, at 8 p. c. be £7 . 10 . 11½ ; find the P. W. of the sum.

20. A man bought an estate for £2000 and sold it immediately for £2287. 10*s*. payable at the end of 5 months. If the use of the money be reckoned at 4 per cent. per annum, what is now his gain per cent. ?

21. £259. 7*s*. is due 4 years hence and £173. 18*s*., 5 years hence : what sum at the present time is equivalent to both these sums, calculating interest at 3½ per cent. ?

22. What sum must be paid now in order that a person may receive ₨2000 at the end of every year for the next 4 years, the rate of interest being 5 per cent. ?

LI. COMMERCIAL DISCOUNT.

296. A **bill** is a promise (in writing) to pay a certain sum of money at the end of a certain time.

Example. Each of the following is a bill : a **Bill of Exchange** or **Hundi** (which is a document in which one person directs another to pay to him or to some other person, a sum of money at the end of a certain time) ; a **Promissory Note** (which is a document in which one person promises to pay another a sum of money at the end of a certain time).

297. In wholesale business transactions, when a man buys goods, he does not usually pay for them in cash. Suppose Messrs. P. C. Dwadash Shreni & Co. of Aligarh purchase books from Messrs. Sanyal & Co. of Calcutta of the value of ₨5050 on 7th March, 1929. It may be agreed between the parties that payment will be made in 4 months' time. The document showing the liability of Messrs. P. C. Dwadash Shreni & Co. to pay, may be either a Promissory Note or a Bill of Exchange.

The following are the forms of the two documents which are drawn up on stamped paper, the value of the stamp depending on the nature of the documents :

Promissory Note.

ALIGARH,

R5050/-

March 7th, 1929.

Four months after date we promise to pay to Messrs. Sanyal & Co., or order, the sum of Five Thousand and Fifty Rupees for value received.

P. C. Dwadash Shreni & Co.

Bill of Exchange.

CALCUTTA,

R5050/-

March 7th, 1929.

Four months after date pay to us, or order, the sum of Five Thousand and Fifty Rupees for value received.

Sanyal & Co.

To Messrs. P. C. Dwadash Shreni & Co.,
Aligarh.

The bill is forwarded to Messrs. P. C. Dwadash Shreni & Co. for their "acceptance". This is done by their writing across the bill "accepted, payable at the Imperial Bank of India, Aligarh. P. C. Dwadash Shreni & Co." It is then returned to Messrs. a nyal & Co.

There is a *custom*, which has the force of law, by which a bill (*if not payable on demand*) always runs *three days* (called the **days of grace**) beyond the time specified. Thus the above bill drawn on the 7th March, at 4 months would be **nominally** due on the 7th July, but **actually** due on the 10th. Moreover, **calendar months** are always reckoned, so that a bill drawn on the 31st January, at 3 months, would be nominally due on the 30th April and actually on the 3rd May.

The above bill is, therefore, legally due on July 10th and Messrs. Sanyal & Co. cannot claim the money till July 10th,

when they will present it to the Imperial Bank of India and receive ₨5050. The whole transaction then closes.

It may be that Messrs. Sanyal and Co. want money before July 10th. They will then take the bill to a Banker (or a Bill Broker) who will purchase (or *discount*) the bill. On July 10th he will present the bill to the Imperial Bank of India and receive ₨5050. Now the question arises as to how much the banker (or bill broker) should pay to Messrs. Sanyal & Co. Naturally he would not like to lose over the transaction, and so would not pay the full amount of the bill but something less. If he pays the true Present Value of ₨5050, he will neither be a loser nor a gainer; but he will not take all this trouble for nothing. So usually he deducts from the face value of the bill *the simple interest* on it for the number of days the bill has yet "to run", including the 3 days of grace. This simple interest is called the **Commercial** or **Banker's Discount.**

Example. A bill for ₨5050 drawn on the 7th March at 4 months **is discounted** (*i.e*, *sold*) on the 28th April at 5 per cent.; how much does the holder of the bill receive, interest being deducted?

The bill is nominally due on the 7th but actually due on the 10th July; therefore the bill has still to run from 28th April to 10th July, that is, for 73 days or $\frac{1}{5}$ of a year (including *one only* of the days named).

Now, the banker's discount = the simple interest on ₨5050 for 73 days at 5% = ₨50. 8*a*. The banker pays for the bill (₨5050 − ₨50. 8*a*.) or ₨4999. 8*a*. He keeps the bill and on July 10th he receives ₨5050.

The Present Value of ₨5050 for 73 days at 5% is ₨5000 and the *true* discount is ₨50. Hence if the banker pays for the bill ₨4999. 8*a*. instead of ₨5000, he makes a profit of 8*a*. This is the **banker's profit.** It is equal to the *difference* between the commercial and true discount.

Note 1. We thus see that a banker in purchasing (or discounting) a bill *obtains a small advantage* by deducting interest instead of discount.

The mathematical discount is called **True Discount.**

Note 2. In Arithmetic 'Discount' is always understood to mean *true* discount (and not *commercial* discount.) Therefore in working examples true discound is always to be calculated unless commercial discount is expressly mentioned.

Note 3. In working an example the 3 days of grace should be added *only* when the information given in the question is sufficient to enable us to determine the *exact number of days* that must elapse before the bill falls due, *and not otherwise.*

Note 4. The purchaser of a bill may sell it at any time before it is due. In this case also, the second purchaser deducts interest on the amount for the time the bill has still to run adding the *three days of grace.*

298. A *second kind of commercial discount* (which has no reference to time) is the deduction which is made by a tradesman for immediate payment of his bill. Thus when a tradesman gives notice upon his bill that he will allow 10 per cent. discount for immediate payment, he deducts ₹10 for every ₹100 in the amount of the bill The calculation of this discount is therefore the same as of finding the simple interest on the amount of the bill for 1 year at 10 per cent.

EXAMPLES. 190.

1. Find the difference between the commercial and true discount on a bill of ₹6002. 8*a.*, due in 4 months, at 6¼ per cent.

2. A bill is drawn for £250 on June 12th at 5 months, and is discounted on Sep. 3rd at 5 per cent. ; how much does the holder of the bill receive, banker's discount being allowed ?

3. Find the banker's discount on a bill of £730 drawn on July 31st at 2 months and discounted on Sep. 3rd at 4 per cent

4. What does a bill-discounter give as the present worth of a bill for ₹91. 4*a.* drawn on Sep. 4th at 5 months and discounted the same day at 6¼ per cent. ?

5. A bill of ₹182. 8*a.*, nominally due on the 15th of May, is discounted on the 23rd April of the same year at 3 per cent ; what does the banker gain thereby ?

6. A bill is drawn for £365 on March 31st at 3 months and discounted on June 13th at 4 per cent. ; how much more was charged than the true discount ?

7. The difference between the commercial and true discount on a bill for 7½ months at 5 per cent. is ₹9 ; find the amount of the bill.

8. The amount of a tradesman's bill is ₹375 ; if he allows 10 per cent. discount, how much does he accept for immediate payment ?

9. A tradesman accepts £40 for immediate payment of a bill for £50 ; what rate of discount does he allow ?

10. If the credit price of five copies of a book is equal to the cash price of six copies of the same book, what is the *rate of discount* ? [Cf. Question 16, Ex. 189]

11. A tradesman's prices are 25 p. c. above the cost price; if he allows his customers a discount of 10 p. c. on his bill, what profit does he make?

12. How much per cent. must a tradesman add on to the cost price of his goods, that he may make 20 per cent. profit after allowing his customers a discount of 10 p. c. on his bill?

LII. EQUATION OF PAYMENTS.

299. When several sums are due from one person to another, payable at different times, we may be required to find the time at which they may all be paid together, so that neither the creditor nor the debtor may lose. The time so found is called the **equated time** of payment.

We give below a rule for finding the *equated time*, which will be found sufficiently accurate for all practical purposes

RULE. Multiply each debt by the number of months [or days] after which it is due; then divide the sum of the products by the sum of the debts; the quotient will be the number of months [or days] in the equated time.

Example. If ₨400 be due from A to B at the end of 8 months, and ₨600 at the end of 10 months, when may both sums be paid in a single payment?

Number of months in the equated time $= \frac{400 \times 8 + 600 \times 10}{400 + 600} = 9\frac{1}{5}$. *Ans.*

EXAMPLES. 191.

1. ₨200 is due in 5 months and ₨400 in 8 months; find the equated time of payment.

2. ₨450 is due 2 months hence, ₨400 is due 3 months hence and ₨250 is due 4 months hence; what is the equated time?

3. Find the equated time of payment of £600, one-half of which is due in 6 months, $\frac{1}{3}$ in 9 months, and the rest in a year.

4. A owes B a debt payable in $4\frac{7}{12}$ months, but he pays $\frac{1}{4}$ in 3 months, and $\frac{1}{3}$ in 4 months: when ought the remainder to be paid?

5. A owes B on the 10th of April ₨900 due 40 days hence; he pays ₨400 on the 10th of May and ₨300 on the 20th of the same month: on what date ought he to pay the rest?

LIII. STOCKS.

300. Stock is the name given to the money borrowed by any Government to meet national expenses, or to the *Capitals of Trading Companies.*

The money borrowed by a Government is called the **National** or **Public Debt.** The money lent to the Government is said to be in **Government Securities** or **Government Promissory Notes** in India, and in the **Funds** in England. A part of the National Debt in England is called the **Consolidated Annuities** or **Consols.**

When any Government raises capital by borrowing, it reserves to itself the option of paying off the principal at any future time, but promises to pay the interest at fixed periods. In India and England the interest is paid *half-yearly.*

The Capital of a Trading Company is divided into **Shares,** generally of R100 or £100 each ; those who join the company by buying one or more of these shares are called **Shareholders.** The shareholders are not required to pay the full price of their shares at once, but they have to pay it in instalments, as the business of the company progresses and **Calls** are made. The part of the capital of the company, which has thus been paid at any time, is called the **Paid-up Capital.** The profits of the company are divided periodically among the shareholders ; and the moneys thus received are called **Dividends.**

When all the capital of a company has been subscribed and the company is in need of more capital, it is not usual to issue more shares like those issued at first. The company generally borrows money at a fixed rate of interest and agrees to pay the interest on this money before any dividend on the original shares is paid. Money so borrowed is called the **Preference Stock** of the company, the original capital being called the **Ordinary Stock.**

The *bonds* which are given by Joint-Stock Companies, Municipalities and similar other bodies for *borrowed capital* are called **Debentures.**

301. Stock is transferable by sale ; but its price varies from a variety of causes. When the *market value* of **R**100 stock is **R**100 cash, the stock is said to be **at par** ; when **R**100 stock is sold for **R**98, it is said to be **at a discount of 2 per cent.,** or, **at 2 below par** ; when it is sold for **R**102, it is said to be **at a premium of 2 per cent,** or, **at 2 above par.**

Purchases and sales of stock are usually made through *Brokers* who generally charge $\frac{1}{8}$ per cent. *on the stock bought or sold.* Thus, if the market value of **R**100 stock is **R**$97\frac{1}{2}$, the purchaser has to pay **R**$(97\frac{1}{2}+\frac{1}{8})$ and the seller receives **R**$(97\frac{1}{2}-\frac{1}{8})$.

Note. By "the 3 per cents." or "3 per cent. stock" is meant a stock, on R100 (or £100) of which is paid a dividend of R3 (or £3) per annum.

N. B. Unless the brokerage is mentioned, it need not be taken into consideration in working examples in stocks.

302. *Example* 1. What is the cost of R1500 stock in the 4 per cents. at $97\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent.?

Cost of R100 stock = R$(97\frac{7}{8}+\frac{1}{8})$ = R98,

∴ .... R1500....... = R98 × 15 = R1470. *Ans.*

Example 2. How much stock at $97\frac{1}{2}$ (brokerage included) can be bought for R390?

Amount of stock bought for R$97\frac{1}{2}$ = R100,

∴ ....R1 = R$\frac{100}{97\frac{1}{2}}$,

∴ ....R390 = R$\frac{100\times 390}{97\frac{1}{2}}$

= R$\frac{100\times 390\times 2}{195}$

= R400. *Ans.*

N. B. It is obvious that we have nothing to do with the rate of interest in any of the two above examples.

EXAMPLES. 192.

1. Find the cost of R2000 of 4 per cent. stock at 95.

2. Find the cost of £250 in the 3 per cent. consols at 3 below par, brokerage being $\frac{1}{8}$ p. c.

3. How much money can be obtained from the sale of R4500 stock in the Calcutta Municipal Debentures at R12 premium? (Brokerage $\frac{1}{8}$ p. c.)

4. Find the price of the 4 per cents. when R800 stock can be purchased for R750. (B. $\frac{1}{8}$ p. c.)

5. Find the price of the $4\frac{1}{2}$ per cents. when R1700 is obtained from the sale of R1600 stock. (B. $\frac{1}{8}$ p. c.)

How much stock can be purchased by investing

6. R1350 in the 4 per cents. at R10 discount?

7. R5062. 8*a*. in the 5 per cents. at $12\frac{3}{8}$ above par? (B. $\frac{1}{8}$ p. c.)

8. £6909. 18*s*. in the consols at $92\frac{7}{8}$? (B. 2*s*. 6*d*. per cent.)

9. A person lays out R3750 in the purchase of 4 per cent. Govt. Securities at $93\frac{5}{8}$ and afterwards sells at $95\frac{5}{8}$; what profit does he make, the usual brokerage being charged on each transaction?

10. A person buys £1000 3 per cent. stock at $98\frac{5}{8}$, and sells out at $96\frac{5}{8}$; how much does he lose by the transaction? (B. $\frac{1}{8}$%).

11. A person bought Russian 5 per cent. stock at 72, and sold it when the price had risen to $75\frac{3}{4}$, thereby clearing £65; how much money did he lay out?

12. A person holds £4800 consols; if he sells out at $87\frac{3}{8}$ and invests the proceeds in the $2\frac{1}{2}$ per cents. at 81, how much of the latter stock will he hold?

13. A person invested £5330 in the 3 per cents. at 91, and when they had risen $1\frac{3}{4}$ per cent. he sold out and invested the money in the stock of the Dominion of Canada, at $102\frac{1}{2}$; how much Canadian stock does he hold?

Example 3. What annual income will be derived from R3725 of $4\frac{1}{2}$ per cent. stock?

Income from R100 stock = R$4\frac{1}{2}$,

∴ .... R1 .... = R$\frac{9}{2\times 100}$,

∴ .... R3725....... = R$\frac{9\times 3725}{2\times 100}$ = R167. 10*a.* *Ans.*

N. B. This is merely a case of finding the interest, where the given stock is the principal.

Example 4. What annual income will be derived from R2042. 8*a.* invested in the 4 per cent. Govt Securities at 102 (B. $\frac{1}{8}$%)?

Cost of R100 stock = R$102\frac{1}{8}$,

∴ Income on R$102\frac{1}{8}$ money = R4,

∴ .... R1 .... = R$\frac{4\times 8}{817}$,

∴ .... R$2042\frac{1}{2}$......... = R$\frac{4\times 8\times 4085}{817\times 2}$ = R80. *Ans.*

Example 5. A person transfers R8000 stock from 4 per cent. Govt. Securities at $98\frac{5}{8}$ to 6 per cent. Municipal Debentures at $131\frac{5}{24}$; find the alteration in his income, the usual brokerage being charged on each transaction.

Income from the 4 per cents. = R$8000\times\frac{4}{100}$ = R320.

Money obtained from the sale of 4 per cents. = R$8000\times\frac{98\frac{1}{2}}{100}$.

Income from R$131\frac{1}{3}$ invested in 6 per cents. = R6,

∴ ....R1 .... = R$\frac{6}{131\frac{1}{3}}$.

∴ ....R$\frac{8000\times 98\frac{1}{2}}{100}$.............................. = R$\frac{6\times 8000\times 98\frac{1}{2}}{131\frac{1}{3}\times 100}$

= R360.

∴ **The alteration in income is R360 − R320, or R40 *increase.***

Example 6. How much money must a person invest in the $4\frac{1}{2}$ per cent. Preference Stock of the O. R. Ry. Co. at $94\frac{1}{2}$ (brokerage included) to obtain an annual income of ₨600 ?

Money to be invested for ₨$4\frac{1}{2}$ income = ₨$94\frac{1}{2}$,

∴ .... ₨1 .... = ₨$\frac{94\frac{1}{2}}{4\frac{1}{2}}$,

∴ .... ₨600.......... = ₨$\frac{94\frac{1}{2} \times 600}{4\frac{1}{2}}$

= ₨12600. *Ans.*

Example 7. Find the price of 4 per cent. stock when from the investment of ₨3900 a person obtains an annual income of ₨160, brokerage being neglected.

Cost of stock producing ₨160 income = ₨3900,

∴ .... ₨1 .... = ₨$\frac{3900}{160}$,

∴ .... ₨4 .... = ₨$\frac{3900 \times 4}{160}$

= ₨$97\frac{1}{2}$. *Ans.*

EXAMPLES. 193.

1. Find the half-yearly dividend on ₨3500, 4 per cent. stock.

2. What annual income will be derived from ₨37250 of $4\frac{1}{2}$ per cent. stock, after paying an income tax of $4p$. in the ₨ ?

3. What amount of $3\frac{3}{4}$ per cent. stock must be bought to produce a quarterly income of £375 ?

4. What annual income will be derived from the investment of ₨5910 in the $4\frac{1}{2}$ per cents. at $98\frac{3}{8}$? (B. $\frac{1}{8}$%)

5. A person invests £25935 in 3 per cent. stock at 90. If the first year's dividend be invested in the same stock at 91, and the dividend for the second year at 95, what will be his income for the third year ?

6. If I invest ₨16420 in the E. I. Ry. Preference Stock which pays 5 per cent. and is at $102\frac{1}{2}$, what will my clear income be, after paying an income-tax of $5p$. in the ₨ ? (B. $\frac{1}{8}$ p. c.)

7. If I lay out ₨2400 in the $4\frac{1}{2}$ per cents. at 96, and after receiving the half-year's dividend sell out when they have sunk to 94, how much do I gain ?

8. A person bought Imperial Bank shares at 113, and after receiving the half-year's dividend at the rate of 12 per cent. per annum sold out at $117\frac{1}{2}$, and made a profit of ₨178. $8a$. in all ; how many shares did he buy ?

9. If a person invest ₨18810 in the 4 per cents. at $104\frac{1}{2}$, at what price must he sell out after receiving the half-year's dividend to make a profit of ₨450 ?

10. A person transfers £11000 from the 4 per cents. at 92 to the 5 per cents. at 110 ; find the alteration in his income.

11. How much stock can be purchased by the transfer of R4000 stock from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 96, and what change in annual income will be produced by the transfer ?

12. A person invested R5600 in the 5 per cent. Calcutta Municipal Debentures at par, and after receiving the half-yearly dividend he sells out at R$2\frac{1}{2}$ premium, and invests the entire proceeds in the 4 per cent. Government Securities at $95\frac{5}{32}$; what change is made thereby in his income ?

13. A person laid out R14500 in the $3\frac{1}{2}$ per cents. at $72\frac{1}{2}$, and when they had fallen to 68 he sold out and invested the money in the 4 per cents. at $75\frac{5}{8}$; find his gain or loss in income.

14. A person has annual income of R480 from stock in the 4 per cents. ; this stock he sells out at $95\frac{7}{8}$ and invests the money in a railway stock (paying 5 p. c.) at $119\frac{9}{16}$; find the alteration in his income. (B. $\frac{1}{8}$ p. c.)

15. How much money must a person invest in the 3 per cent. consols at $91\frac{3}{8}$ to obtain an annual income of £1000 ? (B. $\frac{1}{8}$ p. c.)

16. How much must a person invest in the 4 per cents. at $93\frac{3}{4}$ in order to have a clear income of R940 after paying an income-tax of 4*p*. in the R ?

17. How much 3 per cent. stock at par must a man sell in order to purchase enough 4 per cent. stock at $114\frac{1}{16}$ to produce an income of R252, a brokerage of $\frac{1}{8}$ p. c. being charged on each transaction ?

18. Find the price of the 4 per cents. when the investment of R3750 in them produces an income of R160.

19. What is the price of the $4\frac{1}{2}$ per cents. when a man has an income of R270 by investing R7800 in them ? (B. $\frac{1}{8}$. p. c.)

20. A man invests £1570 in the New 4 per cent. Egyptian Annuities, and has thereupon a clear annual income of £76, after paying an income-tax of 1*s*. in the £ ; find the price of the Annuities. (B. $\frac{1}{8}$. p. c.)

***Example* 8.** What rate of interest is obtained on money invested in the 4 per cents. at $79\frac{7}{8}$? (B. $\frac{1}{8}$. p. c.)

Interest obtained on R80 money = R4,

∴ ....R20 ....= R1,

∴ ....R100 ....= R5.

∴ Rate of interest is obtained is 5 per cent.

***Example* 9.** At what price (including brokerage) would a person have to purchase the $4\frac{1}{2}$ per cents. to get 5 per cent. for his money ?

$$R5 = \text{interest on } R100 \text{ money},$$
$$\therefore \quad R1 = \ldots\ldots\ldots\ldots\ R20 \ \ldots\ldots\ldots,$$
$$\therefore \quad R4\tfrac{1}{2} = \ldots\ldots\ldots\ldots\ R90 \ \ldots\ldots\ldots ;$$
$\therefore$ the stock must be bought at 90.

***Example* 10.** What is the better stock to invest in, 4 per cents. at 95 or $4\frac{1}{2}$ per cents. at 105 ?

In the first case, interest on R95 money $= R4$,

$$\therefore \quad \ldots\ldots\ldots\ldots R1 \quad \ldots\ldots\ldots = R\tfrac{4}{95}.$$

In the second case, ...R105 .... $= R\frac{9}{2}$,

$$\therefore \quad \ldots\ldots\ldots\ldots R1 \quad \ldots\ldots\ldots = R\tfrac{9}{210}.$$

It will be found that $\frac{9}{210}$ is greater than $\frac{4}{95}$; and therefore the second is the better investment.

***Example* 11.** A person finds that if he invests his money in the 4 per cents. at 98 his income will be R42 less than if he invests it in the 5 per cents. at 112 ; find the sum to be invested.

In the first case, income from R1 $= R\frac{4}{98}$;

In the second case,............... R1 $= R\frac{5}{112}$;

$\therefore$ difference of income from $R1 = R\frac{5}{112} - R\frac{4}{98} = R\frac{3}{112 \times 7}$.

Now, $R\frac{3}{112 \times 7}$ = difference of income from R1,

$$\therefore \quad R1 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ R\frac{112 \times 7}{3},$$
$$\therefore \quad R42 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ R\frac{112 \times 7 \times 42}{3}$$

or R10976. *Ans.*

EXAMPLES. 194.

What rate of interest is obtained by investing in the

1. 4 per cents. at 90 ?

2. 3 per cents. at 70 ? (B. $\frac{1}{4}$ p. c.)

3. A person buys £800 3 per cent. consols at 85, and £500 more when they are at 97 ; how much per cent. will he get for his money after deducting an income-tax of 7*d.* in the £ ?

4. What rate of interest do I get upon my money, if I buy Railway Shares of R75 each (which pay 4 per cent.) at 85 and pay an income-tax of 4*p.* in the R ?

5. At what price would a person have to purchase the 4 per cents. to get $5\frac{1}{2}$ per cent. on his money ?

6. What is the price of stock, when the 4½ per cents. pay interest at the rate of 6 p. c. on the money invested ? (B. ⅓ p. c.)

7. When the 4 per cents. are at 88, what ought to be the price of the 4½ per cents. to give the same rate of interest ?

8. A man invested in the 4 per cents. ; if, after deducting an income-tax of 6*p.* in the rupee, he obtained 4½ per cent. interest on the money invested, at what price did he buy ?

9. If Bank stock bought at 14 per cent. discount pay 6½ per cent. on the investment, how much per cent. would it pay if it were bought at 28 per cent. premium ?

10. Which is the better investment, 4 per cents. at 82 or 5 per cents. at 102 ?

11. Which is the better stock to invest in, 3½ per cents. at 82½ or 4 per cents. at 100½ ? (B. ⅛ p. c.)

12. Find the difference per cent. in income between investing in the 4 per cents. at 88 and 4½ per cents. at 90.

13. A person finds that if he invests his money in the 4½ per cents. at 96 his income will be greater by ₨10 than if he invests it in the 4 per cents. at 88 ; find the money to be invested.

14. By investing a certain sum of money in the 3 per cents. at 75 a man gets £5 . 13 . 4 less in income than he would get by investing the same sum in the 3½ per cents. at 84 ; find the sum invested.

303. *Example.* A man invests £1560 partly in 3 p. c. stock at 81 and partly in 4½ p. c. stock at 135, and derives an annual income of £55. How much does he invest in each kind of stock ?

Income from £1 invested in the first stock $= £\frac{3}{81} = £\frac{1}{27}$,

.... £1 .... second...... $= £\frac{4\frac{1}{2}}{135} = £\frac{1}{30}$.

And the average income obtained from £1 $= £\frac{55}{1560} = £\frac{11}{312}$.

∴ Each pound invested in the first stock gives $£\frac{1}{27} - £\frac{11}{312}$ or $£\frac{15}{27 \times 312}$ or $£\frac{5}{9 \times 312}$ more than the average income ; and each pound invested in the second stock gives $£\frac{11}{312} - £\frac{1}{30}$ or $£\frac{18}{30 \times 312}$ or $£\frac{3}{5 \times 312}$ less than the average income.

Hence from Art. 268 we get the following proportion :

money invested in the 1st stock : money invested in the 2nd stock $= \frac{3}{5 \times 312} : \frac{5}{9 \times 312} = \frac{3}{5} : \frac{5}{9} = 27 : 25$.

Dividing £1560 in the ratio of 27 : 25, we get the sums £810 and £750.

∴ The man invests £810 in 3% stock and £750 in 4½% stock.

Or, better thus, *algebraically*,

Suppose he invests £x in 3% stock, £y in $4\frac{1}{2}$% stock;

$$\text{then} \qquad x \times \frac{3}{81} + y \times \frac{\frac{9}{2}}{135} = 55,$$

$$\text{or} \qquad x \times \frac{1}{27} + y \times \frac{1}{30} = 55,$$

$$\text{or} \qquad \left.\begin{array}{r} 10x + 9y = 14850 \\ x + y = 1560 \end{array}\right\}$$

Also

Solving the equations, we have $x = 810$, and $y = 750$.

Thus he invests £810 in 3% stock, and £750 in $4\frac{1}{2}$% stock.

MISCELLANEOUS EXAMPLES. 195.

1. A person invested money in the 4 per cents. when they were at 95, and some more when they were at 90; find the advantage per cent. of the second purchase over the first.

2. A person invests R16600 in the 3 per cents. at 83, and when the funds have risen 7 per cent. he transfers $\frac{2}{3}$ of his capital to railway stock at $67\frac{1}{2}$; what dividend ought the latter to pay that he may thereby increase his income by R50?

3. Which is the better investment, £1256 in the $3\frac{1}{2}$ per cents. at 87, or in the railway shares at £89 per share, the dividends in the latter case being $3\frac{3}{4}$ per cent. on the sum invested?

4. A person possesses £3200 3 per cents., which he sells at $99\frac{3}{4}$; he invests the proceeds in railway shares at £56 a share, which shares pay 5 per cent. interest on £45, the amount paid on each share. By how much is his income altered by the transaction?

5. A person has R5000 stock in the 3 per cents. which he sells and re-invests in the $3\frac{1}{2}$ per cents. at $87\frac{1}{2}$ and increases his income by R5; find the price of the 3 per cents.

6. By selling £1500 3 per cents. at 95 and re investing it I increase my income by £15 a year. If the dividend on the new shares is 8 per cent., what is the price of them?

7. What sum must be invested in the 3 per cents. at 90 to amount in $23\frac{1}{2}$ years at simple interest to £3210 cash; the price of the stock remaining unchanged? How many years sooner would the amount be realized if the price of the stock rose to 96?

8. A gentleman in India has been receiving 12 per cent. on his capital; he goes to England, invests it in the 3 per cents. at $94\frac{3}{8}$, and his income in England is £2400 a year; what was his income in India? (£1 = R10).

9. How much 3 per cent. stock must be sold at $87\frac{1}{2}$ to pay the present worth of ₨1645. 14*a*. due 10 months hence, at $3\frac{3}{4}$ per cent. ?

10. Municipal Debentures are at 119 when the Government Securities are at $93\frac{1}{2}$, what should be their price when the Government Securities are at $71\frac{1}{2}$?

11. What is the price of the 4 per cents. when $\frac{1}{27}$ of the sum invested is received as annual interest after deducting an income-tax of 4 pies in the rupee ?

12. A person invests ₨23800 partly in a $4\frac{1}{2}$ per cent. stock at $97\frac{1}{2}$ and partly in the 3 per cents. at par ; if he holds twice as much 3 per cents. as $4\frac{1}{2}$, find the income that he obtains from the whole investment.

13. A man having money invested in the 3 per cents., from which he derives an income of £864, sells out at 90, and invests in shares that pay 5 per cent. interest ; if his income be now increased by £336, at what price does he buy the shares ?

14. What sum must have I invested in the $3\frac{1}{2}$ per cents. at 91 if, after investing £4000 more in the 3 per cents at 75, and paying an income-tax of 7*d*. in the £ on my total gross receipts, I find my net income to be £524. 5*s*. ?

15. A person who has a certain capital calculates that if he invest half his capital in the 3 per cents. at 90, and half in the 4 per cents. at par, his total income will be ₨1100 ; what is his capital ?

16. *A* invests £3500 in buying equal amounts of 3 per cents. at $78\frac{1}{8}$ and 6 per cents. at $109\frac{3}{8}$. *B* invests the same sum, half in one stock and half in the other. Find (i) the difference in their incomes, (ii) the ratio of their rates of interest.

17. Four per cents. are at 95, and $4\frac{1}{2}$ per cents. are at 105. One person buys ₨200 stock in each, and another person invests ₨200 in each : compare the rates of interest obtained by the two on their whole investments.

***18.** A shareholder receives one year a dividend of 10 per cent. on his stock and pays an income-tax of 4 pies in the rupee. The next year he receives a dividend of 12 per cent. and pays an income-tax of 5 pies in the rupee. If his income is ₨394. 5 . 4 more in the latter than in the former year, how much stock does he hold ?

19. 20 shares in a company are worth ₨1600 when the dividend is at the rate of 5 per cent. ; how many share ought to be worth ₨960 when the dividend is at 6 per cent. ?

20. A person invested ₨2800 in the purchase of 4 per cents. at 90 and $4\frac{1}{2}$ per cents. at 95. If his total income is ₨130, how much of each stock did he buy ?

21. A man invests £1600 in the 4 per cent. stock at 80 and $7\frac{1}{2}$ per cent. stock at 125; what sums must he invest in the respective stocks to make $5\frac{3}{4}$ per cent. on his money?

22. A person, by selling 4 per cents. at 87 and investing the proceeds in the 5 per cents. at 96, finds that his incom is increased by R17 : how much 4 per cents. did he sell?

23. 4 per cent. stock, bought at $95\frac{7}{8}$, is held for 6 months at the end of which time the interest is paid; it is then sold at the same price at which it was bought: find the rate per cent. per annum of interest obtained for the money used. (Usual brokerage).

24. A person invests R255 in the 4 per cents. at 85, and sells part of his stock when they have risen 5 per cent. and the remainder when they have fallen 8 per cent.; he lost R11 by the transaction : how much stock did he sell out at first?

25. 5 per cent. stock is sold at 108, and with the proceeds 4 per cent. stock is bought at $91\frac{1}{8}$; after a time 4 per cent. stock is sold at $95\frac{3}{8}$ and the original stock purchased at 109, leaving a profit of R109 on the transaction : find the amount of 5 per cents. sold.

26. If the 3 per cents. be at 95, and the Government offer to receive tenders for a loan of £5,000,000, the lender to receive £5,000,000 stock in the 3 per cents. together with a certain sum in the $3\frac{1}{2}$ per cents., what sum in the $3\frac{1}{2}$ per cents. ought the lender to accept?

27. The present income of a railway company would justify a dividend of 6 per cent., if there were no preference share; but as £50,000 of the stock consists of such shares which are guaranteed $7\frac{1}{2}$ per cent. per annum, the ordinary shareholders get only 5 per cent. : find the amount of the ordinary stock of the company.

28. A person buys 6 per cent. bonds, the interest on which is payable yearly and which are to be paid off at par 1 year after the time of purchases; if money be worth 5 per cent., what price should be given for the bonds?

LIV. EXCHANGE.

304. Exchange means the giving or receiving of a sum of money of one country equal in value to a given sum of money of another country.

The **par of exchange** between two countries denotes the *intrinsic* value of a coin of one country, as estimated in terms of a coin of the other country.

The **course of exchange** is the *actual* or *marketable* value at any time of a coin of one country, as estimated in terms of a coin of the other country.

Thus, the quantity of gold in the English Sovereign being 1·261 times the quantity of gold in the French Napoleon, at par of exchange £1 is equal to 1·261 Napoleons; but in the course of exchange £1 may be equal in value to a little more or less than 1·261 Napoleons.

Arbitration of exchange is the determination of the rate of exchange, called the **arbitrated rate**, between the first and last of a given number of places, when the rates of exchange between the first and second, the second and third, etc., of these places are known.

305. Money transactions between one country and another are usually carried on by means of **Foreign Bills of Exchange** or briefly **Foreign Bills.**

The following is the usual mode of proceeding:

Suppose I want to transmit £100 to a merchant in London. I go to a banker and buy a bill for the given amount, payable in London, at the current rate of exchange; I then send the bill to the merchant in London, who presents it to the person on whom it is drawn and receives the amount.

306. The following table gives the principal foreign monetary systems.

France Belgium Switzerland	1 franc	= 100 centimes	$= 9\frac{2}{3}d.$ (France to Roumania)
Italy ...	1 lira	= 100 centesimi	
Spain ...	1 peseta	= 100 centimos	
Greece ...	1 drachme	= 100 lepta	
Servia ...	1 dinar	= 100 paras	
Bulgaria ...	1 leva	= 100 stotinkis	
Roumania ...	1 ley	= 100 banis	
Germany ...	1 mark	= 100 pfennige	$= 11\frac{3}{4}d.$
Austria ...	1 florin or gulden	= 100 kreuzers	$= 1s.\ 11\frac{3}{4}d.$
Turkey ...	1 Turkish pound	= 100 piastres	$= 18s.\ 0\frac{3}{4}d.$
Holland ...	1 florin	= 100 cents	= 1*s.* 8*d.*
Portugal ...	1 milreis	= 1000 reis	= 4*s.* 6*d.*
Sweden Norway Denmark	1 crown	= 100 ore	$= 1s.\ 0\frac{3}{4}d.$
United States	1 dollar ($)	= 100 cents	= 4*s.* 2*d.*
Russia ...	1 rouble	= 100 kopecks	= R1. 12. 3.
China ...	1 tael = 10 mace	= 100 candareens	= R3.
Japan ...	1 yen	= 100 sen	= R2. 7. 6.

[In the above table the equivalents given in English and Indian money are the pre-war rates. The Great War caused the most violent fluctuations in the exchanges throughout the

world. The rates have gradually become more or less steady and the present tendency is towards the pre-war level. For further information consult Jackson's Commercial Arithmetic.]

Note. In the countries whose names have been printed in italics in the above table, as in India, the standard coins are *silver*; in England the standard coin is *gold*; hence the value of the Rupee, etc., in English money varies with the amount of silver which can be bought for a gold sovereign. For some years past the value of silver as compared with gold has been steadily declining. A few years ago a Rupee was equal in value to about 2*s.*; now it is equal to 1*s.* 6*d.*

Example 1. Calculate the *par of exchange* between the sovereign and the rupee, supposing pure gold to be worth 15 times its weight of pure silver, having given that $46\frac{29}{40}$ sovereigns are coined from 1 lb. Troy of standard gold, $\frac{11}{12}$ fine, and that a rupee weighs 180 grains of silver and is $\frac{11}{12}$ fine.

The sovereign weighs $\frac{12 \times 20 \times 24}{46\frac{29}{40}}$ gr. or $\frac{12 \times 20 \times 8 \times 40}{623}$ gr.;

and therefore it contains $\left(\frac{12 \times 20 \times 8 \times 40}{623} \times \frac{11}{12}\right)$ gr. or $\frac{20 \times 8 \times 40 \times 11}{623}$ gr. of pure gold.

The rupee weighs 180 gr.; and therefore it contains $(180 \times \frac{11}{12})$ gr. or 165 gr. of pure silver, which is equivalent to $\frac{165}{15}$ gr. or 11 gr. of pure gold.

Now the number of rupees equivalent to a sovereign is the same as the number of times 11 gr. is contained in $\frac{20 \times 8 \times 40 \times 11}{623}$ gr.

Hence the sovereign $= \frac{20 \times 8 \times 40 \times 11}{623 \times 11}$ rupees

$= 10{\cdot}27\ldots$ rupees.

Example 2. Find the relation between the rupee and the shilling as determined from the intrinsic value of the two coins; having given that a rupee weighs 180 grains, and is $\frac{11}{12}$ fine; and that 1 lb. Troy of silver, $\frac{37}{40}$ fine, is coined into 66 shillings.

We find, as in the preceding example, that the rupee contains 165 gr. of pure silver. The shilling contains $\left(\frac{12 \times 20 \times 24}{66} \times \frac{37}{40}\right)$ gr. or $\frac{24 \times 37}{11}$ gr. of pure silver.

$\therefore$ 1 rupee $= (165 \div \frac{24 \times 37}{11})$ shillings

$= 2{\cdot}043\ldots$ shillings.

Example 3. Exchange ₨550 for English money at 1*s.* 8*d.* per rupee.

₨1 $=$ 1*s.* 8*d.*,

$\therefore$ ₨550 $=$ 1*s.* 8*d.* $\times$ 550

$=$ £45. 16*s.* 8*d.* *Ans.*

Example 4. Determine the *course of exchange* between India and England, when Indian money is at a discount of 25 p. c., having given that at par 1 rupee = 2 shillings.

[Indian money being at a discount of 25 p. c. means that it is worth 25 p. c. less English money than it would be if it were at par.]

At par ₨1 = 2*s*.,

∴ at 25 p. c. disc. ₨1 = 2*s*. − $\frac{1}{4}$ of 2*s*.

= 1*s*. 6*d*.

∴ The course of exchange is 1*s*. 6*d*. per ₨1.

Example 5. If the rate of exchange between Calcutta and London is at 1*s*. 9*d*. per rupee, and that between London and Paris is at 25 francs per £1, what is the *arbitrated rate of exchange* between Calcutta and Paris?

₨1 = 1*s*. 9*d*. = £$\frac{7}{80}$ = $\frac{7}{80}$ × 25 francs = $2\frac{3}{16}$ francs. (See Art. 244.)

∴ The required rate is $2\frac{3}{16}$ francs per rupee.

Example 6. The course of exchange between London and Calcutta is 1*s*. 6*d*. per rupee, between London and New York is 4*s*. 4*d*. per dollar, and between Calcutta and New York is ₨3. 2*a*. per dollar. If a merchant of Calcutta remit a debt of 350 dollars to New York through London, instead of direct, does he gain or lose thereby? And by how much?

₨3. 2*a*. × 350 = ₨1050 + ₨43. 12*a*. = ₨1093. 12*a*. = the amount the merchant has to pay for 350 dollars when remitted direct.

$4s.\ 4d. \times 350 = £\frac{13}{3\times 20} \times 350 = £\frac{13\times 35}{6}$ = value of 350 dollars in English money.

$₨1 = 1s.\ 6d. = 1\frac{1}{2}s. = £\frac{3}{2\times 20} = £\frac{3}{40}$, ∴ $£1 = ₨\frac{40}{3}$

$$\therefore\ £\frac{13\times 35}{6} = ₨\frac{13\times 35}{6} \times \frac{40}{3} = ₨\frac{13\times 700}{9} = ₨\frac{9100}{9} = ₨1011.\ 1\frac{7}{9}a.$$

= the amount the merchant has to pay for 350 dollars when sent through London.

Hence the gain = ₨1093. 12*a*. − ₨1011. $1\frac{7}{9}$*a*. = ₨82. $10\frac{2}{9}$*a*.

= ₨82 10*a*. $2\frac{2}{3}$*p*.

EXAMPLES. 196.

1. Convert ₨3782 to English money, the course of exchange being 1*s*. $5\frac{1}{2}$*d*. per ₨.

2. Exchange £329. 7*s*. 6*d*. for Indian money at ₨11. 4*a*. per £.

3. A Spanish pistole is worth 15*s*. and an Austrian ducat 9*s*. 5*d*.; how many ducats are equivalent to 226 pistoles?

4. A French Napoleon or 20 franc piece is worth £·79; find, to the nearest farthing, the value in English money of 123·21 francs.

5. A bill bought in Calcutta at 1*s*. 6*d*. a rupee, is sold in New York at 4*s*. 3*d*. a dollar: determine the course of exchange between New York and Calcutta.

6. If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden; how many gulden can I get in exchange for £11?

7. Find the arbitrated rate of exchange between Vienna and Calcutta in rupees for 1 florin, when the exchange between Calcutta and London is ₨3 for 5*s*., between London and Paris is 25 francs for £1, between Paris and Berlin 5 francs for 4 marks, and between Berlin and Vienna 2 marks for 1 florin.

8. If a thaler is equivalent to 40 kreuzers, 10 silber-groschen and half a gulden, and if 30 silber-groschen make a thaler and 60 kreuzers make a gulden, how many gulden are worth 8 thalers?

9. If ₨1 in England exchanges for 1*s*. 5½*d*., and if £1 in India exchanges for ₨13. 5*a*. 6*p*., how much do you lose in ₨960 by the two exchanges?

10. A person in Calcutta wishes to remit a debt of 240 dollars to New York when the exchanges are 1 dollar = ₨2. 13*a*., ₨1 = 1*s*. 6*d*. and 25*s* = 6 dollars. Is it more advantageous for him to remit directly to New York or circuitously through London?

11. A merchant in London is indebted to one at Petrograd 15000 roubles: the exchange between Petrograd and London is 50*d*. per rouble, between Petrograd and Amsterdam 91*d*. Fl. per rouble, and between Amsterdam and London 36*s*. 3*d*. Fl. per £ sterling. What difference will it make if the London merchant is drawn upon through Amsterdam or direct?

12. If in London I get £1 for 25 francs 20 centimes, what shall I gain or lose per cent. by taking French money into Bavaria when the exchange is 11 gulden 40 kreuzers for £1, and 8 gulden 20 kreuzers for a Napoleon? (1 Nepo. = 20 fr.; 1 fr. = 100 centimes; 1 guld. = 60 kreuz.)

13. The Indian bazar maund is equal to 82$\frac{2}{7}$ lb. Avoir., and the rupee is equal to 2*s*. If 1 md. of wheat cost ₨3, what will be the price in English money of 1 cwt.?

14. Exchange 380 dollars for English money when it is at a discount of 5 per cent., given that at par 1 dollar = 4*s*. 2*d*.

15. Exchange ₨660 for English money it is at a premium of 10 per cent., it being given that at par ₨1 = 1*s.* $10\frac{1}{2}$*d.*

16. If India exchanges with England at a loss of 15 per cent. when the course of exchange is 1*s.* 5*d.* per ₨, what is the par of exchange?

17. A merchant in Calcutta wishes to remit to London ₨900, a rupee being equal to 2*s.*; for what sum in English money must he draw his bill when bills on London are at a premium of $12\frac{1}{2}$ per cent.?

18. I pay ₨5,1000 to a bank for a bill of exchange payable in London. The rate of exchange is 1*s.* $10\frac{1}{2}$*d.* for the rupee, and the bank changes me 2 per cent. on the amount payable in England. How much will my agent in London receive?

19. A person in London owes another in Petrograd 460 roubles, which must be remitted through Paris. He pays the requisite sum to his broker when the exchange between London and Paris is 23 francs for £1, and between Paris and Petrograd 2 francs for one rouble. The remittance is delayed until the rates of exchange are 24 francs for £1, and 3 francs for 2 roubles. What does the broker gain or lose by the transaction?

20. The exchange of Calcutta on London at 3 months is 1*s.* $4\frac{1}{8}$*d.* per ₨; find the exchange at sight, reckoning 5 per cent. per annum.

21. Calculate the par of exchange between the gold mohur, weighing 180 grains, $\frac{11}{12}$ fine, and the U. S. eagle, weighing 258 grains, $\frac{9}{10}$ fine.

22. Calculate the par of exchange between the Napoleon and the rupee, supposing pure gold to be worth 15 times its weight of pure silver; being given that $16197\frac{1}{2}$ grains of French standard gold, $\frac{9}{10}$ fine, is coined into 155 Napoleons, and that a rupee contains 180 grains of silver, $\frac{11}{12}$ fine.

23. From 3465 grains of fine silver are coined 14 thalers; find the value of a thaler, when a pound Troy of Indian standard silver, of which 11 parts out of 12 are fine, is worth ₨32.

24. If 1 lb. of English standard silver, of which 37 parts in 40 are pure silver be worth 62*s.*, find the value of a Hyderabad rupee which weighs 7 dwt. 17 gr., and has a fineness of 30 parts in 31.

25. The gold coinage of one nation contains 1 part of siver to 11 parts of gold; that of another nation, 1 part of silver to 23 parts of gold. It is found that 59 of the first weigh as much as 123 of the second. The intrinsic value of silver is one-sixteenth that of gold Determine the par of exchange.

LV. INVOICES AND ACCOUNTS.

307. (i) *Specimen of an Invoice.*

Calcutta, April 23, 1929.

Charles Smith, Esq.,

Bought of William Moran & Co.

7, Bankshall Street.

	R	a.	p.
8 yd. of flannel at R1. 4a. per yd.	10	0	0
10 yd. of calico at 3a. 6p. per yd.	2	3	0
2 pairs of gloves at R1. 9a. 9p. per pair	3	3	6
R	15	6	6

(ii) *Specimen of an Account.*

Calcutta, June 30, 1929.

Charles Smith, Esq.,

To William Moran & Co.,

7, Bankshall Street

1929		R.	a.	p.
April 23,	To goods, as per invoice	15	6	6
May 7,	To ditto	3	7	3
„ 13,	To ditto	9	0	0
June 11,	To ditto		7	6
	R	28	5	3

(iii) *Specimen of a Detailed Account.*

Calcutta, June 30, 1929.

Charles Smith, Esq.,

To William Moran & Co.,

7, Bankshall Street.

1929		R	a.	p
April 23,	8 yd. of flannel at R1. 4a. per yd.	10	0	0
„	10 yd. of calico at 3a. 6p. per yd.	2	3	0
„	2 pairs of gloves at R1. 9a. 9p. per pair ...	3	3	6
May 7,	3 dozen stockings at R6 per doz.	18	0	0
„ 13,	13 yd. of linen at 8a. 6p. per yd.	6	14	6
June 12,	20 yd. of carpet at R3. 8a. per yd.	70	0	0
„	4 pairs of socks at R1 per pair	4	0	0
	R	114	5	0

Note. Invoices and Accounts are called **Bills.** Each separate entry in a bill is called an **item.** When an account is sent to a buyer it is said to be **rendered.**

LVI. METRIC SYSTEM AND DECIMAL COINAGE.

308. The **Metric System** of weights and measures, which originated in France, has been introduced to a greater or less extent into almost all the countries of Europe. It is also nearly always used in scientific treatises.

The Tables of weights and measures in the metric system are constructed upon one uniform principle, by attaching the following prefixes to each of the *units*.

Greek Prefixes.				Latin Prefixes.			
Deca	means	10	times.	**Deci**	means	10th	part of
Hecto	"	100	"	**Centi**	"	100th	" "
Kilo	"	1000	"	**Milli**	"	1000th	" "
Myria	"	10000	"				

[The abbreviations for the Greek prefixes used for the higher denominations are written with a capital letter while those for the Latin prefixes used for the lower denominations are written with a small letter.]

In this system the fundamental unit of length is the **Metre,** whence the system is called the *metric* system. The metre is equal to 39·370... inches, and was originally taken to be the ten-millionth part of a quarter of the terrestrial meridian. An error has, however, been since found in the measurement of the terrestrial meridian, and the metre therefore is not exactly the length it was stated to be. The metre was computed to be 39·37079... inches. The latest determination makes it **39·370113... inches,** but the last two figures are uncertain.

Table.

10 millimetres (mm.)	=	1 centimetre (cm.).
10 centimetres	=	1 decimetre (dm.).
10 decimetres	=	1 metre (m.).
10 metres	=	1 decametre (Dm.).
10 decametres	=	1 hectometre (Hm.).
10 hectometres	=	1 kilometre (Km.).
10 kilometres	=	1 myriametre (Mm.).

1 metre = $39\frac{3}{8}$ inches nearly = about **$1\frac{1}{11}$ yards** ; 1 kilometre = about 5 furlongs.

1 cm. = two-fifths or four-tenths of an inch nearly ; 1 dm. = 10 cm. = 4 inches roughly ; 1 inch = 2·5400... centimetres.

Note. *Short lengths* are usually expressed in *metres*, *decimetres* and *centimetres*, and *shorter lengths* in *centimetres* and *millimetres* and *long distances* in *kilometres* and *metres.*

309. Decimals and the Metric Table of Length.—We know that the local value of each digit throughout a decimal is ten times the local value of the next digit on the right, and one-tenth of that of the next digit on the left. In the Metric Table of Length the successive denominations are also related in the same way. Hence, with the decimal notation we can express a length measured by the metric system at once in terms of *any* denomination. Taking one metre as unit, we can prepare a table of Metric Linear Measure by writing

metre	under the	units'	column,
deca-metre		tens'	"
hecto-metre		hundreds'	"
kilo-metre		thousands'	"
deci-metre		tenths'	"
centi-metre		hundredths'	"
milli-metre		thousandths'	"

Thousands	Hundreds	Tens	Units •	Tenths	Hundredths	Thousandths
Km.	Hm.	Dm.	metre	dm.	cm.	mm.
5	5	5	5	5	5	5
1		2		3		4
	5		6		7	

Example 1. From the above table we have at once

(i) 5 Km. 5 Hm. 5 Dm. 5 m. 5 dm. 5 cm. 5 mm. = 5555·555 metres.

(ii) 1 Km. 2 Dm. 3 dm. 4 mm. = 1020·304 metres.

(iii) 5 Hm. 6 m. 7 cm. = 506·07 metres.

Example 2. 23564 m. 7 dm. 9 cm. 8 mm. = 23564798 mm.
= 2356479·8 cm. = 235647·98 dm. = 23564·798 m. = 2356·4798 Dm.
= 235·64798 Hm. = 23·564798 Km. = 2·3564798 Mm. = 2 Mm.
3 Km. 5 Hm. 6 Dm. 4·798 m.

EXAMPLES. 197.

(*Oral.*)

State the number of centimetres in

1. 81·25 metres. **2.** 50·17 metres. **3.** 91·29 metres.
4. 65·07 metres. **5.** 92·56 metres. **6.** 27·38 metres.

State the number of decimetres in

7. 32·6 metres. **8.** 90·72 metres. **9.** 65·37 metres.
10. 47·8 metres. **11.** 65·93 metres. **12.** 79·27 metres.

Read off in metres, decimetres, centimetres and millimetres :

13. 5·6 m. **14.** ·652 m. **15.** ·023 m.
16. 3·56 Dm. **17.** ·0066 Dm. **18.** 70·507 m.

Express in metres :

19. (i) 3015 mm., (ii) 5800 mm., (iii) 500 cm.,
(iv) 8751 cm., (v) 70 Hm., and (vi) 115 dm.

Express in centimetres :

20. (i) 382 m., (ii) 5001 m., (iii) 5000 Km.,
(iv) 3 mm., (v) 805 mm., and (vi) 23907 dm.

Express in kilometres :

21. (i) 1075 m., (ii) 5078 mm., (iii) 318 cm.,
(iv) 80000 m., (v) 710 Hm., and (vi) 567808 dm.

Read off as a decimal of a metre :

22. 6 m. 8 dm. **23.** 9 m. 6 cm. **24.** 5 m. 4 dm. 3 cm.
25. 8 Dm. 4 m. 2 dm. **26.** 6 dm. 3 cm. 8 mm.
27. 8 m. 5 cm. 3 mm. **28.** 5 Km. 6 Dm. 8 cm.
29. 7 Hm. 3 m. 9 mm. **30.** 1 Dm. 1 dm. 1 mm.

Read off as a decimal of one kilometre :

31. 6 Km. 3 Dm. 8 m. **32.** 8 Km. 4 Hm. 1 m.
33. 305 metres. **34.** 9 m. 35 cm.

Read off as a decimal of one centimetre :

35. 5 m. 3 cm. 4 mm. **36.** 1 dm. 8 mm.

37. 2 Dm. 3 m. 4 dm. 5 cm. 6 mm.

38. 4 Km. 6 Dm. 8 dm. 2 mm.

Express in metres :

39. (i) (34·25 × 10) cm., (ii) (429·708 × 100) cm., (iii) (78·5 ÷ 10) mm., and (iv) (500 ÷ 1000) mm.

Express in centimetres :

40. (i) (38·57 × 10) m., (ii) (108·705 × 1000) m., (iii) (·0007 ÷ 10) mm., and (iv) (78·067 ÷ 100) mm.

Express in millimetres :

41. (i) (300 × 10) m., (ii) (587 × 100) cm., (iii) (5 ÷ 100) cm., and (iv) (·008 ÷ 1000) cm.

310. The unit of area is the **square metre.** In measuring land the unit used is a *square decametre*, called an **are**, and the only multiple and submultiple used are the *hectare* (= 100 ares = a square hectometre) and the *centiare* (= $\frac{1}{100}$ of an are = a square metre).

TABLE.

100 sq. millimetres (mmq.)	=	1 sq. centimetre (cmq.).
100 sq. centimetres	=	1 sq. decimetre (dmq.).
100 sq. decimetres	=	1 sq. metre (mq.).
100 sq. metres	=	1 sq. decametre (Dmq.).
100 sq. decametres	=	1 sq. hectometre (Hmq.).
100 sq. hectometres	=	1 sq. kilometre (Kmq.).
100 sq kilometres	=	1 sq. myriametre (Mmq.).

1 centiare (ca.)	=	1 sq. metre.
100 centiares	=	1 are (a.) [= 1 sq. decametre.]
100 ares	=	1 hectare (ha.) [= 1 sq. Hm.]

1 sq. metre = 1550·0 sq. inches ; 1 sq. inch = 6·4516 sq. centimetres.

1 are = 1076·4 sq. feet nearly ; 1 hectare = 2½ acres roughly.

Example 1. 2 Dmq. 64 mq. 9 dmq. 34 cmq. = 2640934 cmq. = 26409·34 dmq. = 264·0934 mq = 2·640934 Dmq. = ·02640934 Hmq. = ·0002640934 Kmq.

Example 2. 73204 ca. = 732·04 a. = 7·3204 ha. = 7 ha. 32 a. 4 ca.

311. The unit of volume is the **cubic metre.** The multiples of the cubic metre are seldom used. In measuring wood the cubic metre is called a **stere,** and 10 steres make a *decastere.*

TABLE.

1000 cu. millimetres	=	1 cu. centimetre.
1000 cu. centimetres	=	1 cu. decimetre.
1000 cu. decimetres	=	1 cu. metre.

1 cu. metre = 1 stere ; 10 steres = 1 decastere.
1 cu. metre or stere = 35·3 cu. feet (nearly).

Example. 27·03567 cu. m. = 27035·67 cu. dm. = 27035670 cu. cm = 27 cu. m. 35 cu. dm. 670 cu. cm.

312. The unit of capacity, both for liquids and dry goods, is the **litre**, and is equal to a *cubic decimetre.*

TABLE.

10 millilitres (ml.)	=	1 centilitre (cl.).
10 centilitres	=	1 decilitre (dl.).
10 decilitres	=	1 litre (lit.).
10 litres	=	1 decalitre (Dl.).
10 decalitres	=	1 hectolitre (Hl.).
10 hectolitres	=	1 kilolitre (Kl.).

Since 1 litre = 1 cubic decimetre, 1000 litres = 1 kilolitre, and 1000 cubic decimetres = 1 cubic metre, ∴ 1 kilolitre = 1 cubic metre.

1 litre = 61·024... cu. inches = 1·759... pints = $1\frac{3}{4}$ pints nearly ; 1 kilolitre = 35·3 cu. feet (nearly).

Example. 3025·407 lit. = 30254·07 dl. = 302540·7 cl. = 3025407 ml. = 302·5407 Dl. = 30·25407 Hl. = 3·025407 Kl. = 3 Kl. 2 Dl. 5 lit. 4 dl. 7 ml.

313. The unit of weight is the **gram** (or *gramme*) which is the weight of a *cubic centimetre* of distilled water at its maximum density.

TABLE.

10 milligrams (mg.)	=	1 centigram (cg.).
10 centigrams	=	1 decigram (dg.).
10 decigrams	=	1 gram (gr.).
10 grams	=	1 decagram (Dg.).
10 decagrams	=	1 hectogram (Hg.).
10 hectograms	=	1 kilogram (Kg. or Kilo.).
10 kilograms	=	1 myriagram (Mg.).

Since 1 litre = 1000 cubic centimetres, and 1 kilogram = 1000 grams, ∴ the weight of a litre of water = 1 kilogram. The weight of a kilolitre (1 cubic metre) of water is 1000 kilograms and is called a *tonneau de mer* or *millier*. A *quintal* = 100 kilograms.

1 gram = 15·432... grains or $15\frac{1}{2}$ grains roughly; 1 kilogram = 2·2046... lb. Avoir. = $2\frac{1}{5}$ lb. Avoir. nearly.

314. The Metric units, with their relations with one another and their equivalents in British units, are collected together below for ready reference.

I. Unit of *Length* is the **Metre** = 39·370113... inches
= $39\frac{3}{8}$ inches nearly.

II. Unit of *Surface* is the **Are** = 1 sq. decametre
= 1076·4 sq. feet nearly
= $\frac{1}{40}$ acre roughly.

III. Unit of *Volume* is the **Stere** = 1 cu. metre
= 35·3 cu. feet nearly.

IV. Unit of *Capacity*, both for liquids and dry goods,
is the **Litre** = 1 cu. decimetre
= 61·024... cu. inches
= 1·759... pints or $1\frac{3}{4}$ pints nearly
= ·22 gallons nearly.

V. Unit of *Weight* is the **Gram** = the weight of a cu. centimetre of distilled water at 4°C.
= 15·432... grains
= $15\frac{1}{2}$ grains nearly
= ·0022... lb. Avoir.

315. French Money.

10 centimes (c.)	=	1 decime.
10 decimes	=	1 franc (fr.).

Accounts are kept in francs and centimes only; thus "32·78 francs" is read 32 francs 78 centimes.

The *Franc* is a silver coin composed of 9 parts of silver and 1 part of copper, and weighs 5 grams. It is equal to $9\frac{3}{5}d.$ nearly. The *Napoleon* is a gold coin = 20 francs.

THE PROPOSED DECIMAL COINAGE OF GREAT BRITAIN.

10 mils (m.) = 1 cent (c.); 10 cents = 1 florin (f.); 10 florins = £1.

316. The great advantage of a decimal system of weights and measures is, as we have seen, that a compound quantity can be

reduced to a simple quantity, and *vice versa*, without going through the processes of multiplication and division. Hence compound rules are replaced by the corresponding simple rules.

Example 1. Express 7 hectares 34 ares 6 centiares as a decimal of a sq. kilometre.

7ha. 34a. 6ca. = 73406 ca. = 73406 sq. metres = 734·06 sq. decametres = 7·3406 sq. hectometres = ·073406 sq. kilometres.

Example 2. A wheel makes 1230 revolutions in passing over 2 kilometres 5 hectometres 9 metres 2 decimetres; what is its circumference?

2 Km. 5 Hm. 9 m. 2 dm. = 2509·2 m.; 2509·2 ÷ 1230 = 2·04;
∴ the circumference reqd. = 2·04 metres = 2 metres 4 centimetres.

Example 3. A cubic foot of alcohol weighs 94 lb.; find the weight of a litre in grams, supposing a litre to be equal to ·035 cu. ft. and a gram 15·43 grains.

Weight of a litre of alcohol = ·035 × 94 lb.

$$= \cdot 035 \times 94 \times 7000 \text{ grains}$$

$$= \frac{\cdot 035 \times 94 \times 7000}{15\cdot 43} \text{ grams}$$

$$= 1492\cdot 5\ldots \text{ grams.}$$

Example 4. Cloth is sold at 21 fr. 80 c. per metre; what is the corresponding price per yard in English money, if £1 be worth 25 fr. 25 c.? [1 metre = 39·37 inches.]

$$1 \text{ yard} = 36 \text{ inches} = \frac{36}{39\cdot 37} \text{ metres};$$

$$\therefore \text{ cost of } 1 \text{ yard} = \frac{36 \times 2180}{39\cdot 37} \text{ centimes}$$

$$= £\frac{36 \times 2180}{39\cdot 37 \times 2525}$$

$$= 15s.\ 9\tfrac{1}{2}d. \text{ nearly.}$$

Example 5. Add together 42·7 m., 32·8 dm., 47·9 cm. and express the result in metre, dm. etc.

42·7 m. =	42·7	metres
32·8 dm. =	3·28	"
47·9 cm. =	·479	"
The required sum =	46·459	metres

= 46 metres 4 dm. 5 cm. 9 mm.

Example 6. Add together £3. 7f. 2c. 3m., £9. 2f. 0 c. 4m. and 7f. 3c.

Mils

3723
9204
 730
13657 mils = £13. 6f. 5c. 7m. *Ans.*

Example 7. Multiply 7f. 9c. 3m. by 32.

Mils

 793
  32
1586
2379
25376 mils = £25. 3f. 7c. 6m. *Ans.*

EXAMPLES. 198.

1. Add together and express the result in metre, dm., etc.

(i) 5·7 m., 35·9 dm., 68·6 cm. (ii) 7·5 m., 92·8 dm., 72·5 cm.

(iii) 56 dm., 59·8 cm., 62 mm.

(iv) 18·3 dm., 45·9 cm., 53 mm. (v) 192·3 cm., 253 mm.

2. Subtract

(i) 3·72 m. from ·532 Dm. (ii) 9·82 m. from ·623 Km.

(iii) 15·93 cm. from ·429 Dm.

3. Multiply

(i) 32 cm. 5 mm. by 7. (ii) 74 dm. 3 cm. by 9.

(iii) 36 cm. 8 mm. by 6. (iv) 14 dm. 4 cm. by 5.

and in each case express the result in (*a*) metres, dm., etc. (*b*) as decimal of a metre.

4. Add together

(i) £4. 7f. 3c. 4m., £9. 3f. 4m. and 9f. 4c.

(ii) £7. 4f. 5c. 3m., £4. 9f. 6m., £3. 2c. 7m. and 5c. 8m.

(iii) £3. 7f. 4c. 3m., £9. 4f. 3c. 7m., £4. 3c. 6m. and 3f. 4c. 8m.

(iv) £4. 9f. 5c. 2m., £5. 6f. 2c., £9. 3f. 2c. 4m. and 5f. 3m.

(v) £7. 3f. 4c. 2m., £6. 3f. 5m., £8. 6c. and £8. 5f. 2m.

5. Subtract

(i) £4. 3f. 2m. from £7. 2f. 3c.

(ii) £3. 2f. 7c. 4m. from £6. 3f. 2m.

(iii) £7. 3f. 5m. from £9. 4f. 3c. 2m.

(iv) £6. 4f. 6c. 7m. from £9. 3f. 2m.

(v) £3. 6f. 9m. from £5. 2f. 3c.

6. Multiply

(i) 6f. 9c. 3m. by 25. (ii) £3. 7f. 2c. 4m. by 37.

(iii) £8. 1f. 4m. by 49. (iv) £7. 2f. 3c. 2m. by 25.

(v) £3. 7f. 5m. by 38.

Reduce

7. 2305000 millimetres to kilometres.

8. 304007 centimetres to kilometres, etc.

9 1203270 millimetres to decametres, etc.

10. 75 kilometres 7 decametres 3·05 metres to millimetres.

11. 3070·5086 decametres to kilometres, etc.

12. 23 sq. kilometres 6 sq. decametres 7 sq. metres to sq. metres.

13. 50 sq. kilometres 6 sq. hectometres 4 sq. metres to sq. decametres.

14. 40740 centiares to hectares, etc.

15. 8 hectares 7 ares to centiares.

16. 36·307 sq. hectometres to hectares, etc.

17. 3012035 cu. centimetres to cu. metres, etc.

18. 5 cu. metres 27 cu. decimetres 4 cu. centimetres to cu. millimetres.

19. 40700302 millilitres to kilolitres, etc.

20. 3040600 centigrams to myriagrams, etc.

21. 1375 centimes to francs, etc.

22. A man walks 7·92 kilometres in 2 hours; how many metres does he walk in a second?

23. The circumference of a bicycle wheel is 4 metres 8 centimetres; how many times will it revolve in going 16·83 kilometres?

24. If 25 horses eat 676 kilo. 575 gr. of corn in 9 days, how long will 240 kilo. 560 gr. serve 16 horses?

25. The weight of 226 equal parcels is 1 tonneau 921 kilograms; find the weight of each.

26. If 27 decalitres 8 centilitres of wine cost 67 francs 52 centimes; find the cost of 15 litres.

317. Miscellaneous Examples.

Example 1. Express £7. 15s. 7½d. in decimal coinage.

$$\begin{array}{r|l} 4 & 2\cdot0 \\ 12 & 7\cdot5 \\ 20 & 15\cdot625 \\ & £7\cdot78125 \end{array}$$

$£7\cdot78125 = £7.$ 7f. 8c. 1·25m. *Ans.*

Example 2. Express £9 3f. 9c. 8m. in £. s. d.

$$\begin{array}{r} £9\cdot398 \\ 20 \\ \hline s.\ 7\cdot960 \\ 12 \\ \hline d.\ 11\cdot520 \end{array}$$

∴ £9. 3f. 9c. 8m. = £9. 7s. 11·52d.

Example 3. Assuming a metre to be $39\frac{3}{8}$ inches, find the nearest whole number of litres in one cubic foot. [C. U. 1911.]

$$1 \text{ m.} = 39\tfrac{3}{8} \text{ inches} = \tfrac{315}{8} \text{ in.}$$

$$= \tfrac{315}{8 \times 12} \text{ ft.} = \tfrac{105}{32} \text{ ft.}$$

$$1 \text{ cu. metre} = \left(\frac{105}{32}\right)^3 \text{ cu. ft.}$$

But 1 cu. metre = 1000 cu. dm.

= 1000 litres.

$$\therefore \left(\frac{105}{32}\right)^3 \text{cu. ft.} = 1000 \text{ litres,}$$

$$\therefore \quad 1 \text{ cu. ft.} = 1000 \times \left(\frac{32}{105}\right)^3 \text{ litres}$$

$$= \frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{10}} \times \overset{2}{\cancel{10}} \times 32 \times 32 \times 32}{\underset{21}{\cancel{105}} \times \underset{21}{\cancel{105}} \times \underset{21}{\cancel{105}}} \text{ litres}$$

$$= \frac{262144}{9261} \text{ litres}$$

= 28·3 litres

= 28 litres (approximately).

Example 4. A metre is 39·37 inches. If this is a ten-millionth part of the distance of the equator from the pole, how far is it round the world, the measurement being taken along a meridian? (Give the result in miles, furlongs, yards, etc.)

Let x in. be the distance of the equator from the pole, then $4x$ in. is the distance round the world.

Now, $\frac{x}{10000000} = 39 \cdot 37$,

$\therefore \quad x = 393700000,$

$\therefore \quad 4x = 1574800000.$

```
                     12|1574800000 inches
                      3|131233333 ft.........4 inches
                     ( 2|43744444 yd.......1 ft.
      220            { 11|21872222
(220 yds. = 1 fur.   ( 10| 1988383...........9 }   (3×11×2)+(9×2)
                        8| 198838 fur......3   }   or 84 yd.
                           24854 miles...6 fur.
```

$\therefore$ The distance reqd. $= 4x$ in. $=$ 24854 mi. 6 fur. 84 yd. 1 ft. 4 in. nearly.

Example 5. The third class railway fare in France is 5 centimes per kilometre and in England 1*d.* per mile. Given that 1 yard $=$ 0·9144 metre and £1 $=$ 25·17 francs, find (in English money) the difference of the fares for a journey of 100 miles in the two countries, correct within a farthing. [C. U. 1921.]

In England the fare for 100 miles $=$ 100*d.* $=$ 8*s.* 4*d.*

1 yard $=$ ·9144 metres,

$\therefore$ 100 miles $= 1760 \times 100$ yd.

$= 1760 \times 100 \times \cdot 9144$ metres

$= 176 \times 914 \cdot 4$ metres.

Fare for one kilometre or 1000 metres $=$ 5 centimes (in France)

$= \frac{5}{100}$ francs

$= £\frac{5}{100} \times \frac{1}{25 \cdot 17}.$

$\therefore$ the fare for 1 metre $= £\frac{5}{1000 \times 100 \times 25 \cdot 17}.$

$\therefore$ the fare for $176 \times 914{\cdot}4$ metres $= £\dfrac{5 \times 914{\cdot}4 \times {\cdot}176}{1000 \times 100 \times 25{\cdot}17}$

$$= \frac{5 \times 9{\cdot}144 \times {\cdot}176}{25{\cdot}17} \times 20s.$$

$$= \frac{100 \times 9{\cdot}144 \times {\cdot}176}{25{\cdot}17}s.$$

$$= \frac{914{\cdot}4 \times {\cdot}176}{25{\cdot}17}s.$$

$$= \frac{9144 \times 176}{251700}s.$$

$= 6s.\ 4d.\ 2{\cdot}9f.$

$\therefore$ The difference $= 8s.\ 4d. - 6s.\ 4d.\ 2{\cdot}9f.$

$= 1s.\ 11d.\ 1{\cdot}1f. = 1s.\ 11d.\ 1f.$ nearly.

Example 6. A decimetre is equal to 3·937 inches and a cubic inch of water weighs 252·45 grains. Express a kilogram in pounds Avoir. correct to two decimal places. [One pound Avoir. = 7000 grains.]

One kilogram = 1000 gram

= the wt. of a litre of water

= the wt. of a cubic decimetre of water

= the wt. of $(3{\cdot}937)^3$ cubic inches of water

$= (3{\cdot}937)^3 \times 252{\cdot}45$ grains

$= \dfrac{(3{\cdot}937)^3 \times 252{\cdot}45}{7000}$ lb. (Avoir.) $= \dfrac{(3{\cdot}937)^3 \times {\cdot}25245}{7}$ lb.

$= 2{\cdot}20$ lb.

[$3{\cdot}937 = 4$ approximately. $\therefore$ $4 \times 4 \times 4 = 64$. This is between 10 and 100; so it will affect the last two figures in the product. Hence to find the result correct to 2 places of decimals, the calculation should be carried on up to 4 decimal places.]

·25245	·9940	3·9133
7393	7393	7393
·7574	2·9820	11·7399
·2272	·8946	3·5220
·0076	·0298	·1174
·0018	·0069	·0274
·9940	3·9133	7)15·4067
		2·2009

EXAMPLES. 199.

1. An estate containing 30 hectares 50 ares is divided into 1000 fields of equal area ; find the area of each.

2. How much wheat at 19 francs 55 centimes per hectolitre ought to be given in exchange for 312 hectolitres 80 litres of barley at 1 franc 25 centimes per decalitre ?

3. Express a yard in terms of the metre, supposing a metre to be equal to 39·37 inches.

4. Express a kilometre as a decimal of a mile, if a metre be 39·37 inches.

5. The standard height of the barometer is 760 min. Find this height in inches. [1 metre = 39·3701 inches.]

6. Express a pound Avoir. in grams, a gram being equal to 15·43 grains.

7. If a cubic inch of air weigh ·31 grains, what will be the weight in grains of a litre of air, having given that a cubic metre is equal to 35·3 cubic feet, and a gram 15·43 grains.

8. A gallon of water weighs 10 lb. ; find its volume in cubic centimetres, supposing a kilogram to be equal to $2\frac{1}{5}$ lb.

9. Mahogany is 55 lb. to the cubic foot ; find the weight of a decastere of Mahogany in tonneaux and kilograms, supposing a cubic metre to be 35·3 cubic feet, and a kilogram $2\frac{1}{5}$ lb.

10. An inch is 2·54 centimetres, and a kilogram is 2·2 lb. ; find the pressure of the atmosphere in grams per sq. centimetre, supposing it to be 15 lb. Avoir. to the square inch.

11. If a kilolitre be 220 gallons, find the value in English money, of a pint of a liquid which is worth 33 francs the decilitre, 1200 francs being equal to £47.

12. A decimetre is equal to 3·937 inches, and a cubic inch of water weighs 252·45 grains. Express a kilogram in pounds Avoir. correct to two decimal places.

13. A gallon contains 277·274 cubic inches, a cubic decimetre is 61 cubic inches and a kilogram is $2\frac{1}{5}$ lb. ; calculate the weight in pounds of a gallon of water.

14. The area of a room is 43·68 sq. metres, and its length is 832 centimetres ; find its breadth.

15. Find the length of a piece of ribbon which has a surface of 15·75 sq. dm., and which is 1·5 cm. broad.

16. Find the value per metre of cloth, when a piece 37 m. 2 dm. 5 cm. long is worth 186 fr. 25 c.

17. Find the number of cubic centimetres (1) in a litre, (2) in a stere.

18. A plank is 3 m. long, 5 dm. broad and 2·5 cm. thick; find the volume of the plank.

19. A room is 7·24 metres long, 4·21 metres broad, and contains 121921600 cubic centimetres of air; find the height of the room.

20. Find the weight of a litre of water in grams.

21. Given that 1 lb. = 7000 grains, and 1 gram = 15·4 grains, find the number of grams in an ounce, correct to 2 decimal places.

22. *Specific gravity* of a substance being the ratio of the weight of any volume of the substance to the weight of the same volume of water, find the specific gravities of mercury and alcohol, having given that a decalitre of mercury weighs 136 kilograms and a centilitre of alcohol weighs 8 grams.

23. A plate of iron 55 cm. long and 43 cm. broad weighs 26961 grams. Find the thickness of the plate, if iron is 7·6 times as heavy as water.

24. If a gallon of water weighs 70,000 grains, and 1 kilogram = 15432 grains, how many times can a litre measure be filled from a 3-gallon cask and what decimal of a litre will be left in the cask?

25. If the weight of a cubic foot of water = $62\frac{1}{2}$ lb., and 1 kilogram = 2·2 lb., find the number of cubic feet in a cubic metre.

26. Reduce 5 m. 2 dm. 3 cm. to yards, feet and inches, taking 1 metre as equal to 39·37 inches.

27. Find the area of the field 145 metres long and 84 metres broad, and express it in ares.

28. A cistern 12 metres long and 7 metres wide holds 103656 kilograms of water. Find the depth of the water.

29. Given that a cubic foot of water weighs 1000 ounces, and an inch = 2·54 centimetres, find the nearest whole number of grams in 1 lb.

30. Given that iron is 7·5 times as heavy as water, find the weight in kilograms of a sheet of iron 3·4 metres long, 2·5 metres broad and 1 centimetre thick.

31. A rectangular cistern 3·2 metres long and 2·3 metres broad has a capacity of 11040 litres. Find the depth of the cistern.

32. If 1 franc = 9·4 annas, and 1 kilogram = 1·07 seers, find in francs the price of a kilogram of an article which costs a rupee a seer.

33. A sq. yard = ·84 sq. metre, and £1 = 25 francs. An estate measuring 1848 hectares is sold for five million francs. What is this in pounds per acre? [1 acre = 4840 sq. yd.]

34. A room is 18 metres long and 9 metres broad. Find its area in square yards (correct to two places of decimals), taking a metre to be equal to 39·37 inches.

35. A room is 10 feet 6 inches long, and 5 feet 4 inches broad. Find its area in square metres, taking a metre as equal to 39·37 inches.

36. Given 1 metre = 39·37 inches:

(i) Express an inch in centimetres correct to two places of decimals.

(ii) Express a sq. metre in sq. inches correct to two places of decimals.

(iii) Find the nearest whole number of cubic inches in a litre.

(iv) Find the nearest whole number of litres in a cubic foot.

37. Given 1 inch = 2·5 centimetres approximately, the weight of a cubic foot of water = $62\frac{1}{2}$ lb., and 1 lb. = 7000 grains, find the nearest whole number of grains in a gram.

38. Given 1 gram = 15·43 grains approximately, and 1 tola = 180 grains, express a seer of 80 tolas in grams, correct to two places of decimals.

39. Find the weight of a hectolitre of mercury which is 13·6 times as heavy as water.

40. Given 1 gram = 15·432 grains, 1 lb. = 7000 grains, and 1 seer = 14400 grains, shew that, approximately, 5 kilograms = 11 lb., and 14 kilograms = 15 seers.

41. Given 1 metre = 39·37 inches, shew that 981 centimetres = 32 feet nearly.

42. The driving wheel of a locomotive is 12·5 metres in circumference, and it makes 2·5 revolutions in a second; how long will it take to travel 100 miles, if 1 mile = 1·6 kilometres.

43. Find the value of

0·34 kilograms + 9·4 grams + 600 milligrams as the decimal of a pound; given 1 gram = 15·432 grains, and 1 lb. = 7000 grains.

44. Find, to the nearest litre, the content of a tank 3·21 metres long, 2·15 metres broad, and 54 centimetres deep.

LVII. GRAPHICAL METHOD.

SEC. I.—GRAPHICAL REPRESENTATION OF STATISTICS.

318. It is often found convenient and attractive to represent the facts and figures of certain phenomena by graphs; such representation gives a vivid and clear conception of the nature of those phenomena according to the data furnished.

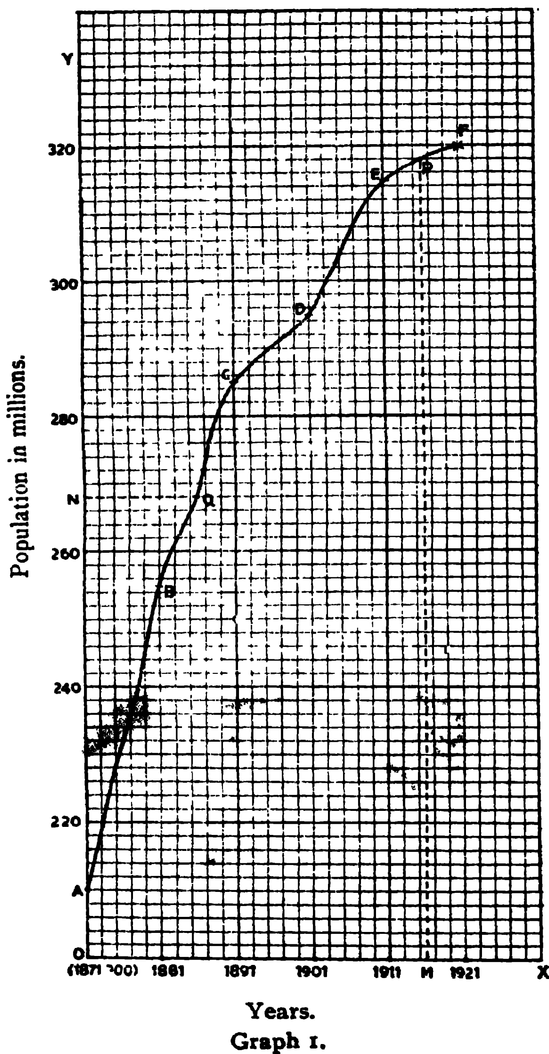
The following examples will explain how some statistical figures may be represented by graphs :—

Example 1. The following table in round figures gives the growth of population in India during the years 1871 – 1921. Represent this graphically, and from the graph drawn find the probable population in the year 1916 as also the probable year in which the population was 268 millions.

Year.	Population (in mill.)	Year.	Population (in mill.)
1871	210	1901	295
1881	255	1911	315
1891	285	1921	320.

Here, population less than 200 millions and years less than 1871 are not wanted. Hence let the length of a side of a small square measured horizontally represent two years and an equal length measured vertically represent 2 millions. The origin is taken at the point (1871,200) and the meaning of the figures in the diagram (Graph No. 1) is thus clear. Now population in 1871 which is 210 millions is represented by a point on OY which is obtained by counting 5 divisions (because 210 mill. – 200 mill. = 10 mill. = 5 divs.) from O along OY; mark the point. This point is 5 divs. above the level OX which represents 200 millions. Now consider the population in 1881 which is 255 mill. Hence it should be represented by a point half of (255 – 200) or $27\frac{1}{2}$ divs. above the line OX along the vertical line representing the year 1881; mark the point. Or, since the vertical line OY has been marked off 10 divs. apart, we can see from the graph at once the position of the line parallel to OX representing 260 mill. Hence by counting $2\frac{1}{2}$ divs. only (representing 5 mill.) *below* this line along the vertical line representing the year 1881 we can obtain and plot the point which shall represent the population 255,000,000 in 1881 without much trouble.

Similarly for the years 1891, 1901, 1911 and 1921, the points representing populations for those years can be plotted by counting $42\frac{1}{2}$, $47\frac{1}{2}$, $57\frac{1}{2}$ and 60 divisions respectively above the line OX representing the successive populations along the verticals, of the respective years.


Y
320
300
280
N
260
240
220
A
O
Population in millions.
B
Q
G
D
E
P
F
(1871 200)
1881
1891
1901
1911
M
1921
X
Years.

Graph 1.

Now draw through these points free-hand a *continuous* curve, as represented in the figure. The resulting curved line is the required graph which shows the nature of the phenomenon we are investigating. The data furnished regarding the growth of the population in India no doubt gives an idea, but this graphical representation gives a *vivid* and *ready* grasp of that idea like a picture of some phenomenon or happening.

Now, it is evident from the graph that the vertical corresponding to the year 1916 (marked M in the figure, which is $2\frac{1}{2}$ divs. from 1911) is about 59 small divisions which means that the population was probably 200+118 or 318 millions in that year. This is represented by PM, and P is the point (1916,318). Again, the point on the curve which corresponds to 268 millions is Q, whose abscissa is found to be 1886. This is represented by NQ, and Q is the point (1886,268).

Note. It may be reasonably supposed that the growth of population is *gradual* and not sudden ; hence is the necessity of drawing a *continuous* curve, that is one not having sharp turns.

Example 2. The rainfall at a certain place for a number of consecutive years is given in the following table. Represent graphically. What is the average (or normal) rainfall ? Show in a graph the deviations in each year from the normal rainfall indicating increase by the plus sign and decrease by the minus sign.

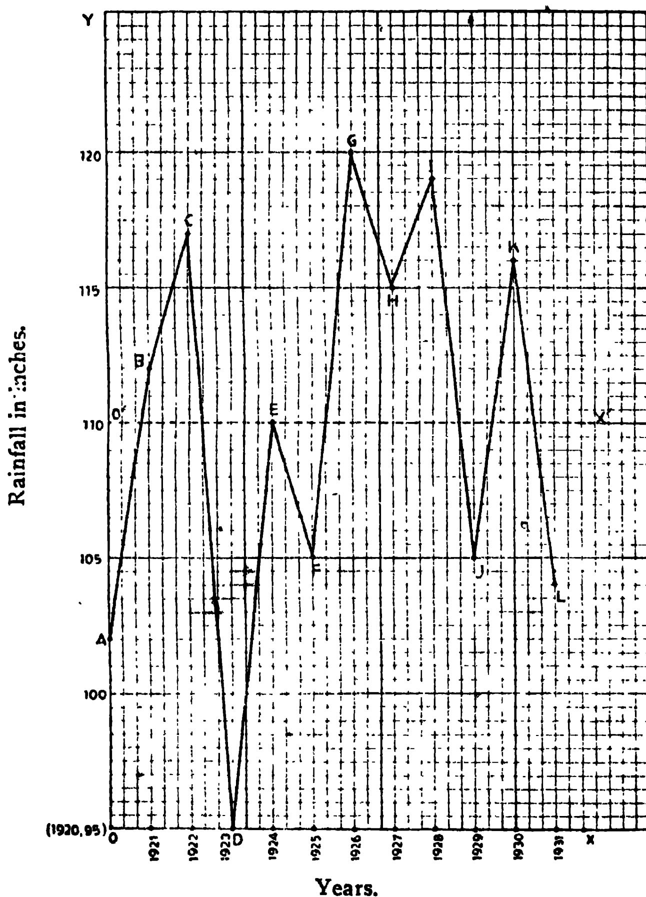
Year.	Rainfall in inches.	Year.	Rainfall in inches.
1920	102	1926	120
1921	112	1927	115
1922	117	1928	119
1923	95	1929	105
1924	110	1930	116
1925	105	1931	104.

Let years be represented along OX, taking 3 divs. for each interval of 1 year beginning from 1920 at the origin and let the rainfall be represented along OY, taking 2 divs. to represent 1 inch of rainfall beginning with 95 inches at O which is the lowest on record.

The same could have been done in the first example with respect to population taking O of OY to represent 210 millions of population, that being the lowest on record during the years to be considered. But we did not do that only to show that we can begin at any point we like according to our choice and

convenience and also for the neatness of the graph to be obtained.

The origin is thus at the point (1920,95) and the meaning of the figures in the diagram (Graph No. 2) is thus clear.



Graph 2.

In the year 1920 the rainfall is 102 in. and taking 2 divs. along OY to represent rainfall of 1 inch, we plot the point representing 102 inches of rainfall on OY counting 14 divs. above OX or counting 4 divs. above the point marked 100 in. on OY. We proceed as in the previous example and complete the graph by joining the successive points. The resulting graph is shewn as annexed.

The normal rainfall in this place is obtained by dividing all the rainfalls from 1920 to 1931 by 12, which is the number of years. It is in this case 110. If we take the new origin at 110, and draw the new X-axis $O'X'$ (see Graph No. 2), the variations from the normal would manifestly be exhibited.

Note. The rainfall in a place being purely casual, the curve is necessarily zigzag or *broken*, and not continuous as in Graph No. 1.

Example 3. Table below shows the growth of expenditure on Education in the principal provinces of British India in round figures in lakhs of rupees from 1917—1927.

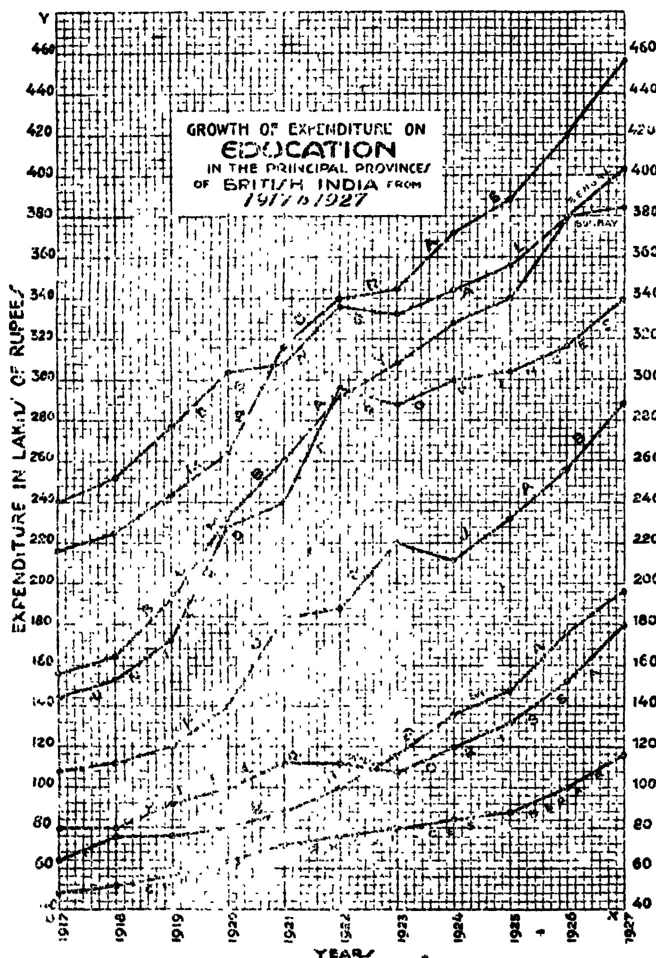
Draw the several graphs representing the growth of expenditure on education in each of the said provinces.

	C. P. & Berar.	Burma.	B. & O.	Pun.	U. P.	Bom	Mad.	Ben.
1917	43,	64,	80,	1,08,	1,44,	1,56,	2,16,	2,40.
1918	52,	70,	80,	1,12,	1,52,	1,64,	2,24,	2,52.
1919	56,	75,	92,	1,20,	1,72,	1,92,	2,44,	2,76.
1920	64,	80,	1,00,	1,40,	2,28,	2,32,	2,64,	3,04.
1921	72,	88,	1,12,	1,84,	2,40,	2,60,	3,16,	3,08.
1922	76,	1,00,	1,12,	1,88,	2,96,	2,92,	3,40,	3,36.
1923	80,	1,16,	1,08,	2,20,	2,88,	3,08,	3,44,	3,32.
1924	84,	1,36,	1,20,	2,12,	3,00,	3,28,	3,72,	3,44.
1925	88,	1,48,	1,32,	2,32,	3,04,	3,40,	3,88,	3,56.
1926	1,00,	1,76,	1,52,	2,56,	3,16,	3,80,	4,20,	3,80.
1927	1,16,	1,96,	1,80,	2,88,	3,40,	3,84,	4,56,	4,04.

Let years be represented along OX, taking 7 divisions for each year beginning with the year 1917 and expenditure in lakhs of rupees be represented along OY taking 1 division to represent 4 lakhs of rupees.

After plotting the points representing the successive expenditures for these years in each province, we complete the graph required for that province by joining the points consecutively by straight lines.

The origin is thus at (1917, 40) and the meaning of the figures in the diagram (Graph No. 3) is thus clear.



Graph 3.

The student will do well to follow the plotting of points according to the data given and thus get a good exercise while verifying the annexed graph.

Example 4. In an observatory the temperature of a certain day was recorded as follows :—

6 A. M.	75°F.	6 P. M.	100°F.
8 A. M.	78°F.	8 P. M.	94°F.
10 A. M.	82°F.	10 P. M.	90 F.
1 Noon	87°F.	12 P. M.	80°F.
2 P. M.	97°F.	2 A. M.	79°F.
4 P. M.	102°F.		

Represent graphically the temperature of the day at different times during the day and read off from the graph the *probable* temperature of the day at 11 A. M. and 11 P. M.

Let OX represent the different hours of the day, 2 divisions representing 1 hour, beginning with 6 A. M. and let OY represent the corresponding temperatures, 2 divisions representing also 1°F. beginning with 75°F. which is the lowest on record. The origin is thus at (6 A. M., 75°) and the meaning of the figures in the diagram (Graph No. 4) will be clear.

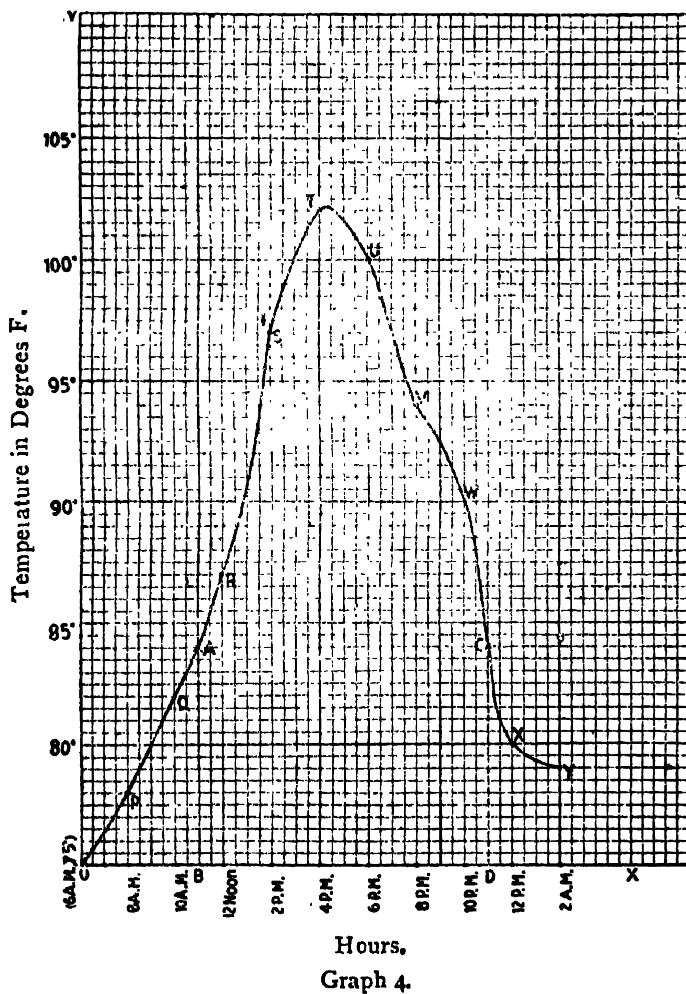
Plot the points according to the data given as in the previous examples and join them *continuously* by drawing freehand the curve $O\ P\ Q\ R\ S\ T\ U\ V\ W\ X\ Y$ as represented in the figure, for reasons previously explained in *Ex. 1*. The student will also perceive that for the same reasons the maximum temperature recorded *i.e.*, 102°F., is most probably not the true maximum but it is rather something higher before it falls down to 100°F., after 2 hours.

To find the temperatures at 11 A. M. and 11 P. M. we draw verticals through those hours represented by the points B and D on OX to meet the curve at two different points. The distances of these points A and C from the line OX are measured which give the respective temperatures of the day at those two hours.

From the graph they appear to be 84° each.

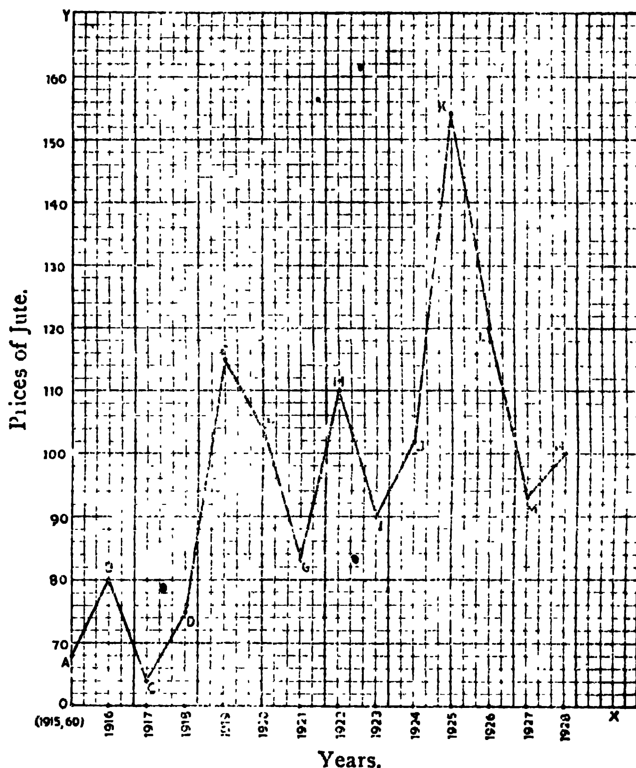
Example 5. The wholesale price in Calcutta of Jute in 1914 is taken to be 100. Prices in the successive years are tabulated below compared to the above. Represent graphically.

1915	68	1922	110
1916	80	1923	90
1917	64	1924	102
1918	75	1925	154
1919	115	1926	120
1920	104	1927	93
1921	83	1928	100.


Y
105°
100°
95°
90°
85°
80°
Temperature in Degrees F.
T
U
S
V
W
R
A
Q
C
X
Y
P
C
(6A.M. 75°)
8A.M.
10A.M.
B
12 Noon
2P.M.
4P.M.
6P.M.
8P.M.
10P.M.
D
12P.M.
2A.M.
X
Hours.

Graph 4.

The graph (No. 5) is shown here. The student should follow the plotting of points.



Graph 5.

EXAMPLES. 199a.

1. The yearly income of a person for 6 consecutive years is given below ; represent them graphically.

1925	₨3,000	1928	₨3,300
1926	₨3,100	1929	₨3,200
1927	₨3,150	1930	₨3,250.

2. The average annual prices of wheat in the United Provinces for a period of successive years are given below noted against each year. Draw a graph showing the fluctuations during those years. The prices given are for each 1 maund of the commodity.

Year.	Average price of 1 md.	Year.	Average price of 1 md.
1925	R5	1929	R3-8-0
1926	R4-8-0	1930	R3.8-0
1927	R4-4.0	1931	R3-4-0
1928	R4-0-0	32	R3-0-0

3. The temperature of an in-door patient in a certain hospital was recorded as given in the table below; draw a graph showing the variations in temperature of the patient at different hours of the day and read off the prob ble temperatures of the patient at 11 A. M. and 11 P. M.

8 A. M.	98°F.	6 P. M.	103°F.
10 A. M.	99°F.	8 P. M.	101°F.
12 Noon	103°F.	10 P. M.	100°F.
2 P. M.	103·5°F.	12 A. M.	99 F.
4 P. M.	104°F.	2 A. M.	97 F.

4. The average time of sun-rise (in hours and minutes) at Calcutta during each month is given below; represent graphically the variations throughout the year.

Month.	Hours of sun-rise.	Month.	Hours of sun-rise.
January	6-46 min.	July	5-26 min.
February	6-37 „	August	5-37
March	6-14 „	September	5-48
April	5-45 „	October	6-0
May	5-25 „	November	6-17
June	5-18 „	December	6-36

5. The average monthly rainfall (in inches) of Tainta in 1933 is given in the following table. Draw a graph showing the variations in that year.

Month.	Rain-fall. (in inches)	Month.	Rain-fall. (in inches)
January	·5	July	6·5
February	·3	August	3·7
March	·6	September	2·8
April	1·8	October	3·7
May	2·1	November	1·3
June	5·9	December	·2.

6. The population of a town which was 20,400 in 1914 increased to as shown below in the next successive years ; draw a graph showing the nature of the increments.

Year.	Population.	Year.	Population.
1915	20,900	1921	16,300
1916	11,600	1922	18,300
1917	12,400	1923	20,500
1918	13,800	1924	22,900
1919	14,470	1925	25,400
1920	15,500	1926	29,400.

7. The following table gives the Hindu and Mahomedan populations of Serajganj (in round figures) from 1923 to 1932 :—

Year.	Hindu.	Maho.	Year.	Hindu.	Maho.
1923	10,000	12,100	1928	13,000	17,500
1924	10,200	12,700	1929	13,400	18,000
1925	10,900	13,300	1930	14,200	19,000
1926	11,500	14,500	1931	14,300	19,300
1927	12,000	16,000	1932	14,700	19,900.

Draw the two graphs showing the variations in the populations of the two communities concerned during the periods mentioned.

8. The following table shows the new business effected by Indian life offices since 1922 in each year and the total business remaining in force at the end of the year.

Year.	New business written during the year.	Total business remaining in force at the end of the year.
1922	5,64 lakhs.	37 crores.
1923	5,85 ,,	39 ,,
1924	6,89 ,,	42 ,,
1925	8,15 ,,	47 ,,
1926	10,35 ,,	53 ,,
1927	12,77 ,,	60 ,,
1928	15,41 ,,	71 ,,
1929	17,29 ,,	82 ,,
1930	16,50 ,,	89 ,,
1931	17,76 ,,	98 ,,

Draw graphs showing the two kinds of business done during those years.

9. The price of cotton in Calcutta in 1914, is taken to be 100; the prices for the subsequent years are noted below compared to the above. Draw a graph showing the fluctuations.

Year.	Prices compared to that of 1914 which is taken to be 100.	Year.	Prices compared to that of 1914 which is taken to be 100.
1915	89	1922	191
1916	121	1923	244
1917	174	1924	272
1918	309	1925	205
1919	230	1926	147
1920	152	1927	167
1921	143	1928	167.

10. The rate of premium of life insurance in a certain company to realise R2,000 payable at death is given in the following table for different ages of the policy-holder at which the policies are issued.

Age in years.	Annual Premium (to the nearest anna).
20	R60. 4*a*.
25	R67. 7*a*.
30	R76. 14*a*.
35	R84. 12*a*.
40	R99. 10*a*.
45	R120. 8*a*.
50	R145. 10*a*.

Show graphically the variations in the rates of the premium at different ages of the policies effected and from the graph find out the probable premiums required to be paid by intending persons 27, 38 and 47 years of age.

11. Exports from India of raw and manufactured jute from 1919-20 to 1933-34 are given below. Represnt graphically the quantities of the two commodities exported in those years.

Year.	Export from India (in thousands of tons). Raw jute.	Manufactured.
1919-20	592	652
1920-21	472	833
1921-22	467	641
1922-23	578	672
1923-24	660	747
1924-25	696	812
1925-26	647	811
1926-27	708	860
1927-28	892	885
1928-29	898	911
1929-30	807	958
1930-31	620	766
1931-32	587	663
1932-33	563	680
1933-34	748	672.

12. Total expenditure on education in British India from the Government and semi Government funds in every fifth year from 1877-78 to 1917-18 and annually thereafter to 1926-27 is furnished in round figures in the following table; represent graphically.

Year.	Total Expenditure (in lakhs of rupees.)	Year.	Total Expenditure (in lakhs of rupees.)
1877-78	1,60	1918-19	13,20
1882-83	2,00	1919-20	14,80
1887-88	2,80	1920-21	17,20
1892-93	3,20	1921-22	18,80
1897-98	3,60	1922-23	19,20
1902-03	4,20	1923-24	20,40
1907-08	6,40	1924-25	21,60
1912-13	9,60	1925-26	23,20
1917-18	11,60	1926-27	24,80.

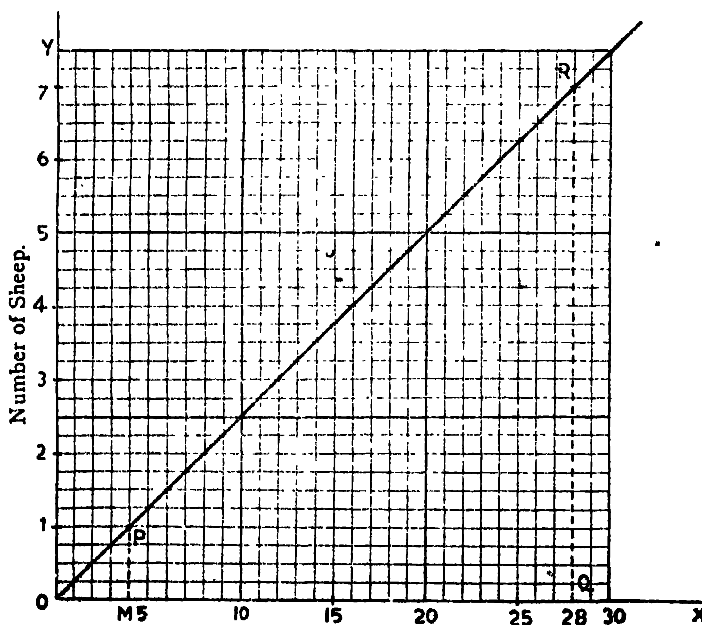
SEC. II—GRAPHICAL PROBLEMS.

319. Some arithmetical problems can be solved graphically. But it should be remembered that by this method only an approximate result is obtained which must be verified by the arithmetical solution giving an exact or correct answer. The following examples will explain the method of Graphical solution.

Example 1. If one sheep costs ₨4, how many sheep cost ₨28 ?

Let prices be represented along OX, taking 1 div. to represent ₨1 and sheep be represented along OY, taking 4 divs. for 1 sheep.

Since 1 sheep costs ₨4, we count 4 divs. from O along OX to get at the length OM representing ₨4 and then count 4 divs. up along the vertical (ordinate) through M to get at the point representing 1 sheep (4 divs. = 1 sheep.). (See Graph No. 6.)



Price in rupees.

Graph 6.

Let the point be marked P. Join OP and produce it. The straight line OP represents the Graph, the length of the ordinate of every point on which gives the number of sheep which may be bought by the number of rupees represented by the length of OX cut off between O and the feet of the respective ordinates.

To find the number of sheep worth ₨28, count 28 divisions from O along OX and mark the point Q. From Q draw QR parallel to OY to meet the straight line OP at R. Then the length QR gives the number of sheep required. By counting QR we find it to contain 28 divs. Hence the number of sheep $=28 \div 4=7$.

Example 2. A can row a boat along a river at the rate of 4 miles per hour, and B can row at the rate of 3 miles per hour. If they both row the boat jointly, find the time they will take to reach at a point 24 miles from the starting place. Also find at what time B will be at a distance of $17\frac{1}{2}$ miles from the starting place when rowing alone.

Let hours be represented along OX, taking 7 small sub-divisions equal to one hour, and let miles be represented along OY, taking 1 subdivision = 1 mile. Then the meaning of the figures (see Graph No. 7) along the lines of the diagram will be clear.

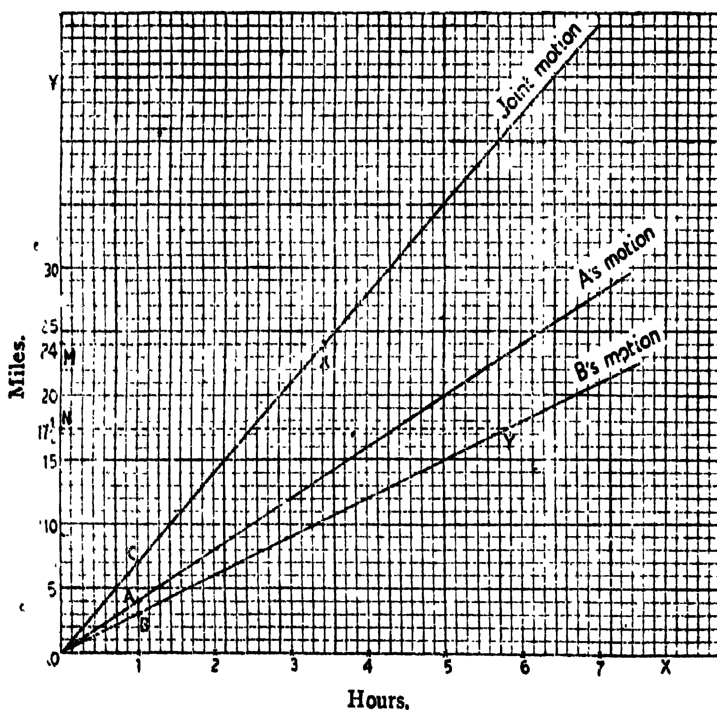
Let A be the point (1 hour, 4 miles). This of course is obtained by taking 7 divisions along OX, representing one hour, and then proceeding vertically through $1A$ equal to 4 divisions representing 4 miles. Join OA and produce it. Then this st. line is the graph of A's motion. Similarly, if we take B at the point (1 hour, 3 miles) and join OB and produce it we get the graph for the motion of B's boat.

Now to get the graph of their combined motion, we must add to the ordinate $A1$ of A, a length $1C$ equal to the ordinate $B1$ of B, thus getting a point C whose ordinate is the sum of the ordinates of A and B. Join OC and produce it, which is the graph of their joint motion.

Next, count 24 miles along OY, and we get the point M. Draw the horizontal MX through M cutting the graph for joint motion at X. This represents the abscissa of X, and on reckoning it we find it to be also 24 divisions. Thus the time taken to reach 24 miles will be $\frac{24}{7}$ or $3\frac{3}{7}$ hours.

Again, count $17\frac{1}{2}$ miles along OY and find the point marked N in the figure. Draw the horizontal line NY through N, cutting the graph for B's motion at Y. This represents the abscissa of Y, and on counting it in the diagram, it is found to

be about $40\frac{1}{2}$ small divisions. Thus the time taken by *B* alone to reach $17\frac{1}{2}$ miles will be $\frac{40.5}{7}$ or 5.8 hours, nearly.



Graph 7.

Note. The result obtained by graph is necessarily approximate but results which are within ·1 of the unit may be regarded as fairly correct. The student is advised also to use graph paper of good quality in which the lines are equally spaced and do form squares and not rectangles.

Example 3. *A* can do a piece of work in 3 days and *B* can do it in 4 days. In how many days will both of them together take to do it ?

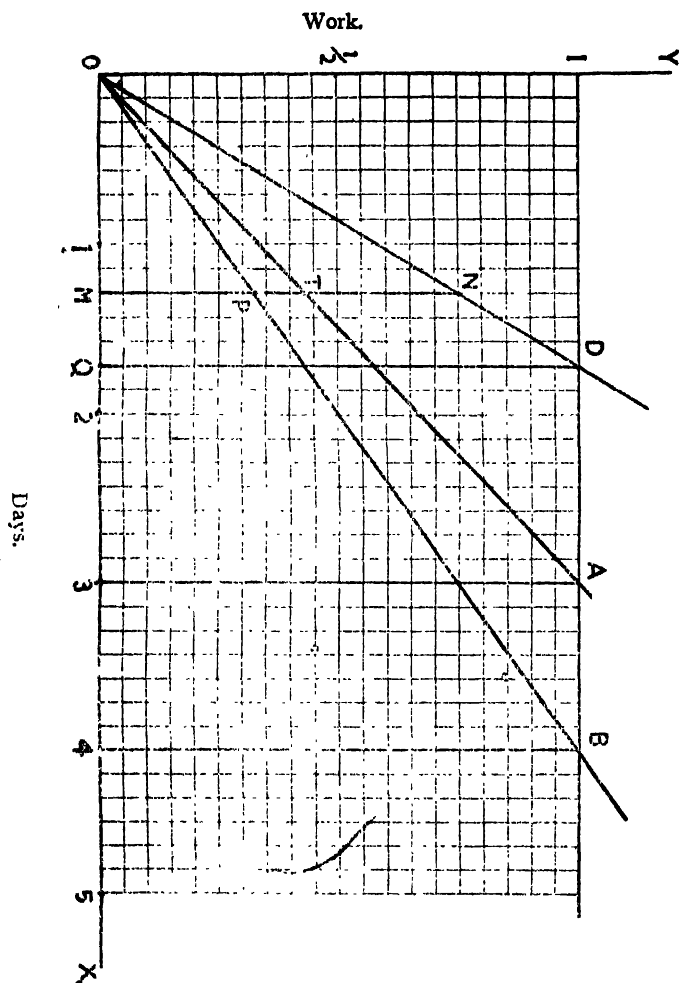
Let days be represented along *OX*, taking 7 divs. to represent 1 day and work along *OY*, taking 20 divisions to represent the whole or one work. Through 1 on *OY*, draw a straight line 1 *DAB* parallel to *OX* to represent 1 work. Since *A* can do the work in 3 days, count 21 divs. along *OX* to represent 3 days and then draw the vertical which meets the line of 1 work at *A*. Join *OA* ; this straight line gives the graph of *A*'s rate of work. Similarly since *B* can do the work in 4 days, count 28 divs. along *OX*, and then draw the vertical which meets the line of 1 work at *B* Then *OB* gives the graph of *B*'s rate of work. (See Graph No. 8).

To find the graph of their combined rates, add to the ordinates of the points on the line *OA* the corresponding ordinates of those on *OB*. This is effected as follows. Take any point *P* on *OB*, whose distance from *OX* is given by *PM*. Then produce *MP* to meet *OA* at *T*. Produce *MT* to *N* so that $PM = TN$. Join *ON* and produce it to meet the line of 1 work in *D*. Then *OD* gives the graph of their combined efforts. From *D* draw *DQ* perpendicular to *OX*. Count *OQ*, which gives the time required to complete the work jointly $= \frac{12 \text{ divs.}}{7 \text{ divs.}}$ days $= 1\frac{5}{7}$ days.

Example 4. In a game of skill, *A* can give *B*, and *B* can give *C*, 10 points out of a game of 50 ; how many should *A* give *C* ?

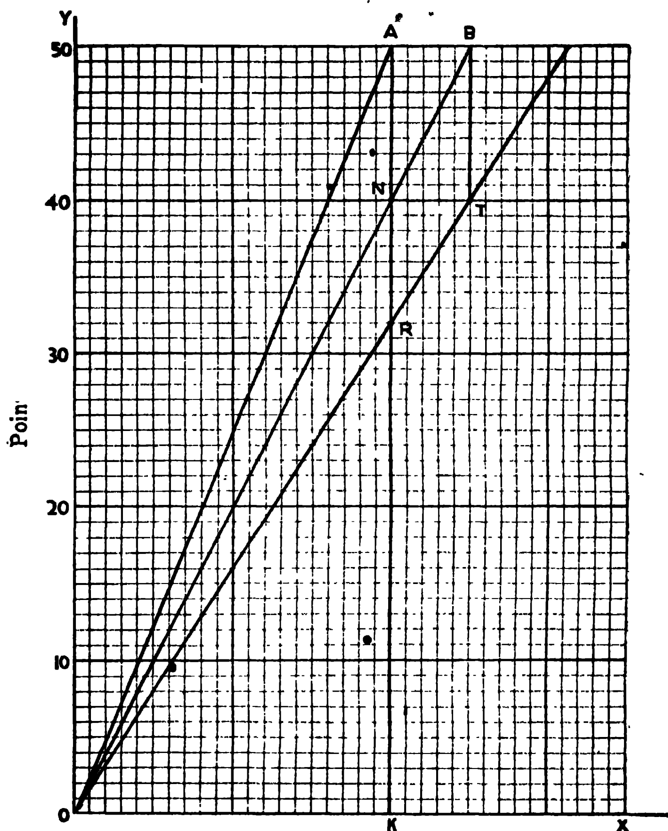
[**Note.** "*A* can give *B* 10 points out of a game of 50" means that while *A* makes 50 points *B* can make (50 − 10) or 40 points."]

Let *OK* along *OX* represent the time *A* takes to obtain 50 points and 1 div. along *OY* to represent 1 point. Through the point on *OY* representing 50 points draw a straight line *ABC* parallel to *OX* representing the line of 50 points. Let the



Graph 8.

vertical through *K* meet this line at *A*. Then *OA* represents the graph of the rate at which *A* gains points. (See Graph No. 9).



Time.

Graph 9.

Take N, 10 divs. below A, on AK. Join ON and produce it to cut the line of 50 points at B. This line gives the graph of the rate at which B gains points.

Now take T 10 divs. vertically below B. Join OT. Let it cut the line of 50 points at C and the vertical AK at R. This line gives the graph of the rate at which C gains points.

Now it is evident from the graph, in the time OK, A's points are represented by the length AK, that of B by the length NK and that of C by the length RK which is equal to $(NK-NR)$ $=40-8=32$ divs., as by counting NR it is found to contain 8 divs.

Hence A can give C $(50-32)$ or 18 points in a game of 50 points.

EXAMPLES. 199b.

1. If the price of 1 watch is ₨5, draw a graph showing the relation of the number of watches and their prices. Read off from your graph the price of 6 watches.

2. Draw the graph from which you can read off the price of 1, 2 etc. articles when those for 5 articles are given to be ₨25.

3. A man walks on foot at the rate of 4 miles per hour. Draw the graph of his motion and from the graph find out how far he travels in 4 hours, and at what time he will be at a distance of 22 miles from where he starts.

4. If a man's average annual income is ₨2000, find out his average monthly income from the graph which represents his average annual income.

5. If in a certain year a man's income is assessed at 6*p*. in the rupee, find out graphically the income-tax payable on an income of ₨2000. The next year he pays an income-tax of ₨93·75 according to the above rate; find out his income this year.

6. 1 metre $=39 \cdot 370$ inches; draw a graph showing their relations and read off from the graph in inches the value of 1·5 metres and in metres the value of 78·74 inches.

7. Find graphically the amount of ₨100 at simple interest after 4 years calculated at ₨6 per cent. per annum.

8. A can do a piece of work in 4 days and B in 5 days. How many days would both of them working together take to do it? Also find out how much of the work they can do working together in a single day. Use graphs.

9. When all working together, a certain piece of work can be finished by *A*, *B* and *C* in 3 days. *A* takes 9 days and *B* 7 days to finish it when working alone. Find out from the graph the time *C* takes to finish the job alone ?

10. A monkey climbs up a greased pole at the rate of 10 ft. per minute, but climbs down 3 ft. the next minute; find graphically the time the monkey will take to reach at the top of the pole which is 49 ft. high.

11. *A* can give *B* 25 yards' and *B* can give *C* 30 yards' start in a race of 150 yards. Find graphically how many yards' start should *A* give *C* in the same race so that *A* may finish off 10 yards leading.

12. In a race of 140 yards, *A* can give *B* 20 yards' and *C* 32 yards' start; find graphically how many yards' start *B* can give *C*.

13. A cistern can be filled by one pipe in 20 minutes and by a second pipe in 15 minutes, while it can be emptied by a third pipe in 12 minutes. If, when the cistern is empty, all the three pipes are set running, find graphically when the cistern will be filled.

14. Two friends take loans of R200 each, one at simple interest and the other at compound interest at 10 per cent. per annum.

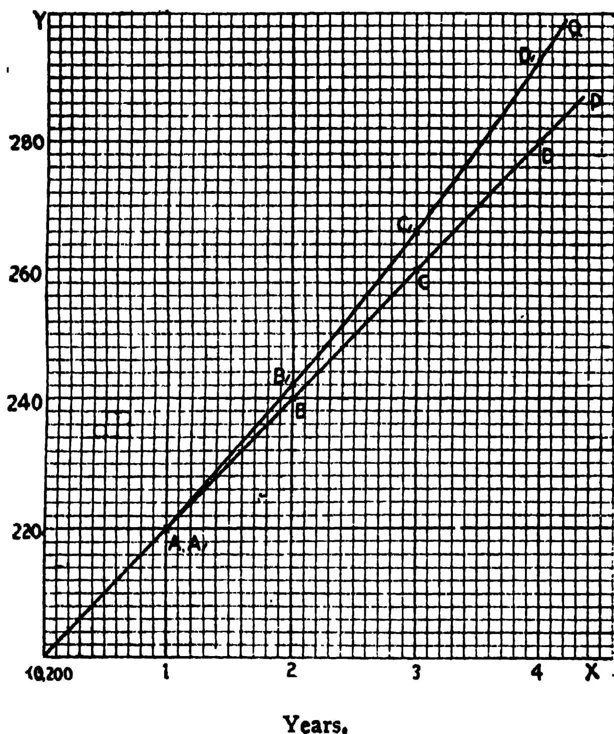
Draw graphs showing amounts at the end of 1, 2, 3 and 4 years in both the cases.

Let us at first compute the amounts in the two cases :—

Years.	Amount at Simple Interest.	Amount at Compound Interest.
	R	R
0	200	200
1	220	220
2	240	242
3	260	266·2
4	280	292·82

Let years be represented along *OX*, 10 divisions representing 1 year and amounts be represented along *OY*, taking 1 div. to represent R2 or 10 divs. to represent R20, beginning with R200 at *O*.

Plot the points accordingly and we see that the amounts at simple and compound interests are shown by the graphs *OP* and *OQ* respectively at different periods of which *OP* is a straight and *OQ* a curved line. (See Graph No. 10).



Years.

Graph 10.

LVIII. PROBLEMS IN HIGHER ARITHMETIC.

320. *Example* 1. A person has a number of oranges to dispose of; he sells half of what he has and 2 more to A, $\frac{1}{3}$ of the remainder and 4 more to B, $\frac{1}{4}$ of the remainder and 6 more to C; by which time he has disposed of all he had. How many had he at first?

When he had given $\frac{1}{4}$ of his oranges to C he had 6 left; therefore this is $(1-\frac{1}{4})$ or $\frac{3}{4}$ of the number he had before C came, and therefore he had $6\times\frac{4}{3}$ or 8 before C came; therefore he had $(8+4)$ or 12 before he had given 4 oranges to B; but this is the number he had left when he had given $\frac{1}{3}$ of his oranges to B; therefore this is $(1-\frac{1}{3})$ or $\frac{2}{3}$ of the number he had before B came, and therefore he had $12\times\frac{3}{2}$ or 18 before B came; therefore he had $(18+2)$ or 20 before he had given 2 oranges to A; but this is the number he had left when he had given $\frac{1}{2}$ of his oranges to A; therefore he had 20×2 or 40 before A came: that is, he had 40 oranges at first.

Example 2. The expenses of a family when rice is at 12 seers for a rupee are R80 a month; when rice is at 15 seers for a rupee the expenses are R77 a month; what will they be when rice is at 18 seers for a rupee?

The prices of a seer of rice in the three cases are R$\frac{1}{12}$, R$\frac{1}{15}$ and R$\frac{1}{18}$ respectively; $\therefore$ the price of a seer is first reduced by R$(\frac{1}{12}-\frac{1}{15})$ or R$\frac{1}{60}$, and finally by R$(\frac{1}{15}-\frac{1}{18})$ or R$\frac{1}{90}$. Now, when the saving on a seer of rice is R$\frac{1}{60}$ the total saving is R$(80-77)$ or R3; $\therefore$ when the saving on a seer is R$\frac{1}{90}$ the total saving will be R$\frac{3\times 60}{90}$ or R5. $\therefore$ The reqd. expenses = R$(80-5)$ = R75.

Or thus: When the saving on each seer of rice is R$\frac{1}{60}$ the total saving is R3; $\therefore$ the number of seers of rice required by the family per month = R3 $\div$ R$\frac{1}{60}$ = 180; and the price of 180 seers at 12 seers for a rupee is R15; $\therefore$ the other expenses of the family = R$(80-15)$ = R65. Again, the price of 180 seers at 18 seers for a rupee is R10; $\therefore$ the total expenses when rice is at 18 seers for a rupee will be R$(65+10)$ or R75.

Example 3. A labourer was engaged for 36 days, on the agreement that for every day he worked he should have 4*a.*, but that for every day he absented himself he would be fined 2*a.* He received R7. 8*a.* at the end of the time; how many days was he absent?

If he had worked all the 36 days he would have received R9; $\therefore$ through absence he lost (R9 − R7. 8*a.*) or R1. 8*a.* But for each day of absence he actually loses (4*a.* + 2*a.*) or 6*a.*; $\therefore$ the number of days he was absent = R1. 8*a.* $\div$ 6*a.* = 4.

Example 4. I have to be at a certain place in a certain time, and I find that if I walk at the rate of 4 miles per hour I shall be 5 minutes too late, and if at the rate of 5 miles per hour I shall be 10 minutes too soon; what distance have I to go?

If I walk 4 miles an hour I require 15 minutes more time in going the distance than if I walk 5 miles an hour. And in walking 1 mile I require 3 minutes more at the former rate than at the latter. Hence I have to go a distance of 5 (*i.e.*, $15 \div 3$) miles.

Example 5. I have a certain sum of money to be distributed among a certain number of boys, and I find that if I give R3 to each I shall spend R4 too little, but that if I give R5 to each I shall spend R6 too much. How much have I to spend?

If I give R5 instead of R3 to each I require R2 more per head and (R4 + R6) or R10 more on the whole; ∴ the number of boys = R10 ÷ R2 = 5; and ∴ I have to spend (R3 × 5 + R4) or R19.

Example 6. A lb. of tea and 4 lb. of sugar cost 5*s.*; but, if sugar were to rise 50 per cent. and tea 10 per cent., they would cost 6*s.* 2*d.*; find the cost of the tea and the sugar per lb.

If both tea and sugar were to rise 50 p. c., the cost of 1 lb. of tea and 4 lb. of sugar would be 7*s.* 6*d.*; but tea rises only 10 p. c., ∴ 40 p. c. of the cost of a lb. of tea = 7*s.* 6*d.* − 6*s.* 2*d.* = 1*s.* 4*d.*; ∴ the cost of a lb. of tea = 3*s.* 4*d.*; ∴ the cost of 4 lb. of sugar = 5*s.* − 3*s.* 4*d.* = 1*s.* 8*d.*; and ∴ 1 lb. of sugar costs 5*d.*

Example 7. Three tramps meet together for a meal; the first has 3 loaves, the second 2, and the third, who has his share of the bread, pays the other two 5*d.*; how ought they to divide the money?

Each eats $\frac{5}{3}$ loaves; ∴ the first has given $(3 - \frac{5}{3})$ loaves and the second $(2 - \frac{5}{3})$ loaves to the third; ∴ the 5*d.* given by the third ought to be divided in the ratio of $(3 - \frac{5}{3})$ to $(2 - \frac{5}{3})$, *i.e.*, of 4 to 1; ∴ the first will take 4*d.*, and the second 1*d.*

Example 8. The sum of the ages of A and B is now 45 years, and their ages 5 years ago were as 3 is to 4: find their present ages.

5 years ago the sum of the ages of A and B was 35 years; if 35 years be divided in the ratio of 3 to 4, the parts are 15 years and 20 years. ∴ The present age of A is (15 + 5) or 20 years, and that of B is (20 + 5) or 25 years.

Example 9. A is twice as old as B, and 4 years older than C; the sum of their ages is 71 years: find the age of each.

If C were as old as A, the sum of the ages of A, B and C would be 75 years; now, dividing 75 in the ratio of 2, 1 and 2, we find that the parts are 30, 15 and 30; ∴ A's age is 30 years, B's 15 years, and C's (30 − 4) or 26 years.

Example 10. *A* and *B* begin business with equal capitals. At the end of the year *A* has gained ₨600, and *B* has lost $\frac{1}{10}$ of his capital ; *A* has then twice as much as *B*. Find how much each had at first.

($\frac{9}{10}$ of *B*'s capital) × 2 = *A*'s capital + ₨600,

∴ ($\frac{9}{10}$ of *A*'s capital) × 2 = ...,

∴ $\frac{18}{10}$ or $1\frac{4}{5}$ of *A*'s capital = ...,

i.e., *A*'s capital + $\frac{4}{5}$ of *A*'s capital = *A*'s capital + ₨600,

∴ $\frac{4}{5}$ of *A*'s capital = ₨600,

∴ *A*'s capital = ₨600 × $\frac{5}{4}$ = ₨750. *Ans.*

Or, *algebraically* thus :

Let ₨x be the capital.

Then, by the question, $x + 600 = 2 \times (1 - \frac{1}{10})x = \frac{9}{5}x$,

∴ $\frac{4}{5}x = 600$,

∴ $x = 750$.

Hence the required capital is ₨750.

Example 11. Divide 250 into two parts such that, 3 times the first part and 5 times the second part may be together equal to 950.

3 times the 1st part + 5 times the 2nd part = 950 ; ...(i)

and the 1st part + the 2nd part = 250,

∴ 3 times the 1st part + 3 times the 2nd part = 750 ; ...(ii)

∴ 2 times the 2nd part = 200, [subtracting (ii) from (i)]

∴ the 2nd part = 100 ;

and ∴ the 1st part = 250 − 100 = 150.

Or, *algebraically* thus ;

Let x be one of the parts. Then the other part = $250 - x$.

By the question, $5x + 3(250 - x) = 950$,

∴ $2x = 950 - 750 = 200$,

∴ $x = 100$.

Hence the parts are 100 and 150.

Example 12. Mangoes are bought at ₨10 per 100 ; at what rate per 100 must they be sold that the gain on ₨100 may be equal to the selling price of 250 mangoes ?

₨100 is the cost price of 1000 mangoes ; ∴ (1000 − 250) or 750 mangoes must be sold for ₨100 ; ∴ the selling price of 100 mangoes = ₨100 × $\frac{100}{750}$ = ₨$13\frac{1}{3}$.

Example 13. Two passengers going to the same place have 6 md. of luggage between them, and are charged for excess of luggage ₨4. 8*a*. and ₨3 respectively; had the luggage all belonged to one person he would have been charged ₨8. 4*a*. for excess. How much is allowed free?

₨4. 8*a*. + ₨3 is the charge on 6 md. *less* twice the free allowance, and ₨8. 4*a*. is the charge on 6 md. *less* the free allowance; ∴ the charge on free allowance = ₨8. 4*a*. − (₨4 8*a*. + ₨3) = 12*a*. ∴ (₨8. 4*a*. + 12*a*.) or ₨9 = charge on 6 md.; ∴ 12*a*. = charge on $\frac{1}{2}$ md. Therefore $\frac{1}{2}$ md. is allowed free.

Example 14. Two guns are fired from the same place after an interval of 6 minutes, but a person approaching the place observes that 5 min. 51 sec. elapse between the reports; what was his rate of progress, sound travelling 1125 ft. per second?

In 5 min. 51 sec. or 351 sec. the man travels a distance which sound will travel in (6 min. − 5 min. 51 sec.) or 9 sec. But in 9 sec. sound travels 1125 × 9 ft., ∴ in 351 sec. the man travels 1125 × 9 ft.; ∴ in 1 hour the man travels $\frac{1125 \times 9 \times 60 \times 60}{351 \times 3 \times 1760}$ miles or $19\frac{191}{286}$ miles.

Example 15. ₨49 was divided amongst 150 children, each girl had 8*a*. and each boy 4*a*.; how many boys were there?

If 4*a*. be given to each child, ₨37. 8*a*. will be spent, and the boys will have got their shares. The remaining sum, ₨11. 8*a*., must therefore be distributed amongst the girls only, giving 4*a*. to each. Hence the number of girls is the same as the *number of times* 4*a*. is contained in ₨11. 8*a*.; therefore the number of girls is 46, and therefore the number of boys is 104.

This example may also be solved by the method of Art. 268. Thus: When ₨49 is divided amongst 150 children, each gets $\frac{392}{75}a$. on the average. Hence the question may be put thus—"Each boy is to have 4*a*. and each girl 8*a*.; in what ratio should they be mixed that each may have $\frac{392}{75}a$. on the average?" Therefore by the method of Art. 299, we find that the ratio of the number of boys to the number of girls must be $(8-\frac{392}{75}) : (\frac{392}{75}-4)$ or 104 : 46. But 104 + 46 = 150; ∴ the number of boys = 104, and the number of girls = 46.

Example 16. A free-hold estate is bought at 20 years' purchase; find the rate of interest obtained on the money invested.

["A free-hold estate is bought at 20 years' purchase" means that it is bought for 20 times the yearly rent derived from the estate.]

If the value of the estate is ₨20, the rent is ₨1; ∴ if the value of estate is ₨100, the rent is ₨5. Therefore the rate of interest obtained is 5 p. c.

Example 17. If 36 oxen in four weeks eat up the grass on a field of 12 acres and what grows upon it during the time; and 21 oxen eat up the same in 9 weeks; how many oxen will it maintain for 18 weeks, supposing the grass to grow uniformly during the time?

Origl. growth + 4 wk.'s growth maintains	36 ox for	4 wk.,	
∴	1 ox for	144 wk.;	
also, origl. growth + 9 wk.'s growth............	21 ox for	9 wk.,	
∴	1 ox for	189 wk.	

Hence, subtracting 2nd line from the 4th,

5 wk.'s growth maintains	1 ox for	45 wk.,	
∴ 1 wk.'s growth	1 ox for	9 wk.,	
∴ 16 wk.'s growth	1 ox for	144 wk.;	
but origl. growth + 4 wk.'s growth	1 ox for	144 wk.;	

∴ origl. growth = 12 wk.'s.

Now, 1 wk.'s growth maintains	for	9 wk.	1 ox,
∴ 1 wk.'s growth	for	18 wk.	½ ox,
∴ (12 + 18) or 30 wk.'s growth	for	18 wk.	15 ox.,
i.e., origl. growth + 18 wk.'s growth	for	18 wk.	15 ox.

Answer. 15 oxen.

Or, *algebraically* thus:

Let x = original growth,

y = quantity required to maintain 1 ox for one week,

z = total weekly growth.

Then $36y \times 4 = x + 4z$, ... (i)

$21y \times 9 = x + 9z$. ... ... (ii)

∴ By subtraction, $5z = 45y$,

∴ $z = 9y$. ... ... (iii)

∴ From (i) and (iii) we have $x + 36y = 144y$,

∴ $x = 108y = 12 \times 9y = 12z$.

But the required no. of oxen eat up the grass in 18 weeks;

∴ $x + 18z$ = quantity eaten up in 18 weeks.

But $x + 18z = 30z = 30 \times 9y = 15y \times 18$.

∴ The required number of oxen = 15.

EXAMPLES. 200.

1. A person has a number of oranges to dispose of; he sells half of what he has and one more to A, half of the remainder and one more to B, half of the remainder and one more to C, and half of the remainder and one more to D; by which time he has disposed of all he had. How many had he at first?

2. A thief having stolen some money from the palace of Siraj-Uddowlah was caught on his way back by the head *khoja* who let him off on getting half the money and R20 more; he was caught again by the sentry at the palace gate, who got a third of what he then possessd and R10 more; lastly he was let off by the *kotwal* in his rounds on getting $\frac{1}{4}$ of what he still had and R6 more. The thief came home robbed of all he stole. How much did he steal?

3. The expenses of a family, when rice is at 8 seers for a rupee, are R75 a month; when rice is at 10 seers for a rupee, the expenses are R72 a month (other expenses remaining unaltered): what will they be when rice is at 12 seers for a rupee?

4. A labourer was engaged for 15 days, on the agreement that for every day he worked he should have 6*a.*, but that for every day he absented himself he would be fined [illegible]*a.* He received R4. 2*a.* at the end of the time; how many days was he absent?

5. I have to be at a certain place in a certain time, and I find that if I walk 3 miles an hour I shall be 10 min. too late, and if I walk 4 miles an hour I shall be $7\frac{1}{2}$ min. too soon; what distance have I to go?

6. I have a certain sum of money to be distributed among a certain number of boys; and I find that if I give R2 to each I shall spend R4 too little, but if I give R3 to each I shall spend R3 too much. How much have I to spend?

7. I have a certain sum of money wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny I shall spend 5*d.* too much, if 50 a penny, 10*d.* too little. How much have I to spend?

8. A lb. of tea and 3 lb. of coffee cost 5*s.*; but, if coffee were to rise $33\frac{1}{3}$ p. c. and tea 50 p. c., they would cost 7*s.* Find the cost of tea and coffee per lb.

9. 3 lb. of tea and 4 lb. of sugar cost 8*s.*; but, if sugar were to rise 25 p. c. and tea were to fall 25 p. c., they would cost 7*s.* Find the cost of tea and sugar per lb.

10. Three tramps meet together for a meal; the first has 3 loaves, the second 4, and the third, who has his share of the bread, pays the other two 7 half-pence; how ought they to divide the money?

11. Two settlers in New Zealand own adjoining farms of 700 and 500 acres respectively. They unite their farms, taking at the same time a new partner who pays £1200 on the understanding that $\frac{1}{3}$ of the land will in future belong to each. How is the £1200 to be divided between the original owners ?

12. The sum of the ages of *A*, *B* and *C* is now 90 years, and their ages 10 years ago were as 3 : 4 . 5 ; find their present ages.

13. *A* is twice as old as *B*, and 5 years older than *C* ; the sum of their ages is 45 years ; find the age of each.

14. Divide ₨80 between *A*, *B* and *C* in such a manner that *A* may get 3 times as much as *B*, and *B* ₨10 more than *C*.

15. *A* and *B* begin business with equal capitals. At the end of the year *A* has gained ₨130, and *B* has lost $\frac{1}{15}$ of his capital ; *A* has then twice as much as *B*. Find how much each had at first.

16. *A* and *B* begin business with equal capitals. At the end of a certain time *A* has gained $\frac{1}{4}$ of his capital, and *B* has lost ₨200 ; *B* has now $\frac{1}{3}$ of what *A* has. How much had each at first ?

17. Divide 155 into two parts such that, twice the first part and 3 times the second part may be together equal to 370.

18. Divide 100 into two parts such that, $\frac{1}{2}$ of one part and $\frac{1}{3}$ of the other part may be together equal to 40.

19. Divide 350 into two parts such that, 3 times the first part and $\frac{1}{3}$ of the second part may be together equal to 250.

20. Mangoes are bought at ₨5 per 100 ; at what rate per 100 must they be sold that the gain on ₨100 may be equal to the selling price of 400 mangoes ?

21. Sugar is bought at 4*a.* per seer ; at what rate per seer must it be sold that the gain on ₨10 may be equal to the selling price of 8 seers ?

22. Two passengers going to the same place had 8 md. of luggage between them, and were charged for excess of luggage ₨8 and ₨4 respectively ; had the luggage all belonged to one person he would have been charged ₨14 for excess. Find how much is allowed free, and how much luggage each had.

23. Two guns are fired from the same place after an interval of 10 minutes, but a person approaching the place observes that 9 min. 30 sec. elapse between the reports ; what was his rate of progress, sound travelling 1121 ft. per second ?

24. Two guns are fired from the same place at an interval of 15 minutes, but a person going away from the place hears the reports at an interval of 15 min. 30 sec. ; if sound travels 1125 ft. per second, find his rate of travelling per hour.

25. Two guns are fired from a place at an interval of 28 minutes, but a person approaching the place, at the rate of $13\frac{10}{11}$ miles an hour, hears the reports at an interval of 27 min. 30 sec. Find the velocity of sound per second.

26. Cannons are fired at regular intervals in a town, and a person riding towards it at the rate of 9 miles an hour hears the reports at intervals of 15 minutes; at what intervals must the cannons have been fired, sound travelling 1120 ft. per second?

27. Cannons are fired at intervals of 10 minutes in a town towards which a passenger train is approaching at the rate of 30 miles an hour; if sound travels 1136 ft. per second, find at what intervals the reports will be heard by the passengers.

28. R60 was distributed among 50 children, each girl had R2 and each boy R1; how many boys were there?

29. 35 fruits, consisting of mangoes and oranges were bought for R2. 8*a.*; if the mangoes cost 2*a.* each and the oranges 6*p.* each, find the number of oranges bought.

30. A lump composed of gold and silver measures 6 cu. inches and weighs 100 oz.; if a cu inch of gold weighs 20 oz and an equal bulk of silver 12 oz., find the weight of gold in the mixture.

31. 19 grains of gold or 12 grains of silver displace one grain of water. If a ring composed of gold and silver, weighs 88 grains and displaces 5 grains of water, how many grains of silver does it contain?

32. A farmer has oxen worth £12. 10*s.* each, and sheep worth £2. 5*s.* each; the number of oxen and sheep being 35, and their value £191. 10*s.* Find the number he had of each.

33. If an income-tax of 7*d.* in the £ on all incomes below £100 a year, and 1*s.* in the £ on all incomes above £100 a year realises £18750 on £500000, how much is raised on incomes below £100 a year?

34. How many years' purchase should be given for a free-hold estate so as to get 5 per cent. for the money?

35. An estate is bought at 25 years' purchase for R40,000, one-fourth of the purchase-money remaining at mortgage at 6 per cent. The cost of collecting rents is R100 per annum. What interest does the purchaser make on his investment?

36. If 10 oxen in 5 weeks eat up the grass on a field of 7 acres and what grows upon it during the time, and 11 oxen eat up the same in 4 weeks, how many weeks' growth is on the field?

37. If 20 oxen in 4 weeks eat up the grass on a field of 2 acres and what grows upon it during the time; and 17 oxen eat up the same in 10 weeks; how many oxen will it maintain for 5 weeks, supposing the grass to grow uniformly during the time?

38. In a certain meadow there is a crop of 525 stones of grass, which grows uniformly. If 11 oxen turned in would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day?

39. If 25 horses eat the grass of 35 acres of one field in 11 days, in what time would 20 horses eat the grass of another field of 56 acres, where there is at first twice as much grass per acre as in the former field, the growth of the grass being neglected. What must be the ratio of the rates of the growth of the grass in the two fields so that your result may be accurately true?

40. A well is fed by a spring which flows continuously and uniformly into it. When there are 10,000 cu. ft. of water in the well, 7 men can empty it in 20 days; and when there are 15,000 cu. ft. of water in the well, 5 men can empty it in 50 days. How many cu. ft. of water flow into the well in one day?

41. A cistern has one supply-pipe (A) and 2 equal waste-pipes (B, C) attached to it. A is opened, and when the cistern is partially filled B is also opened, and the cistern is emptied in 3 hours. Had C been opened along with B the cistern would have been emptied in 1 hour. How long after A was B opened?

42. A cistern has two pipes attached to it, one to supply and one to draw off. If both the pipes are opened together, the cistern is filled in 9 hours; but if the waste-pipe is opened one hour after the supply-pipe, the cistern is filled in 7 hours. In what time can the supply-pipe fill the empty cistern?

43. A leaky cistern is filled in 5 hours with 30 pails of 3 gallons each, but in 3 hours with 20 pails of 4 gallons each, the pails being poured in at intervals. Find how much the cistern holds, and in what time the water would waste away.

EXAMPLES FOR EXERCISE. 201a.

(First Series).

1. State in words 10030200720021.

2. Find the value of 66674 − 9545 − 201 + 843 − 8761.

3. Reduce £49. 6*s.* 2¼*d.* to farthings.

4. Find the prime factors of 51425.

5. Reduce $\frac{18877}{20008}$ to its lowest terms.

6. Find the sum and difference of 23·001 and ·0414.

7. Find the value of $\frac{3}{8}$ of R7. 7*a.* 7*p.*

8. Write in words 3200103102 according to the Indian numeration.

9. The greatest prime number known is expressed by 1251^2+2920^2; find the number.

10. What sum will remain when four bills, amounting to R5 . 7 . 6, R3 . 4 . 9, R2 . 15 . 3, and R10 . 13 . 3 respectively, have been paid out of R25 ?

11. Find the G. C. M. of 23791 and 8029.

12. Subtract $14\frac{51}{140}$ from $16\frac{4}{35}$.

13. Multiply ·038 by ·0042, and divide ·03217 by 6·25.

14. Find the value of ·00625 of £1.

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15. Subtract one crore five lacs three thousand and twenty from twenty-nine million twelve thousand and four.

16. Multiply 765389 by 64164 in 3 lines.

17. I go to town with £9. 1*s*. 3*d*. What have I left after buying a dozen chairs at 13*s*. $7\frac{1}{2}$*d*. each ?

18. Find the L. C. M. of 9669 and 16115.

19. Add together $\frac{14}{17}$, $3\frac{1}{3}$, $1\frac{5}{51}$ and $\frac{7}{27}$.

20. Express as a decimal $·0003+\frac{817}{3125}-·00849+\frac{361}{800}$.

21. Reduce $\frac{3}{5}$ of $\frac{5}{13}$ of 19*s*. 6*d*. to the fraction of $\frac{3}{8}$ of $\frac{4}{17}$ of £1. 8*s*. 4*d*.

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22. Express 944 in Roman notation, and CDXCIX in Arabic notation.

23. Multiply 387659 by 85672 in 3 lines.

24. How many cows at R10. 14*a*. each can I buy with the proceeds of selling 87 horses at R115. 2*a*. each ?

25. Simplify $\dfrac{6\frac{3}{4}-1\frac{5}{11}}{2\frac{1}{3}+1\frac{3}{7}}$.

26. Multiply ·006134 by 80·032, and divide the result by ·0032.

27. Reduce $(8\div1\frac{1}{4})$ of 1*p*. to the decimal of R1. 4*a*.

28. If a rupee is worth 2*s*. $0\frac{3}{8}$*d*., and a dollar 4*s*. $4\frac{1}{2}$*d*., find the least number of rupees which makes an exact number of dollars.

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29. What number multiplied by 76 will give the same product as 153 multiplied by 380 ?

30. Find the greatest number which will divide each of 3456, 26244 and 99225 without remainder.

31. Reduce 57 tons 9 cwt. 1 qr. 10 lb. to drams.

32. Simplify $\frac{3}{4} \times \frac{5}{8} \div 1\frac{1}{3}$ of $1\frac{1}{2}$.

33. Find the least fraction which being added to $\frac{1}{3} - \frac{1}{4}$ of $\frac{1}{3} - \frac{1}{6}$ will make the sum an integer.

34. *A* did ·0025 of a piece of work, and *B* ·7855. How much was left undone ?

35. Find the cost of 3·125 yards at £·375 a yard.

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36. What number is the same multiple of 35 that 3456 is of 9 ?

37. If my income is R3500 and I save R507 a year, what is my average daily expenditure ?

38. Simplify $\dfrac{(\frac{1}{2} - \frac{1}{3}) \text{ of } (\frac{1}{4} - \frac{1}{6})}{\frac{1}{2} - \frac{1}{3} \text{ of } \frac{1}{8} - \frac{1}{6}}$.

39. If the sum of $21\frac{5}{7}$ and $3\frac{6}{11}$ be added to the product of $2\frac{1}{4}$ and $\frac{7}{8}$, by how much will the result differ from 28 ?

40. Reduce $3\frac{17}{247}$ to a decimal.

41. Find the vulgar fraction equal to $\cdot 27\dot{8}9\dot{9}$.

42. Find the value of $\frac{3}{8}$ of R3 . 7 . 6 + ·375 of R6 . 8 . 6.

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43. Find the least number which being subtracted from 97856 will make the result divisible by 141.

44. Reduce 3 acres 1 rood 2 perches to square feet.

45. Arrange $\frac{2}{5}$, $\frac{3}{7}$, $\frac{27}{70}$ in order of magnitude.

46. Divide $\frac{2}{3} \div \frac{3}{4}$ of 12 by $\frac{2}{3}$ of $\frac{3}{4} \div 12$.

47. Add $3\cdot 7\dot{2}\dot{5} + \cdot 00\dot{2} + \cdot 27\dot{2}\dot{5}$.

48. Reduce $\cdot 0\dot{3}$ of R3 to the decimal of $\frac{3}{4}$ of R1·5.

49. Find the least number of weeks in which an exact number of half-guineas can be earned, the wages per week being 7·5 shillings.

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50. What is the least number which being added to 30321 will make the sum divisible by 681 ?

51. A bill of £6. 1*s.* 11*d.* has to be paid by several persons in equal shares ; if three of them together pay £1. 13*s.* 3*d.*, how many are there to share the cost ?

52. Simplify $2\frac{10}{17} \times 1\frac{33}{35} \div \frac{87}{28} \times 2\frac{18}{65}$.

53. Divide 352·95624 by ·000504.

54. Express $1\cdot 4 \div 1\cdot \dot{1}\dot{3}$ as a decimal.

55. Reduce ·543 of 19*s.* $3\frac{1}{4}$*d.* to pence.

56. Find the greatest unit of time by means of which 2 hr. 3 min. and 1 hr. 4 min. 30 sec. can both be expressed as integers.

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57. I multiply a number by 36 and divide the result by 12 and obtain 374181 as quotient. What was the number ?

58. *A* and *B* together have R36. 13*a*. 9*p*., and *A* has R3. 3*a*. 3*p*. more than *B* ; find how much *B* has.

59. Reduce $\frac{5184}{10008}$ to its lowest terms.

60. Express $3\frac{19}{22}$ poles in poles, yards, etc.

61. What are the nearest integers to $8\frac{9}{16}$ and $7\frac{5}{11}$?

62. Find the difference between the product and quotient of 5·312 by ·0125.

63. Simplify $(2\cdot3\dot{6}\dot{4}-1\cdot6\dot{9}\dot{7})+1\cdot\dot{3}\times(2\cdot\dot{4}+7\cdot\dot{5})$.

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64. If in a division sum the divisor be 7 times and the quotient 5 times the remainder, what is the dividend when the remainder is 360 ?

65. Reduce 300,003,840 grains to pounds Troy.

66. Find the cost of 13724 articles at R8. 0*a*. $7\frac{1}{2}$*p*. each.

67. Multiply $7\frac{1}{8}+6\frac{2}{3}$ by $2\frac{1}{13}-\frac{33}{52}$.

68. What fraction of a journey of 15 miles have I gone on reaching a place $6\frac{2}{3}$ miles distant ?

69. By what must $15501\frac{3}{5}$ be divided that the quotient may be $459\frac{1}{2}$?

70. If a metre be 39·37 inches, how many metres make 3 miles ?

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71. When 2080400 is divided by a certain number, the quotient is 381 and the remainder 1664. What is the number ?

72. Reduce 67501 inches to poles, etc.

73. If $2\frac{1}{18}$ tons cost R964. 3*a*. 8*p*., what is the cost of 1 ton ?

74. Simplify $\frac{3-4\frac{1}{7}+2\frac{1}{4}}{3\times2\frac{1}{3}-4\frac{1}{7}}\div\frac{6\frac{1}{4}\text{ of }4\frac{4}{5}}{11\frac{1}{3}-6\frac{7}{10}}$.

75. Divide equally amongst 5 boys $\frac{5}{7}$ of £4. 2*s*. $1\frac{1}{4}$*d*.

76. Divide ·7029 by ·0165.

77. What decimal of R3. 7*a*. must be taken from R4. 15*a*. to leave R2·5 ?

78. If when a number is divided continuously by 5, 6 and 7, the remainders are 2, 3 and 4 respectively, what would be the remainder if the number were divided by 210 ?

79. If 1 md. cost R11. 1*a*., find the cost of $\frac{70}{231}$ of a md.

80. The 1st of January 1893 was on a Sunday ; on what day of the week did 10th February fall in the year 1894 ?

81. Find the value of $\frac{7\frac{6}{11}}{8\frac{7}{22}} \div \frac{2\frac{5}{6}}{7\frac{1}{2}}$ of $\frac{81\frac{4}{11}}{5\frac{1}{12}}$.

82. If from a rope 7 ft. long as many pieces as possible are cut off, each $1\frac{1}{3}$ ft. long, what fraction of the whole will be left ?

83. Reduce $\cdot\dot{1}4285\dot{7} + \cdot\dot{8}5714\dot{2} - \cdot\dot{2}8571\dot{4}$ to a vulgar fraction.

84. Simplify $\frac{1\cdot5}{\cdot075} \times \frac{3\cdot25}{1\frac{1}{2}}$.

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85. Find a number such that if it be added 35 times to 25 the sum will be 25540.

86. If a person spends in 4 months as much as he earns in 3, how much can he lay by annually, supposing that he earns £250. 10*s*. every 6 months ?

87. Simplify $\frac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$.

88. How many steps does a man whose length of pace is 32 inches take in $4\frac{3}{4}$ miles ?

89. Divide ·75445 by ·00625.

90. How many inches are there in ·1215625 of a mile ?

91. Subtract ·432 of an acre from $2\frac{1}{2}$ roods, expressing the result in sq. yards and the decimal of a sq. yard.

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92. A man buys 100 md. of rice ; he loses as much by selling 60 md. at R3 a md. as he gains by selling the rest at R4. 4*a*. a md. Find the cost price of a md.

93. By what prime numbers may 109 be divided so that the remainder may be 4 ?

94. Add $\frac{6479}{6510} + \frac{4347}{4410} + \frac{6881}{6930}$.

95. How many times can ·053 be subtracted from 14·578, and what will be the magnitude of the remainder ?

96. Express $\cdot2\dot{3}\dot{6}$ of 4*a*. 7*p*. + $\cdot51\dot{6}$ of 10*a*. as the decimal of R1. 4*a*.

97. Simplify $\frac{(3\cdot2 - 2\cdot9) \times 147}{\cdot003 \times \cdot0005}$.

98. Three bells toll at intervals of 1·2, 1·8 and 2·7 seconds respectively, beginning together; how often will each toll before their tolling together again?

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99. The remainder after a division is 97, the quotient is 521, and the divisor is 9 more than the sum of both; what is the dividend?

100. Two pieces of cloth of the same length cost £5. 11*s*. 9*d*. and £7. 4*s*. respectively; the price of the first was 3*s*. $1\frac{1}{2}$*d*. per yard: what was the price of the second per yard?

101. Divide $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{7}$ of 42 by the sum of $2\frac{1}{3}$ and $4\frac{5}{8}$.

102. Simplify $\frac{1}{2}[2-\frac{1}{2}\{2-\frac{1}{2}(2-\frac{1}{2})\}]$.

103. Reduce $\frac{29}{65}$ to a decimal.

104. Multiply 28·8 by 25·3 and divide the product by 6·48.

105. The distance between two wickets was marked out for 22 yd., but the yard measure was $\frac{5}{12}$ of an inch too short; what was the actual distance?

106. If a number of articles at R4. 0*a*. $5\frac{1}{4}$*p*. each cost R7059. 14*a*. $11\frac{1}{4}$*p*., how many are there?

107. Simplify $\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ of $\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}$ of $\frac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{6}+\frac{1}{7}}$ of 117.

108. Find the value of $\frac{\cdot426\times\cdot426-\cdot174\times\cdot174}{\cdot426-\cdot174}$ of R1. 4*a*.

109. Subtract 5·142857 from $5\cdot\dot{1}4285\dot{7}$.

110. Divide 1·00625 by 132·5 to five places of decimals.

111. Reduce 4 hr. 48 min. to the decimal of 6 hr.

112. A man owns $\frac{3}{16}$ of a house, and sells ·1351 of his share; what fraction of the house does he still own?

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113. How many revolutions will be made by a wheel, which revolves at the rate of 243 revolutions in 3 min., while another wheel revolving 374 times in 11 min. makes 544 revolutions?

114. Multiply 10 sq. yd. 4 ft. 76 in. by 132.

115. Reduce to its lowest terms [illegible].

116. Find the least number which, when divided by each of $\frac{7}{12}$, ·25 and $\cdot0\dot{3}$, gives a whole number as quotient in each case.

117. Simplify $\frac{5\cdot3\dot{4}\times5\cdot3\dot{4}-2\cdot\dot{6}\dot{5}\times2\cdot\dot{6}\dot{5}}{5\cdot3\dot{4}-2\cdot\dot{6}\dot{5}}$.

118. Find, to the nearest pie, the value of ·1234 of R12·5.

119. A kilolitre contains 35·32 cubic feet, and a gallon contains 277·274 cubic inches; find to the nearest integer the number of gallons in a kilolitre.

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120. A farmer has 899 sheep and 493 lambs. He forms them into flocks, keeping sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many flocks will there be altogether?

121. If 257 pounds of tea cost £34. 16*s.* $7\frac{1}{2}$*d.*, find the price of a pound to the nearest farthing.

122. Simplify $\frac{\frac{1}{2} \div \frac{1}{3}}{\frac{1}{4} \div \frac{1}{5}} \div \frac{\frac{1}{6} \div \frac{1}{7}}{\frac{1}{8} \div \frac{1}{9}}$.

123. How many whole cakes will be required for 50 children if each is to have $2\frac{2}{7}$ of $1\frac{4}{11}$ of $2\frac{5}{8}$ of $\frac{4}{9}$ of $\frac{11}{16}$ of $\frac{1}{10}$ of a cake?

124. Find the value of $\frac{\frac{3}{8} \text{ of } \cdot375 - \frac{2}{45} \text{ of } \cdot0\dot{4}}{\cdot375 + \cdot0\dot{4}}$.

125. Find the circulating decimal which will become 2 when multiplied by $2\frac{3}{5} \div 4\cdot5$.

126. A German mark is worth £·04895; find to the nearest farthing the value of 3725·39 marks.

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127. To a certain number I add 2, I multiply the sum by 4, I divide the product by 3, and I take 3 from the quotient; the remainder is 17. What is the number?

128. On what day of the week will Feb. 10 fall in the year 1960?

129. Find the greatest prime number which used as divisor of 12260 will leave remainder 17.

130. Find the value of $\frac{2\cdot8}{\cdot21}$ of $\frac{\text{R}1.\ 5a.\ 4p.}{\text{R}4.\ 2a.\ 8p.}$.

131. What is the number whose half exceeds its fifth part by 6?

132. Simplify $\cdot\dot{4}2857\dot{1} \times \cdot49 \times \cdot20\dot{5}71$[illegible].

133. How many times does a carriage wheel, whose circumference is 17·125 feet, turn round in a distance of $12\cdot\dot{4}\dot{5}$ miles?

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134. Determine the prime factors of 282660 and 40299. Hence deduce the G. C. M. and L. C. M. of these numbers.

135. Find the least integer which, when divided by $1\frac{4}{21}$ and $1\frac{2}{7}$, will give a whole number as quotient in each case.

136. Simplify $\frac{3}{7}$ of $\dfrac{6\frac{5}{12}}{3\frac{2}{3}} - \frac{19}{20}$ of $\dfrac{1\frac{2}{3}}{6\frac{2}{3}} + \frac{2}{5}(\frac{3}{2} \times \frac{1}{6} + \frac{8}{9} \div \frac{2}{3})$.

137. Reduce $\frac{5}{99} + \frac{7}{990} + \frac{16}{990900}$ to a decimal.

138. If a cu. yd. of clay make 460 bricks, each $101\frac{1}{4}$ cu. in., how much does clay contract in baking ?

139. Multiply 324·567 by 13·212 in 2 lines.

140. One pendulum oscillates 6 times in 3·2 seconds, and another pendulum 8 times in 3·6 seconds ; if started simultaneously, how often will they tick together in an hour ?

EXAMPLES FOR EXERCISE. 201b.

(Second Series)

1. Write down the greatest and the least numbers of four digits that you can form with the figures, 3, 0, 2, 1.

2. Simplify $\frac{1}{3}[3+\frac{1}{3}\{3+\frac{1}{3}(3+1\frac{1}{2})\}]\div\frac{1}{8}$.

3. The telegraph posts on a railway line are 66 yards apart : find the smallest number of miles that corresponds to an exact number of posts.

4. A bath is supplied with water from two pipes, one of which can fill it in $12\frac{1}{2}$ min., the other in 15 min. ; there is also a discharging pipe which would empty it, when filled, in 10 min. The first pipe is open alone for 4 min., and then the first and second open together for 1 min. ; if now the third pipe is opened as well, how long will it take to fill the bath ?

5. The wages of *A* and *B* together for 20 days amount to the same sum as the wages of *A* alone for 35 days. For how many days will this sum pay the wages of *B* alone ?

6. A cask contains 5 parts wine and 3 parts water ; how much of the mixture must be drawn off and water substituted in order that the resulting mixture may be half and half ?

7. A person borrows £130 on the 5th of March, and pays back £133. 18*s.* on the 10th October ; find the rate of interest charged.

8. The digits in the units' and lacs' places of a number are 3 and 8 respectively ; what will be the digits in the same places in the remainder when 99999 is subtracted from the number ?

9. A whole number diminished by $\frac{1}{4}$ of itself, when divided by 307 gives a quotient 12 and a remainder 96 ; what is the number ?

10. The length of a rectangular tennis-court is 5 yards longer than its breadth, and its perimeter is 130 yards; find its area.

11. The train which leaves Calcutta at 4-30 P. M. arrives at Burdwan at 8 P. M.; and the train which leaves Burdwan at 4-50 P. M. arrives in Calcutta at 8-30 P. M.: when do they pass each other?

12. The rent of a farm consists of a fixed sum of money together with the value of a certain number of maunds of wheat; when wheat is ₨2 a md. the rent is ₨40; when wheat is ₨2. 4*a*. a md. the rent is ₨42. 8*a*. What will be the rent when wheat is ₨2. 10*a*. a md.?

13. Assuming that the circumference of a circle is to its diameter as 22 is to 7, and that the circumference of the earth is to its diameter as 160 metres to 167 feet, determine to 4 places of decimals the ratio of a metre to a foot.

14. The interest on a given sum of money for one year is £5. 8*s*. 4*d*., the compound interest for two years is £11. 1*s*. Find the rate per cent.

15. If when a number is divided continually by 5, 6 and 8 the remainders are 2, 3 and 4 respectively, what would be the remainder if the same number were divided by 240?

16. Divide 1255 by 1·004 and hence deduce the quotient of 12·55 by 1004 and ·01255 by 1004000.

17. I bought a certain number of chairs for ₨45; also a certain number for ₨28. 2*a*. at the same rate; find the greatest possible price of each chair.

18. A clock which gains $2\frac{1}{2}$ min. in a day, is 3 min. slow at noon on Sunday; when will it show correct time, and what time will it indicate at 6 on Monday evening?

19. A person bought 4 railway tickets to go 60 miles. Two were for the 1st class, one for the 2nd, and the fourth, a half first class ticket, for a child. The cost of a 2nd class ticket was $\frac{2}{3}$ of that of a 1st class, and the whole sum paid was £1 . 11 . 8. Find the price of each ticket, and the rate per mile for the first class.

20. There are two mixtures of wine and water, in the ratios of 3 : 2 and 4 : 5 respectively; if one gallon of the first be mixed with 2 gallons of the second, what fraction of the resulting mixture will be wine?

21. A book sent from England costs me (including 1*s*. 6*d*. postage) 16*s*. 1*d*., my book-seller allowing me two pence in the shilling discount on the published price. What is the published price?

22. What number is the same multiple of 7 that 3975 is of 15?

23. Simplify $\frac{1}{7\frac{1}{4}+6\frac{6}{11}} \div \left(\frac{3}{13}-\frac{2}{9}\right)-\left(\frac{13}{3}+\frac{1}{6}\right)\div\frac{2}{3}$ of $\frac{3}{8}$ of 63.

24. On laying down a bowling green with sods 2 ft. by 9 in., it is found that it requires 120 sods to form one strip extending the whole length of the green, and that a man can lay down one strip and a half each day; find the space laid down by 5 men in 2 days.

25. *A* can do a piece of work in 3 days, *B* can do 3 times as much in 8 days, and *C* 5 times as much in 12 days. In what time will they do it together, supposing them to work at the rate of 9 hours a day?

26. A farmer pays a corn-rent of 5 quarters of wheat and 3 quarters of barley, Wichester measure; what is the money value of his rent, when wheat is at 60*s.*, and barley at 54*s.* per quarter, Imperial measure; 32 Imperial gallons being equal to 33 Winchester gallons?

27. Six coins of equal weight, made of gold and silver mixed, were melted together and re-cast. In one the gold and silver were in the ratio of 2 : 3; in two others, of 3 : 5; and in the rest, of 5 : 4. In what ratio will the gold and silver be mixed in the new coins?

28. A tradesman, selling goods for a certain price to be paid six months hence, offers to give one-tenth more of the same goods for the same price in ready money. What is the rate of discount?

29. Find the greatest and least numbers of 6 digits which are exactly divisible by 239.

30. There is a number, to which 3 is added and $\frac{1}{10}$ of the result taken; to this 5 is added and $\frac{1}{13}$ of the result taken, giving $1\frac{1}{2}$; what is the number?

31. Find all the numbers of 5 digits divisible by 9, which have unity for their first and last digits and 2 for their middle digit. State the principle upon which you proceed.

32. On a stream, *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back again in 5 hr. 15 min., and from *A* to *C* in 7 hr. How long would it take to go from *C* to *A*?

33. If the price of bricks depends upon their magnitude, and if 100 bricks, of which the length, breadth and thickness are 16, 10 and 8 inches respectively, cost R2. 9*a.*, what will be the price of 921600 bricks which are one-fourth less in every dimension?

34. There are two mixtures of wine and water, the quantities of wine in them being respectively ·25 and ·75 of the mixtures. If 2 gallons of the first be mixed with 3 gallons of the second, what will be the ratio of wine to water in the compound ?

35. How much per cent. must be added to the cost price of goods that a profit of 20 per cent. may be made after throwing off a discount of 10 per cent. from the labelled price ?

36. Determine the least number, by which 616 must be multiplied so as to produce a number exactly divisible by 770.

37. Multiply the sum of $2\cdot\dot{4}$ and $7\cdot\dot{5}$ by $1\cdot\dot{3}$, and add the result to the difference of $2\cdot3\dot{6}\dot{4}$ and $1\cdot6\dot{9}\dot{7}$.

38. The floor of a room is 50 ft. long and 40 ft. wide. Find the cost of supplying it with carpet, 2 ft. wide, at R3 per yard, and oil-cloth, 2 yards wide, at R1 per yard ; the oil-cloth to be laid along the sides and ends a yard and a half wide, and the carpet to extend one foot over the oil-cloth everywhere.

39. On a certain evening half an hour after sunset a watch was set at 12 o'clock. The morning following it was 8 minutes past 4 by a common clock when it was 4 minutes past 8 by this watch. Find the time of sunset the previous evening.

40. *A* has shares in an estate to the amount of $(\cdot15 \div \cdot3\dot{6})$ of it. *B* has shares in the same estate to the amount of $\cdot4\dot{7}\dot{2}$ of it. Find the difference in value between the properties of *A* and *B*, when $\cdot0\dot{5}\dot{6}$ of the estate is worth £373·3.

41. Three equal glasses are filled with mixtures of spirit and water ; the proportion of spirit to water in each glass is as follows : in the first glass as 2 : 3, in the second glass as 3 : 4, and in the third as 4 : 5. The contents of the three glasses are emptied into a single vessel ; what is the proportion of spirit and water in it ?

42. If the true discount on a bill of £14641 be £4641 at 10 per cent. compound interest, how many years has the bill to run ?

43. Twenty-fifth part of a certain number is equal to the seventh part of 42 ; what is the number ?

44. Simplify $\frac{2\frac{1}{4}}{13\frac{1}{8}}(4\frac{1}{8}$ of $6\frac{3}{7}+\frac{3}{11})\div4\frac{1}{8}$ of $(6\frac{3}{7}+\frac{3}{11})$.

45. A company of Sepoys proceed in 5 equal rows, and after sometime arrange themselves into 7 equal rows. Find the least number above 1000, which the company may contain.

46. *A* is twice and *B* is just as good a workman as *C*. The three work together for two days, and then *A* works alone for half

a day, and *B* for a day. How long would it have taken *A* and *C* together to complete as much as the three will have thus performed ?

47. A steam-ship whose speed averages 14 miles an hour, reaches a certain port in 12 days ; how many days afterwards will a sailing vessel arrive, which started at the same time and sailed on an average 8 miles an hour ?

48. From a cask of wine $\frac{1}{3}$ is drawn off and the cask is filled up with water ; $\frac{1}{3}$ of the mixture is then drawn off and the cask is again filled up with water ; after this process has been repeated 4 times, what will be the ratio of wine to water in the resulting mixture ?

49. The sum of £2100 is due in 4 years, but it is paid by instalments as follows :—£275 at the end of 2 years, £460 at the end of the 3rd year, £500 at the end of the 4th year, and £600 at the end of the 5th year. What amount should be paid at the end of the 6th year, in order to clear off the balance, simple interest being reckoned at the rate of 5 per cent. per annum ?

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50. Twenty times a certain number is equal to 7 times 40 ; what is the number ?

51. What is the least number of shot each $1\frac{4}{5}$ oz., that will weigh an integral number of pounds ?

52. A *rod of brick work* contains 306 cu. ft. ; find the cost of building a brick wall, 68 yd. by 6 ft. by 2 ft. 2 in., at R18 per rod.

53. How long would a column of men, extending 3420 feet in length, take to march through a street, a mile long, at the rate of 58 paces in a minute, each pace being $2\frac{1}{2}$ feet ?

54. 195 men are employed to work on a railway embankment, $1\frac{1}{2}$ miles long, which they are expected to finish in 4 weeks. But at the end of 1 week it is found that they have finished only 520 yards. How many more men must be engaged to finish it in the required time ?

55. *A* is a cask containing 125 gallons of wine ; *B* is another cask containing 175 gallons of water. 100 gallons are drawn from each, mixed together, and the casks are refilled with the mixture. This operation is once more repeated. Find the ratio of wine to water in each cask now.

56. A person who pays 5*d.* in the £ income-tax finds that a rise of interest from 6 to $6\frac{1}{2}$ per cent. increases his income by £23. 10*s.* What is his capital ?

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57. From a certain number I take 320 ; to the remainder I add 24 ; I multiply the sum by 8, and find that the product is equal to the sum of 304 and 760 ; what is the number ?

58. What decimal of 2·25 units is ·05 of a unit ?

59. A jar can be exactly filled by glasses holding 3 pints each ; it can be exactly emptied again by glasses holding 5 pints each ; given that the capacity of the vessel is between 11 and 12 gallons ; find the exact capacity.

60. Two clocks are set right at noon on Monday. One loses and the other gains 1 min. a day. What time will be indicated by the latter, when the former points 10 h. $49\frac{1}{11}$ m. P. M. on the following Saturday ?

61. Three gardeners working all day can plant a field in 10 days, but one of them having other employment can work only half time. How long will it take them to complete the work ?

62. One vessel contains 20 gallons of wine ; another contains 20 gallons of water. One gallon is taken from each, and poured into the other. This is done 3 times. Find the strengths of the two mixtures.

63. A gentleman bequeaths his property to his children to be so divided that their shares shall be equal on their coming to age at 21, counting interest and discount at 5 per cent. He dies worth £13240, leaving three children aged 23, 21 and 19 respectively. How much should each receive ?

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64. To a certain number I add 7, I multiply the sum by 5, I divide the product by 9, and take 3 from the quotient ; the remainder is 12 : what is the number ?

65. Simplify $(\cdot5+\cdot75)(2\cdot5-\cdot4)\div\left(125+\frac{1}{4\cdot8}\right)$.

66. Find the weight in tons per sq. mile of a rain-fall of 7 inches, having given that a cu. ft. of water weighs 1000 oz.

67. *A*, *B* and *C* are employed on a piece of work. After 15 days *A* is discharged, $\frac{1}{3}$ of the work being done. *B* and *C* continue at the work, and after 20 days more *B* is discharged, $\frac{1}{7}$ more of the work being done. *C* finishes the work in 30 days. In what time would the work have been done, if *A* and *B* had continued to work ?

68. If one man walks 165 miles in 6 days, how far will another man walk in 15 days, if the first man walks $3\frac{3}{4}$ miles in the same time that the other man takes to walk 4 miles ?

69. If 3 cubic inches of iron and 2 cubic inches of water weigh as much as 2 cubic inches of iron and 9 cubic inches of water ; find the ratio of the weight of a cubic inch of iron to that of a cubic inch of water.

70. I buy goods for R600, and sell them directly for R680, giving three months' credit; what is gained per cent. per annum?

71. From the tenth part of a certain number I subtract 10, and find that the remainder is 10; what is the number?

72. $\frac{4}{5}$ of a number exceed the sum of its third and fourth parts by 26; what is the number?

73. Two cog-wheels, having 75 and 130 teeth respectively, are working together; after how many revolutions of the smaller wheel will the teeth which once touch, touch again?

74. A train leaves *P* for *Q*, at the same time that a train leaves *Q* for *P*; the trains meet at the end of 6 hours, the train from *P* to *Q* having travelled 8 miles an hour more than the other. Find the rates of the trains, the distance from *P* to *Q* being 162 miles.

75. If 1000 rupees a month be equivalent to £1112. 10*s*. a year, what is the value of a rupee in English money?

76. Divide £20 among 2 men, 3 women and 4 children, so that each woman gets twice as much as a child, and each man as much as a woman and a child together.

77. If the interest of £253. 2*s*. 6*d*. at 5 p. c. be equal to the discount on £257. 6*s*. 10$\frac{1}{2}$*d*. for the same time and at the same rate, when is the latter sum due?

78. Find a number such that if it be subtracted 25 times from 7201 the remainder will be 951.

79. How many parcels of gold dust, each weighing 17·35 grains, can be made up out of 1 lb. 2 oz. 1 dwt. 3 gr.; and how much will remain over?

80. A room is 20 ft. long, 15 ft. wide and 10 ft. high. There are in it 4 doors, each 7 ft. by 4 ft.; the fireplace is 6 ft. wide and 4 ft. high; a skirting 2 ft. deep runs round the walls. Find the expense of papering the room at 6 annas a sq. yd.

81. If the hands of a clock coincide every 65$\frac{1}{2}$ min. (true time), how much does the clock gain or lose in a day?

82. *A* can copy a certain manuscript in 17 hours by writing at the rate of 3 lines per minute; *B* can copy the same in 24 hours. After 476 lines have been copied by *A*, in what time can *B* finish it?

83. A town contains 12 Hindus to every 3 Mahomedans and to every 3 Christians; if there are 4800 Hindus, find the number of Christians.

84. Two sums, each of £138. 2*s.* 6*d.*, being due, one at the present time and the other 12 months hence, how much ought to be paid 6 months hence to clear off both debts, interest being 4 p. c. per annum.

85. The difference between two numbers is 375, and one of them is 7809 ; what is the other ?

6. Simplify

$\frac{1664}{1105}$ of $\{\frac{7}{10}$ of £$3\frac{3}{14}+6\frac{2}{3}$ of £3. 0*s.* $9\frac{3}{4}$*d.* $-4\frac{13}{31}$ of £3. 2*s.*$\}$.

87. A fruit-seller has 1134 mangoes and 630 oranges. He forms them into heaps keeping the mangoes and oranges separate, and having the same number of fruits in each heap. If these heaps are as large as possible, how many fruits are there in each ?

88. A cistern, the cubic content of which is 360 cu. ft., has two pipes which can empty it in 3 and 4 hours respectively. It has also a third pipe with an orifice of 1 sq. ft., through which water flows into the cistern at the rate of 1 yd. per minute. If all the three pipes be opend together when the cistern is full, in what time will it be emptied ?

89. If 4 men or 6 women can do a piece of work in 20 days, in what time will 3 men and 2 women do it ? On what supposition will the numerator of the fraction in your answer represent the number of *hours* they worked on the day to which the fraction refers ?

90. Divide £1140 among *A*, *B*, *C*, in such a way that *A* may get half as much again as *B*, and *B* half as much again as *C*.

91. A dealer buys 10 horses at ₨400 each, 8 horses at ₨500 each and 4 horses at ₨600 each. He keeps the horses for 6 months, during which time each costs ₨15 a month, and sells them clearing $12\frac{1}{2}$ p. c. on his original outlay after paying all his expenses. Find the average selling price of each horse.

92. A carriage and a horse are together worth ₨1200 ; if the carriage is worth ₨200 more than the horse, how much is the horse worth ?

93. The population of a town is 60,000 ; if the births are 1 in 20, and the deaths 1 in 30 annually, what will the population become in one year ?

94. A cistern, 9 ft. by 6 ft. by 5 ft., is emptied in 15 minutes by a pipe whose cross-section is 36 sq. in. ; how fast does the water flow in the pipe ?

95. A race-course is $2\frac{1}{2}$ miles round. Four men start to walk round it. They walk at the rate of $3\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{1}{2}$ and 5 miles per hour.

How long will it be before they all meet again at the starting point ?

96. 40 lb. Troy of standard gold containing 11 parts in 12 of pure gold, is coined into 1869 sovereigns ; calculate in grains the weight of pure gold in a sovereign.

97. Divide ₨7. 5*a.* into two parts, one of which is $\frac{5}{8}$ of the other.

98. If mangoes be bought at the rate of 13 for a rupee, how must they be sold to gain 30 per cent. ?

99. *A* has £324 ; *B* has £29 less than *A* ; and *C*, if he had £205 more than what he has, would have as much as the double of *A* and *B* together : how much has *C* ?

100. In how many years will the error amount to a day in considering the year to consist of $365\frac{1}{4}$ days instead of 365·242218 ?

101. The circumferences of two wheels measure 168 and 401 inches respectively ; find the largest cogs which can be cut in each that they may work together.

102. The hands of a clock which gains uniformly at the rate of 15″ a day were set at sunset on the evening of the first of the month at 6 o'clock. The true time of sunrise on the 3rd was known to be a quarter to six, but the clock indicated a quarter past six. Find the error made in setting the clock on the 1st.

103. A train travels 30 miles an hour when it does not stop, and 25 miles an hour including stoppages ; in what distance will the train lose one hour by stoppages ?

104. Divide ₨123 among *A*, *B*, *C*, so that as often as *A* gets ₨3 *B* shall get ₨$2\frac{1}{2}$, and as often as *B* gets ₨4 *C* shall get ₨$3\frac{1}{2}$.

105. A merchant buys 4000 maunds of rice, $\frac{1}{5}$ of which he sells at a gain of 5 p. c., $\frac{1}{4}$ at a gain of 10 p. c., $\frac{1}{8}$ at a gain of 12 p. c., and the remainder at a gain of 16 p. c. If he had sold the whole at a gain of 11 p. c., he would have made ₨728 more. What was the cost of the rice per maund ?

106. A man sold 16 oranges to *A*, to *B* 4 more than to *A*, to *C* 5 less than to *B* ; had he sold 3 less to each he would have left only one-third of what he had ; find how many he had at first.

107. Simplify $\left\{\dfrac{1\frac{2}{3} \div 1\frac{3}{4}}{1\frac{4}{5} \div 1\frac{5}{6}} \div \dfrac{1\frac{6}{7} \div 1\frac{7}{8}}{1\frac{8}{9} \div 1\frac{9}{10}}\right\} \div \left\{\dfrac{\frac{1}{7} \div \frac{1}{9}}{\frac{1}{11} \div \frac{1}{13}} \div \dfrac{\frac{1}{17} \div \frac{1}{18}}{\frac{1}{19} \div \frac{1}{20}}\right\}.$

108. A room is 18 ft. long ; and the cost of carpeting it is ₨72. If the breadth of the room were 4 ft. less, the cost would be ₨54 ; find the breadth of the room.

109. *A* can mow $2\frac{1}{2}$ acres of grass in $6\frac{2}{3}$ hours, and *B* $2\frac{1}{3}$ acres in $5\frac{1}{3}$ hours; in what time will they together mow a field of 10 acres, and how many acres will each mow?

110. The cost of 12 md. of wheat and 10 md. of gram is R50 when gram is at R2 per md. What is the price per md. of gram when 8 md. of rice and 6 md. of gram cost R34, the price of rice being $\frac{1}{4}$ higher than that of wheat?

111. Divide R20. 4*a*. among 5 persons so that the share of each (except the first) may be double of the shares of all who come before.

112. A merchant bought a 50-gallon cask of wine for R741. Supposing it to have lost 4 gallons, at what price per dozen bottles (nine bottles holding a gallon) should he sell it in order to gain 15 p. c. upon the whole original cost?

113. A man lost as much by selling 20 chests of tea at R620 per chest as he gained by selling 25 chests at R692 per chest: what did each chest cost him?

114. A man left his property to two sons and a daughter; to the elder son he left $\frac{1}{4}$ of his property, to the younger son $\frac{1}{3}$, and to the daughter the rest, which was R4000 less than what the two sons together received: what was the entire property?

115. Three lines of paling run side by side for a distance of 864 yards. The rails are respectively 4, 6 and 9 feet apart. How often will a person walking outside the palings, on looking across them, see three rails in a line?

116. Three persons, *A*, *B*, and *C*, who can walk respectively 2, 3, and 4 miles per hour, start from the same place *P* at intervals of an hour. *A* starts first, and as soon as *B* has caught him up, *B* returns to the station *P*; find where he will meet *C*.

117. A fraudulent tradesman uses a yard measure one inch too short; what does he gain by his dishonesty in selling 20 yd. of cloth at R1. 2*a*. per yard?

118. *A*, *B*, *C* had each a cup of tea, containing 4 oz., 5 oz. and 6 oz. respectively. They blended their teas and then refilled their cups from the mixture; how much of the teas of *A* and *B* are contained in *C*'s cup?

119. If by selling wine at R6 per gallon I lose 25 per cent., at what price must I sell it to gain 25 p. c.?

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120. A man, having lived at the rate of £300 a year for 6 years, finds himself in debt, and reduces his expenditure £250 a year; he is out of debt in 4 years: what is his income for

121. Express the sum of $\cdot\dot{5}7142\dot{8}$ of a viss, $\frac{3}{8}$ of $\frac{1}{3\frac{3}{8}}$ of $\frac{217}{384}$ of a maund and $\frac{3801}{10188}$ of a cwt. as a decimal of one ton. [One viss = 3 lb. 2 oz.; one maund = $82\frac{2}{7}$ lb.]

122. A rectangular cistern, 12 ft. long, 10 ft. wide and 4 ft. 3 in. deep, is filled with liquid which weighs 2040 lb. How much deep must another cistern be, which will hold 196 lb. of the same liquid, its length being 7 ft. and width 3 ft. 6 in.?

123. *A* can run 100 yd. in 12 sec., and *B* in 13 sec. How much start in distance must *A* give *B* in order that they may run a dead heat?

124. The Fort Barracks are lighted with gas from 100 burners. Find the cost of lighting them per night of 10 hours at the rate of R$5\frac{1}{2}$ for 1000 cu. ft. of gas, assuming that for the first 3 hours each burner consumes 1 cu. in. per second, and during the remainder of the night the light is so reduced that the consumption of gas by each burner is only $\frac{3}{4}$ of that quantity per second.

125. 120 coins consist of crowns, half-crowns and florins; the values of the crowns, half-crowns and florins are as 25 : 10 : 6; how many half-crowns are there?

126. A merchant sells 60 md. of rice at a profit of 8 p. c. and 94 md. at a profit of 10 p. c.; if he had sold the whole at a profit of 9 p. c. he would have received 17 annas less than he actually did: how much per md. did he pay for the rice?

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127. A man having a certain number of mangoes to dispose of, sells half of what he has and one more to *A*, half of the remainder and one more to *B*, half of the remainder and one more to *C*., half of the remainder and one more to *D*; by which time he has only one left; find how many he had at first.

128. Simplify $\frac{3}{5}+\frac{5}{8}-\frac{2}{5}$ of $\frac{7\frac{1}{2}-5\frac{1}{4}}{1\cdot625}+\cdot 054\dot{7}4358\dot{9}$.

129. A dollar being worth 4*s.* 2*d.* and a rouble 3*s.* $1\frac{1}{2}$*d.* find the sum of money which can be paid by an exact number of either dollars or roubles, the number of roubles exceeding the number of dollars by 20.

130. *A* can do a piece of work in 15 days, *B* in 12 days and *C* in 10 days. All begin together; *A* leaves after 3 days, and *B* leaves 2 days before the work is done. How long did the work last?

131. A tank is 300 yd. long and 150 yd. broad; with what velocity per second must water flow into it through an aperture 2 ft. broad and $1\frac{1}{2}$ ft. deep, that the level may be raised 1 ft. in 9 hours?

132. The height of the top of a flag-staff standing on a tower is 110 ft., and the height of the tower is 6 ft. more than 12 times the length of the flag-staff; what is the length of the flag-staff?

133. A merchant buys some cloth at such a price that by selling it at R4. 6*a*. per yd. he will gain 5 p. c. on his outlay. What percentage will he gain or lose if the cloth be sold at R3. 14*a*. per yd.?

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134. I wish to buy an equal number of 3 kinds of toys, worth respectively 1*s*., 1*s*. 6*d*. and 2*s*. 6*d*. each; how many can I get for £10?

135. In a book on Arithmetic an example was printed thus:

"Add together $\frac{1}{6\frac{2}{3}}$, $\frac{1}{5\frac{5}{6}}$, $\frac{1}{\ }$, $\frac{1}{8\frac{3}{4}}$"

the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{541}{840}$; required the missing denominator.

136. Find the side of a square court-yard, the expense of paving which at 3*s*. 9*d*. per sq. yd. was £42. 3*s*. 9*d*.

137. *A* and *B* start at the same time from Calcutta to Hughli and from Hughli to Calcutta respectively, each walking at the rate of 4 miles an hour. After meeting *B*, *A* increases his rate to $4\frac{1}{2}$ miles an hour, and arrives at Hughli in $1\frac{1}{2}$ hours from that time. After meeting *A*, *B* reduces his rate to $3\frac{1}{2}$ miles an hour. In what time will he reach Calcutta?

138. If the rent of a farm of 24 acres be £39, what will be the rent of another farm of 36 acres, 5 acres of the former being worth 6 acres of the latter?

139. A purse contains £8 . 7 . 11, made up of pennies, shillings, half-crowns and crowns, the numbers of which are proportional to 7, 3, 2 and 5 respectively; how many of each coin are there in the purse?

140. Calculate the profit per cent. made by a book-seller, assuming that he pays 11*s*. 4*d*. for a 16-shilling book and receives 25 copies for 24.

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141. A person mixes together 10 lb. of tea at R1. 4*a*. a lb, 12 lb. at R1. 6*a*., and 14 lb. at R1. 8*a*. He reserves 6 lb. of the mixture for himself and sells the remainder at R1. 13*a*. 4*p*. a lb. How much does he gain in money?

142. Multiply ·047321 by 12172·8144, using only 3 lines of multiplication.

143. Three men, the length of whose strides are 2 ft. 6 in., 3 ft. and 3 ft. 6 in., walk a mile. How often do they step together?

144. *A* and *B* start on a bicycle race. *A* has 10 minutes start, during which he goes $2\frac{1}{2}$ miles; *B* rides at the rate of 16 miles an hour. Which will win in a race of 40 miles?

145. If 3 soldiers or 10 coolies can dig 150 cu. ft. of earth in 5 days, how many coolies must be employed to assist 7 soldiers in removing 580 cu. ft. of earth so as to get it done in 4 days?

146. 12*s*. $3\frac{1}{4}$*d*. is divided among men, women and children, whose numbers are proportional to 3, 5 and 7 respectively; if a man receives $5\frac{1}{2}$*d*., a woman $3\frac{1}{4}$*d*. and a child $2\frac{1}{3}$*d*., find the number of men.

147. An article was sold so as to gain 5 p. c. on its cost price If it had been bought at 5 p. c. less, and sold for 1*s*. less, 10 p. c. would have been gained. Find the cost price.

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148. A wine merchant bought 7 gallons of wine at 17*s*. a gallon and 5 gallons at 15*s*. a gallon; he mixed the whole and added some water. The whole mixture he put into quart bottles which cost him 8*s*. 6*d*. and sold each bottle at 4*s*. and gained £1. 17*s*. 6*d*. on the whole. How much water did he mix?

149. Find the value of $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 + $\frac{1}{3}$ of £140. 10*s*. 6*d*. + $\frac{3}{7}$ of 21*s*

150. The weight of water contained in a rectangular cistern, 8 ft. long, 7 ft. wide, is $93\frac{3}{4}$ cwt. Find the depth of water in the cistern, supposing a cu. ft. of water to weigh 1000 oz.

151. 25 men are employed to do a piece of work, who could finish it in 20 days; but the men drop off by 5 at the end of every 10 days: in what time will the work be finished?

152. If 48 men, working 8 hours a day for one week, can dig a trench 235 ft. long, 40 ft. wide and 28 ft. deep; in what time can 12 men, working 10 hours a day, form a railway cutting of 131,600 cu. yards? [A week=6 working days.]

153. The sum of areas of two circles, of which the diameters are as 3 is to 4, is equal to the area of another circle 10 ft. in diameter; find the diameters of the two circles, having given that areas of circles are to one another as the squares of their diameters.

154. A merchant sells sugar to a tradesman at a profit of 50 per cent.; but the tradesman becoming bankrupt pays only 5 annas in the rupee. How much per cent. does the merchant gain or lose by the sale?

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155. How many parcels of 6 lb. and 8 lb. each can a grocer make out of a hogshead of sugar, weighing 4 cwt. 3 qr. 14 lb., so as to have the same number of parcels of each sort?

156. *A* had 10*s*. in his purse, and *B* having paid *A* $2 \times \frac{3\frac{1}{7}}{1\frac{3}{8}}$ of £1. 11*s*. 6*d*., finds that he has remaining $\frac{1}{22}$ of the sum which *A* now has; what had *B* at first?

157. A number is exactly divisible by 11; but when divided by 5, 6 or 8 leaves always the remainder 1: find the least number which satisfies these conditions.

158. A boat's crew row over a course of 2½ miles against a stream, which flows at the rate of 3 miles an hour, in 30 minutes. The usual rate of the stream is one mile an hour. Find the time which the boat would take in the usual state of the river.

159. If the cost of 11 miles of iron rails be R55000 when iron is selling at R95 a ton, what will be the cost of 19 miles of the same rails when iron is selling at R105 a ton?

160. A circular plate of gold, 10 in. in diameter and 2 in. thick, is melted and formed into two other circular plates, each 1 in. thick, whose diameters are as 3 to 4; find the diameters.

161. A man buys goods for R750, and sells ⅓ of them at a loss of 4 p. c.; by what increase per cent. must he raise that selling price in order that by selling the rest at the increased rate he may gain 4 p. c. on the whole transaction?

162. A person gives 53 guineas for 184 gallons of wine; how much water must he add to it, if he wishes to sell it at 5*s*. 3*d*. a gallon and make a profit of 7 half-guineas?

163. A vessel containing 21·84375 gallons of water is emptied by a pitcher which contains when full ·078125 gallon. How many times can the pitcher be filled entirely, and what fraction of a pint will it contain when the last quantity of water is poured into it?

164. A room is 8 yd. long; the cost of carpeting it is R94. 8*a*., and that of papering is R86. 10*a*. If the breadth of the room were 1 yd. more and its height 1 ft. less, the cost of carpeting would be R110. 4*a*. while the cost of papering would remain the same. Find the breadth and height of the room.

165. *A* and *B* run a race; *A* has a start of 40 yd., and sets off 5 min. before *B*, at the rate of 10 miles an hour. How soon will *B* overtake him if his rate of running is 12 miles per hour?

166. If the gas for 5 burners, lighted 5 hours every evening for 10 days, cost R3. 12*a*, what will be the cost of 75 burners which are lighted 4 hours every evening for 15 days?

167. Find the three highest integral numbers whose sum is under a thousand, so that the first may be ⅔ of the second and the second ¾ of the third.

168. A tradesman sells one kind of sugar at 3*a*. per seer and loses 20 p. c., and another kind at 5*a*. per seer and gains 25 p. c. He mixes the two together in equal proportions and sells the mixture at 6*a*. per seer. What is now his gain per cent. ?

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169. Two equal sums are divided, the one among 35 men, and the other among a certain number of women ; each man received ₨1. 4*a*. and each woman 10 annas less ; how many women were there ?

170. Simplify $\dfrac{\frac{4}{9} \text{ of } 1\frac{1}{2} - \frac{2}{3} \text{ of } \frac{5}{8}}{\frac{9}{100}(3\frac{4}{5} - \frac{1}{45} + 5\frac{1}{6})} \div \dfrac{6\frac{7}{8}}{4 + \dfrac{1}{\frac{5}{6} \text{ of } 2\frac{4}{15}}}$

171. Three equal circular wheels revolve round a common horizontal axis ; the first makes a revolution in $5\frac{1}{3}$ minutes, the second in $2\frac{6}{7}$ minutes and the third in $3\frac{3}{7}$ minutes. Three marks, one in each wheel, are in a horizontal line at a certain moment. What is the shortest interval after which they will be in a horizontal line again ?

172. *A* can do a piece of work in 6 hours, *B* in 8 hours and *C* in 10 hours ; how long will it take *C* to complete a piece of work, $\frac{1}{2}$ of which has been done by *A* working 7 hours and *B* working 8 hours ?

173. *A* walks $2\frac{1}{2}$ miles in 40 min., taking exactly a yard each step ; in what time will *B* walk $4\frac{3}{8}$ miles when his stride is 40 in. and he takes 21 steps while *A* takes 22 ?

174. Three persons *A*, *B*, *C*, agree to pay their hotel bills in the proportion 4 : 5 : 6. *A* pays the first day's bill which amounts to £1. 5*s*. 5*d*. ; *B* the second which amounts to £1. 16*s*. 1*d*. ; and *C* the third which amounts to £1. 18*s*. 6*d*. ; how must they settle their accounts ?

175. A person bought a French watch bearing a duty of 25 per cent., and sold it at a loss of 5 per cent. ; had he sold it for £3 more, he would have cleared 1 per cent. on his bargain. What had the French maker for the watch ?

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176. An equal number of men, women and boys earn ₨165 in 6 days. If a woman earns 13*a*. 4*p*. a day, a man 8*a*. more, and a boy 8*a*. less, how many are there of each ?

177. What sum increased by $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{20}$ of itself, amounts to £2463 ?

178. The length, width and depth of a cistern are 8 ft, 5 ft. 4 in. and 4 ft 6 in respectively. How many gallons does it contain, having given that a cubic foot of water weighs 1000 oz. and that a pint of water weighs a pound and a quarter ?

179. *A* and *B* are termini of a railway 144 miles long. A fast train starts from *B* at 9 A. M.; another fast train travelling at the same rate, starts from *A* at 10 A. M. A slow train starts from *B* at 10-20 A. M.; the fast train from *A* meets the other fast train at 11-30 A. M, and the slow train at 12-32 P. M. Find the rates at which the trains travelled.

180. If R1 = 1*s.* $10\frac{1}{2}$*d.*, £1 = 4·84 dollars, and 1 dollar = 5·2 francs, find the value in francs of 10 lacs of rupees.

181. Three merchants, *A*, *B*, *C*, trading with a capital of £3850, find after a certain time that their respective shares are increased by £65 . 7 . 6, £59 8 . 7 and £66 . 13 . 11; how much did *A* subscribe to the original capital?

182. A grocer buys 200 lb. of tea, and sells 180 lb. for the same amount that he gave for the whole. The rest he sells at a profit of 20 per cent. What is his gain per cent. on the whole outlay?

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183. The large wheel of an engine is 20 ft, and the small wheel 12 ft., in circumference. If the large wheel slips on an average 2 inches in every revolution, how many revolutions will the small wheel make more than the large one in going a distance of 12 mi. 1728 yd.?

184. Calculate correctly to 7 places of decimals the value of

$$\frac{1}{9}+\frac{1}{3 \cdot 9^3}+\frac{1}{5 \cdot 9^5}+\frac{1}{7 \cdot 9^7}+\dots\dots\dots$$

185. The circumferences of the wheels of a carriage are $6\frac{3}{14}$ ft and $8\frac{1}{18}$ ft.; what is the least distance in which both the wheels will simultaneously complete an integral number of revolutions? How often will the lowest points of the two wheels at starting touch the ground together in 10 miles?

186. In a 200-yd. race *A* beats *B* by 20 yd., and *C* by 40 yd. By how many yards can *B* beat *C* in a 100-yd. race?

187. On a piece of work 2 men and 5 boys are employed, who do $\frac{1}{2}$ of it in 6 days; after this 1 man and 1 boy more are put on, and $\frac{1}{3}$ more is done in 3 days; how many more men must now be put on if the work is to be completed in 1 day more?

188. *A*, *B*, *C* invest capital to the amount of £800, £600 and £500; *A* was to have $\frac{2}{3}$ of the profits which amount to £330; find *C*'s share of the profits.

189. A tradesman defrauds his customers (i) by an adulteration of the article to the extent of 7 per cent., (ii) by using a balance which indicates 1 lb. when the amount in the other scale is really

15 oz. Which of the two practices is the more fraudulent, and to what extent is the customer cheated when he orders 1 lb. of the commodity?

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190. Find the distance between two towns when R309. 5*a*. 4*p*. is paid for the fare of 17 first class passengers at 1*a*. 8*p*. a mile, of 26 second class at 1*a*. 2*p*. a mile, and of 40 third class at 8*p*. a mile.

191. Find the value of $\left\{\frac{3\frac{1}{2} \text{ of } 5\frac{5}{8}}{2\frac{5}{6} \text{ of } 3\frac{1}{3}} \div \frac{2\frac{2}{11} \text{ of } 1\frac{1}{2}}{3\frac{1}{9} \text{ of } 7\frac{5}{7}}\right\}$ of $\frac{1s.\ 5d.}{4s.\ 7d.}$ of $\frac{2 \text{ ft. } 3 \text{ in.}}{5 \text{ ft. } 5 \text{ in.}}$ of 24 weeks 4 days 19 hours.

192. How many poles of fencing are required to enclose a square park containing 27 ac. 12 po. 1 yd.?

193. *A*, *B*, *C* can do a piece of work in 6, 8, and 10 days respectively. They begin to work together; *A* continues to work till it is finished, *B* leaving off 2 days, and *C* 1 day before the work is completed. In what time is the work finished?

194. If the supply of a number of persons with bread at $7\frac{1}{2}d.$ the loaf for 31 days cost £27. 18*s*.: what will it cost to supply $\frac{2}{3}$ of that number for 20 days at $6\frac{1}{4}d.$ the loaf?

195. *A*, *B*, *C* purchase a farm for R10000, of which *A* pays R4000; they sell it so as to gain a certain sum, of which *B* takes R275 and *C* R175; find *A*'s share of the profit.

196. One company guarantees to pay 5 per cent. on shares of 1000 rupees each; another guarantees to pay $4\frac{5}{8}$ per cent. on shares of 75 rupees each; the price of the former is 1245 rupees and of the latter 85 rupees. Compare the rates of interest which the shares return to the purchasers.

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197. If 5000 people took in hand to count a billion of sovereigns, and beginning their work at the commencement of the year 1852, could each count on the average 100 sovereigns in a minute (without intermission), when would they finish their task?

198. The total area of three estates is 1768 acres. If the areas of the two smaller estates be respectively three-fifths and two-thirds of that of the largest, find the acreage of each.

199. There are 3 pendulums, the first makes 35 beats in 36 seconds, the second 36 beats in 37 seconds, and the third 37 beats in 38 seconds. Supposing they commence together, find how many times they will again beat coincidently in 24 hours.

200. Sound travels at the rate of 1142 ft. per second; what is the distance of the thunder cloud, when the thunder succeeds the lightning at an interval of 9 seconds?

201. If 4 men and 6 women can do a piece of work in 5 days, which 5 men and 10 children can do in 4 days, or 3 women and 4 children can do in 10 days; find (i) how many men, (ii) how many women, (iii) how many children, could do the work in one day.

202. *A* and *B* enter into partnership; *A* puts into the business R5000 more than *B*, who, as acting partner, is to have a salary of R125 a month; at the end of 2 years the gross profits computed at $\frac{1}{5}$ of the capital per annum, are found to be R7000, from which *B*'s salary is to be paid: find each one's share of the net profit.

203. The 3 per cents. are at $85\frac{1}{8}$; what price should the $3\frac{1}{2}$ per cents. bear, that an investment may be made with equal advantage in either stock? And what interest would be derived by so investing 5000*l.*?

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204. Find the least sum of money that must be subtracted from £660. 7*s.* 4*d.* to make the remainder exactly divisible by 39.

205. What decimal must be added to

$$\frac{\frac{5}{2}(\cdot\dot{3}-\cdot2)+\frac{1}{4}(\cdot\dot{1}+\cdot2)}{\frac{1}{7}(2\cdot\dot{3}+\frac{2}{11})+\frac{1}{11}(\frac{11}{6}-\frac{3}{7})} \text{ to produce unity?}$$

206. If gold can be beaten out so thin that one tola will form a leaf of 20 sq. yards, how many of these leaves will make up the thickness of a sheet of paper, the weight of a cu. inch of gold being $52\frac{11}{12}$ tolas and 432 sheets of the paper in thickness going to an inch?

207. A race-course is $\frac{1}{2}$ a mile long: *A* and *B* run a race and *A* wins by 10 yards; *C* and *D* run over the same course and *C* wins by 30 yards; *B* and *D* run over it and *B* wins by 20 yards; if *A* and *C* run over it, which would win, and by how much?

208. Four men are employed to reap a field and after working 5 days they have cut 10 acres; 2 more men are then put on, and the whole is finished in 3 more days. How many acres are there in the field?

209. *A*, *B* and *C* are employed to do a piece of work for R529; *A* and *B* together are supposed to do $\frac{19}{23}$ of the work, and *B* and *C* together $\frac{8}{23}$ of the work: what should *A* be paid?

210. If R16430 be invested in the Govt. $4\frac{1}{2}$ per cent. loan at 106, what is the monthly income derived? Supposing that the loan is paid off at par in 10 years, what would be the rate of simple interest on the sum invested?

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211. 120 tons of coal are purchased for £87 . 16 . 9; find, to the nearest farthing, the price at which they must be retailed per ton so that no loss may be increased; and at that price what profit will accrue?

212. Reduce to a decimal correct to 6 places :

$$\frac{1}{1.3}+\frac{1}{3\,3^3}+\frac{1}{5.3^6}+\frac{1}{7.3^7}+\ldots\ldots\ldots$$

213. Find the greatest unit of time by means of which 11 hr. 31 min. 18 sec. and 23 hr. 4 min. $27\frac{1}{2}$ sec. can both be expressed as integers.

214. A man does $\frac{3}{5}$ of a piece of work in 18 days, and then gets a boy to help him. They work together for 3 days, when the boy leaves, and the man finishes the work in $7\frac{1}{2}$ days more. How long would it take the boy to do the whole ?

215. If 10 horses and 93 sheep can be kept 9 days for £37 . 17 . 6 ; what sum will keep 45 horses and 216 sheep for 40 days, supposing 5 horses to eat as much as 76 sheep ?

216. *A* starts business with R1200, and subsequently admits *B* who brings R1600. At the end of the year *A* receives $\frac{3}{5}$ of the profits ; when was *B* admitted ?

217. A man who has a certain capital calculates that if he invest it in $3\frac{1}{2}$ per cent. stock at 91, his income will be £25 more than if he invest it in 3 per cent. stock at 88. What is his capital ?

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218. A tradesman buys 200 lb. of tea for £16, intending to gain one-fourth of his outlay by sale ; but two pounds' worth at this circulation being damaged, at what price shall he sell the remainder per lb. to gain as much upon the whole outlay as he intended ?

219. Express $\left(\frac{9}{10}+2\frac{1}{2}\right)-\left(2\frac{3}{8}-1\frac{3}{4}\right)\times\left\{\left(5\frac{1}{4}\times7\frac{2}{3}\right)\div16\frac{1}{10}\right\}$ in its simplest form.

220. The diagonal of a square court-yard is 100 ft. ; find the area.

221. Sound travels at the rate of 1140 feet a second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard $14\frac{1}{4}$ miles off ?

222. The length of the minute hand of a church clock is $5\frac{1}{4}$ feet ; what distance will the end of it travel through in 35 days, if 7 times the circumference of a circle be 22 times its diameter ?

223. Three men *A*, *B*, *C*, undertake to complete in 20 days a piece of work for R247. 8*a*. *A* furnishes 10 men for 8 days and 6 men for the remaining days ; *B* furnishes 7 men for 7 days and 12 men for 12 days ; *C* furnishes 15 men who work on alternate days only until the work is completed. Find *A*'s share of the sum.

224. A person having R8,500 in 4 per cent. Govt. bonds sells out when they are at $8\frac{1}{4}$ per cent. discount, and with the amount thus realised purchases 5 per cent. bonds which are at $6\frac{5}{8}$ per cent. premium : what does he gain or lose in annual income by the change ?

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225. A contractor employs 100 men, 40 of whom work 10 hours on week days and only 5 hours on Sundays ; the rest work 8 hours a day. If the wages of the former be 5*p*. per hour and of the latter 4*p*. per hour, what is the amount of wages paid in 4 weeks ?

226. Two chests of tea of the same size and quality are consigned to *A*, *B*, *C*. *A* at first was to have $\frac{1}{8}$ of a chest, *B* $\frac{5}{8}$, and *C* the rest. But *A*, *B* purchase $\frac{8}{11}$, $\frac{2}{11}$ of *C*'s share respectively. How much will each have ?

227. Find the side of the largest square tile, with which a court, 33 yd. 1 ft. 7 in. long and 20 yd. 11 in. broad, can be paved.

228. In a bicycle race of 2 miles over a circular course of 1 furlong, the winner in his last round overtook the second at a point in his 15th round. Their paces were as 159 to 149. At what distance was this point from the winning post ?

229. If 3 men can do as much as 7 boys in a day, how many days will it take 25 boys to finish a piece of work of which 12 men have done a quarter in 13 days ?

230. *A*, *B*, *C* hold a pasture in common for which they pay R16 per month ; they put on it 70, 50 and 40 sheep respectively. *A* sells $\frac{2}{7}$ of his flock to *B* after 4 months, and after 3 months more *C* sells $\frac{2}{5}$ of his to *A*. How much of the rent should each pay at the end of the year ?

231. A person bought 10 Bank of Madras shares at R1540 each and for 5 years got interest on his investment at the rate of $5\frac{1}{2}$ per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction, and what rate per cent. per annum had he for his money ?

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232. A certain number of cows and twice as many sheep were bought for R94. 6*a*. ; the cows cost R10 3*a*. 6*p*. each and the sheep R4. 5*a*. 3*p*. each : how many sheep were bought ?

233. The master of a ship, worth £5161. 3*s*. 9*d*., is himself owner of $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of her. He sells her for $\frac{5}{8}$ of her value ; what is his own share ?

234. The height of a square room is one-half of his breadth, and the cubic content of the room is 108 cu. yd. ; find its dimensions.

235. Two pipes, *A* and *B*, would fill a cistern in 37½ min. and 45 min. respectively. Both pipes being opened, find when the second pipes must be turned off, that the cistern may be just filled in half an hour.

236. If 13 locomotive engines, each of 290 horse-power, working 11 hours a day for 7 days a week, can convey 7315 tons of goods to a distance of 221 miles in a given period, how many hours' work a day for 6 days a week must be done by 7 locomotives of 319 horse-power each, in order to convey 4845 tons of similar goods to a distance of 154 miles in an equal period?

237. How must teas at 2*s.* a lb. and 2*s.* 9*d.* a lb. be mixed so that by selling the mixture at 2*s.* 8*d.* a lb. there may be a gain of 2*d.* per lb.?

238. If I sell 40 shares of R250 each in the Oriental Bank at 121 per cent. premium, how many shares of R1000 each in the Madras Bank at 72 per cent. premium can I buy, and how much will be left?

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239. Equal quantities of sugar, flour and rice were bought for R720. 9*a.*; the price of a md. of sugar is twice as much as that of a md. of flour, and the price of a md. of flour is twice as much as that of a md. of rice; find the cost of the sugar.

240. Find the value of $\frac{6.757}{2.17\dot{4}\dot{2}} \times \frac{.\dot{2}5\dot{9}}{2.78}$ of 12*s.* 9¾*d.*

241. A tea-merchant has a rectangular space for storing tea. It is 15¾ ft. long, 10½ ft. broad and 9⅓ ft. high. He wishes to fill this space with packets of a cubical shape, all of the same size. What is the largest size of such cubical packets that can be made to fill it exactly, and what would be the number of such packets?

242. A hare starts 40 yards before a greyhound and is not seen by him till she has been up 30 seconds. She runs at the rate of 12 and the hound at the rate of 15 miles an hour; how long will the chase last, and what distance will the hound have run?

243. If 3 men and 5 boys can reap 20 acres in 10 days, and if 5 men and 3 boys can reap 34 acres in 15 days, how many boys must assist 9 men, in order that they may reap 45 acres in 9 days?

244. A grocer bought 60 lb. of sugar of two different sorts for R16. 4*a.* The better sort cost 5*a.* per lb., and the worse 4*a.* per lb. Find how many pounds there were of each sort.

245. How much stock in the 3 per cents. must I sell to pay off a debt of £470, the price of the stock being 94⅛, and the commission of ⅛ on £100 of stock being also taken into consideration?

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246. How many four-anna pieces can be coined from 9 lb. of standard silver ?

247. Find, by Practice, the dividend on a debt of £3471, at 13*s.* 7½*d.* in the £.

248. The sides of a square are divided each into 8 equal parts, and lines are drawn through the points of division parallel to the sides. If the area of the square be 256 sq. ft., find the length of the side of each of the smaller squares, into which it is divided.

249. *A* and *B* run a mile race : at first *A* runs 5 yards to *B*'s 4, but after *A* has run half a mile he tries and runs 3 yards in the time in which he at first ran 5, *B* running at his original rate. Which wins, and by how much ?

250. If the carriage of 150 ft. of wood, that weighs 3 stones per ft., cost R30 for 40 miles, how much will the carriage of 54 ft. of wood, that weighs 8 stones per ft., cost for 25 miles ?

251. A greengrocer sells potatoes at 2*s.*, 2*s.* 6*d.* and 3*s.* 6*d.* a bushel, selling equal quantities of the first two kinds ; what quantities of each kind does he sell, if the total quantity sold is 60 bushels, and if the average price obtained is 3*s.* a bushel ?

252. A person invests 1250 gold mohur in the Govt. five per cent. rupee stock at 105. The stock is converted subsequently to 4½ per cents. at 95. Find the difference in his income, each gold mohur being considered equivalent to R17.

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253. If a person whose income is R1825 a year spend R44. 1*a.* a week for the first 20 weeks, to what must he limit his daily expenditure for the rest of the year so as not to be in debt at the end of it ?

254. What number multiplied by itself will give $109\frac{29}{36}$?

255. A cubical block of marble whose edge is 2 ft. is placed within a rectangular cistern 4 ft. long, 3 ft. wide and 2 ft. deep, which is then filled with water ; how many pounds of water must be taken out to reduce the surface 6 in. ? [A cu. ft. of water weighs 62½ lb.]

256. *A* and *B*, can do a piece of work in 2⅔ days, but when *B* works half time the work is done in 4 days. Show that *B* is twice as good a workman as *A*.

257. If 12 men and 5 women can do a piece of work in 8 days of 9 hours each ; how long will it take 3 men and 4 wemen to do a piece of work twice as great, working 8 hours a day, the work of a man being double that of a woman ?

258. Gold is 19 times as heavy as water, and copper 9 times. In what ratio should these metals be mixed that the mixture may be 15 times as heavy as water?

259. When the 3 per cents. were at 90 I found that by selling out and investing in the 4 per cents. at 95 I could improve my income by R243. What was the amount of my stock in the 3 per cents.?

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260. A person has in his drawer 15 piles of rupees, each containing 20; his servant steals them and puts in their place 15 piles, each consisting of 19 double-pice with a rupee at the top. How much does the person lose?

261. A person owes the sum of R31500, and R8500; and his property amounts to R14125 only. How much is he able to pay in the rupee; and what is the loss upon the second debt?

262. A rectangular piece of ground of 243 sq. yd. is one-third as broad as it is long; what is the distance round it?

263. A passenger train going 41 miles an hour, and 431 ft. long, overtakes a goods train on a parallel line of rails. The goods train is going 28 miles an hour, and is 713 ft. long. How long does the passenger train take in passing the other?

264. The distance by rail from Turin to Venice is 420 kilometres, and the first class fare is 56 lire; find at the same rate in Indian money, the fare from Calcutta to Benares, a distance of 480 miles, reckoning 7 lire equal to R3 and 8 kilometres to 5 miles.

265. 40 lb. of coffee, at 2*s*. 6*d*. a lb., were mixed with a certain quantity of chicory at 1*s*. 9*d*. a lb., and the resulting mixture was worth 2*s*. a lb. How many pounds of chicory were there in the mixture?

266. How much money must be invested in the 3 per cent. consols when they are at $92\frac{1}{2}$, to produce the same income as would be produced by R1520 invested in the $3\frac{1}{2}$ per cents. at 95?

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267. If R20 . 7 . 6 be gained by selling an article for R79 . 10 . 9, how much would have been gained or lost by selling it for R59 . 7 . 6?

268. Find, by Practice, to the nearest penny, the rent of 375·3675 acres at £2. 19*s*. $10\frac{1}{2}$*d*. per acre.

269. Determine, by Duodecimals, the area of a rectangle whose adjacent sides are respectively 9 ft. $3\frac{1}{2}$ in. and 6 ft. $4\frac{1}{2}$ in.

270. *A* can beat *B* by 5 yd. in a 100-yd. race, and *B* can beat *C* by 10 yd in a 200-yd. race; by how much can *A* beat *C* in a 400-yd. race?

271. If 210 coolies, in 7 days of 10 hours each, dig a channel,

1 mile long, 6 feet broad and 2 feet deep ; in how many days of 7 hours each should 35 coolies dig a channel, 660 feet long, $7\frac{1}{2}$ feet broad and $2\frac{1}{4}$ feet deep ? And how many cubic feet does each cooly dig in an hour ?

272. The average of eleven results is 30 ; that of the first five is 25, and that of the last five is 28. Determine the sixth result.

273. What amount must be invested in the $4\frac{1}{2}$ per cent. stock at $103\frac{3}{4}$, in order to obtain, after deducting an income-tax of $3\frac{1}{8}$ per cent., a clear income of R4000 a year ?

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274. 4 thalers, 6 half-crowns and 8 florins amount to £2 ; what is the value of a thaler ?

275. A reduction in the income-tax diminishes a tax, which is R15 when the tax is 8 pies in the rupee, by R3 . 12 . 0 ; what is the diminished rate of the tax ?

276. The length of a room is twice its breadth and 4 times its height, and it contains 216 cu. yards of air ; find its length.

277. *A* can reap a field in 5 days, and *B* in 6 days, each working 11 hours a day in what time could they together reap it, working 10 hours a day ?

278. If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hours a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour as 3 men of the second set can do in $1\frac{1}{2}$ hours.

279. The average weight of 5 men is 5 st. 7 lb. ; the average weight is diminished by 7 lb. ; when the weight of a boy is included ; what is the weight of the boy ?

280. A share-holder in a commercial company receives one year a dividend of 5 per cent. on his shares. The next year he receives a dividend of $7\frac{1}{2}$ per cent and finds that he is R412. 8*a.* richer. Find the amount of his shares.

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281. To march at quick step is to take 108 paces of 2 ft. 8 in. per minute ; what rate is this per hour ?

282. A society subscribed R21. 5*a.* 4*p.* to a charity, each member paying as many pies as there were members in the society ; find the number of members.

283. Find, by Duodecimals, the volume of a block of marble, 3 ft. 7 in. long, 2 ft. $3\frac{1}{2}$ in. wide and 1 ft. $2\frac{1}{4}$ in. deep.

284. A train, 880 feet long, overtook a man walking along the line at the rate of 4 miles an hour, and passed him in 30 seconds : the train reached the next station in 15 minutes after it had passed the man. In what time did the man reach the station ?

285. If 40 men and 50 boys can do a piece of work in 6 days, working 6 hours a day, in how many days will 8 men and 20 boys do a piece of work half as large again, working 7 hours a day, assuming that a man does as much work in 3 hours as a boy in 5 hours?

286. The average age of 8 men is increased by 2 years, when one of them, whose age is 24 years, is replaced by a fresh man: what is the age of the new man?

287. If the price of the 4 per cents. just before the payment of a half-yearly dividend be 93, what ought to have been the price 3 months previously, supposing no change in the value of money to have taken place during that interval?

288. The weekly wages at a mill amount to £186. 4*s.* In the mill a certain number of women are employed at 2*s.* 10*d.* a day, five times as many men at 5*s.* 6*d.* a day, and 6 times as many boys at 2*s.* 4*d.* a day: how many men are employed?

289. If the income-tax be 7*d.* in the £ in the first half of the year, and $3\frac{1}{2}$*d.* in the second, what is the net income of a gentleman whose gross annual receipts are £1542 . 10 . 6?

290. An open cistern, made of sheet iron a quarter of an inch thick, is internally $62\frac{1}{2}$ in. long, 36 in. wide and 24 in. deep; find the weight of the cistern when full of water, if iron weighs 7 times as much as water and a cu. ft. of water weighs 1000 oz.

291. In a two-mile race *A* wins, *B* being 22 yd. behind, and *C* 105 yd. behind *B*. By how much would *B* beat *C* in a three-mile race in which *A* does not run?

292. If the wages of 18 coolies for a month amount to R85 when rice is 24 seers per rupee, what ought the daily pay of a cooly to be in proportion when the price of rice is R2. 10*a.* 8*p.* per maund?

293. *A* and *B* started on a race and ran a distance exactly together. Then *B* began to fail and give up the race when he had run 56 yards farther, *A* having gone during the same time 320 yards. The average of the entire distances run by the two men was 1188 yards. What distance had they run together?

294. The £23 shares of one company pay a dividend of £1 per share; the £15 shares of another yield £·725 per share. The market value of the former is £24·92, of the latter £17. Compare the rates of interest returned to the purchasers.

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295. A man bought 100 oranges at 2 a pice, and 100 more at 3 a pice, and mixed and sold the whole at 5 for 2 pice; how much did he lose?

296. Find, by Practice, the cost of fencing 3 mi. 3 fur. 180 yd. 1 ft. 6 in. of road at £479. 15*s.* per mile.

297. An open cistern, made of sheet iron ½ inch thick, is externally 10 in. long, 8 in. broad and 5½ in. deep; find the price of the cistern at R8 per cwt., if a cu. ft. of iron weighs 4½ cwt.

298. *A* does half as much work again as *B* in the same time, and *B* does one-third as much again as *C*; working together they can do a certain work in 5 days; but if after working 2 days *A* leaves off, how long will *B* and *C* take to finish it?

299. When rice is 10 seers the rupee, 7 persons can be fed for 30 days at a certain cost. For how many days can 6 persons be fed at the same cost when rice is 14 seers the rupee?

300. If the daily wages of a labourer rise from 4*a.* 9*p.* to 6*a.*, what percentage of the increase in the price of food and other commodities will cause his position to be unaltered?

301. A person buys 5 shares in a company, and sells three of them at a gain of 10 per cent. and the remaining two at a gain of 16⅔ per cent. The gain on the latter sale is £2 . 19 . 7½ more than on the former. How much did he pay for each share?

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302. A man buys 25 seers of milk at 1*a.* 6*p.* a seer, and sells it at 1*a.* 3*p.* a seer, making a profit of 5 annas; how many seers of water did he add to the milk?

303. Now that the income-tax is 5 pies in the rupee, a person's net income is R374 per mensem; what will it be when the income-tax is raised to 7 pies?

304. Find, by Duodecimals, the area of a square whose side is 12 ft. 8 in. 4 pt.

305. A train starts from *A* at 12 o'clock and runs towards *C*, which is 100 miles distant, at the rate of 30 miles an hour; at the same time the mail coach starts for *C*, from *B*, which is half way between *A* and *C*, and runs at 10 miles an hour; at what distance from *C* will it be overtaken by the train?

306. If 13 solid inches of copper balance 17 of iron, and 15 of iron balance 16 of tin, and 19 of tin balance 12 of zinc, how many solid inches of zinc balance 2470 solid inches of copper?

307. If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman whose net annual receipts amount to R1454. 1*a.*?

308. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 stock when they have risen to 93½, and the remainder when they have fallen to 84¼, and investing the whole proceeds in the 4 per cents. at par he may increase his annual income by £9. 5*s.*?

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309. Divide R115. 2*a*. among 20 boys and 25 girls, so that each boy may receive 12 annas more than each girl; how much will each boy receive?

310. Three-fifths of the square of a certain number is 126·15; what is the number?

311. An open cistern whose capacity is 4320 gallons is externally 14·1137 ft. long, 10·25 ft. wide and 5·16 ft. deep; the sides are $1\frac{1}{2}$ in thick; find the thickness of the bottom, having given that a gallon contains 277·274 cu. inches.

312. *A* and *B* walk a race of 10 miles; *A* gives *B* 20 minutes' start; *A* walks uniformly a mile in $17\frac{1}{2}$ minutes and catches *B* at the 8th mile stone: find by how much *B* lost in time and space.

313. If 17 men can build a wall 100 yd. long, 12 ft. high and $2\frac{1}{2}$ ft. thick, in 25 days, how many men will build a wall twice the size in half the time?

314. In 1861 three towns had populations of 17650, 19600, 18760, respectively. In 1871 the population of the first had decreased 18 per cent., that of the second had increased 21 per cent., while the population of the third had increased by 4690; find the change per cent. in the total population of the three towns.

315. A gentleman invests R5600 in the $5\frac{1}{2}$ per cent. Govt. paper, and derives therefrom an annual income of R275. At what premium was the $5\frac{1}{2}$ per cent. paper at the time he invested?

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316. Find the circumference of the wheel of a locomotive, which makes 5 revolutions in a second, and which performs a journey of 30 miles in 44 minutes.

317. A man has an income of £200 a year; an income tax is established of 7*d* in the £, while a duty of $1\frac{1}{2}d.$ per lb. is taken off sugar; what must be his yearly consumption of sugar that he may just save his income-tax?

318. *A*, *B*, *C* are three spouts attached to a cistern. *A* can fill it in 20 min, *B* in 30, and *C* can empty it in 40 min If *A*, *B* and *C* be opened successively for one minute each, in what time will the cistern be filled?

319. A besieged garrison consists of 300 men, 120 women and 40 children, and has provisions enough for 200 men for 30 days. If a woman eats $\frac{4}{5}$ as much, and a child $\frac{1}{2}$ as much, as a man, and if after 6 days 100 men with all the women and children escape, how long will the remaining provisions last the garrison?

320. The price of rice being raised 50 per cent., by how much per cent. must a house-holder reduce his consumption of that article so as not to increase his expenditure?

321. The owner of 4 per cent. Govt. paper, bringing in R8976 per annum, exchanges it for 5 per cent. paper. His annual interest is increased by R44. What is the increase or decrease of his nominal capital?

322. A bill on London for £175 drawn at 6 months after sight, is purchased at Madras, the rate of exchange being 2*s.* $0\frac{3}{8}$*d.* the rupee. Four months before it becomes due, it is discounted in London at the rate of $2\frac{1}{2}$ per cent. (per annum) discount. What was paid for the bill in Madras, and what does it realise in London?

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323. A man laid out £30 15*s.* in spirits which he bought at 15*s.* a gallon; he retailed them at 17*s.* 6*d.* a gallon, making a profit of £4. 5*s.*: how many gallons must he have lost by leakage?

324. Arrange $\sqrt{2}$, $\sqrt[3]{3}$ and $\frac{4}{3}$ in order of magnitude.

325. Two trains, running at the rates of 25 and 20 miles an hour respectively on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rates as before, a person sitting in the faster train observes that he passes the other in $31\frac{1}{2}$ seconds; find the lengths of the trains.

326. If 6 dollars and 6 roubles are together worth £1. 13*s.* 9*d.*, and 4 dollars and 8 roubles are together worth £1. 11*s.* 8*d.*, what is the value of 6 dollars and 8 roubles?

327. In an examination *A* obtains 10 per cent less than the minimum number of marks required for passing; *B* obtains $11\frac{1}{9}$ per cent. less than *A*; and *C* $41\frac{3}{17}$ per cent. less than the number of marks obtained by *A* and *B* together. Does *C* pass or fail?

328. I have R6500 to invest in public securities. Will it be more to my advantage to invest it in the 5 p. c. Govt. loan which is at $10\frac{3}{4}$ per cent. discount, or to purchage at par Treasury Bills which bear an interest of 3 pies per cent. per diem? Calculate the difference.

329. If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for R540. 12*a.* be negotiated in London for £51. 10*s.*, how much per cent. below par is the rate of exchange?

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330. On Monday January 3, 1888, a man commenced to subscribe for a daily pice paper (published on week days only); what had he spent by June 13th of the same year?

331. A gentleman's income is diminished by £150; but the income-tax being raised from 6*d.* to 7*d.* in the £, he pays the same amount of tax as before; find his present income.

332. *A* and *B* start to run a race; their speeds are as 17 to 18. *A* runs $2\frac{1}{4}$ miles in 16 min. 41 sec.; *B* finishes the course in 34 min.: determine the length of the course.

333. If 5 men and 8 boys reap 9 acres in 10 days, and 4 men and 4 boys reap 3 acres in 5 days, how many acres will 2 men and 3 boys reap in 7 days?

334. To 432 gallons of a mixture of brandy and rum, which contains $8\frac{3}{4}$ per cent. of brandy, some water is added, and the proportion of brandy in the mixture is thereby diminished to $7\frac{1}{2}$ per cent. How much water is added?

335. A person who has £1900 Russian 4 per cent. stock sells out at 104 and devotes £952. 13*s.* 4*d.* to the purchase of 3 p. c. consols at 95, and lends the rest of the sum realised on mortgage. What interest must he ask for his money that his income may be the same as before?

336. If the rate of interest for money be 3 per cent., what should be the rate of exchange for bills payable at sight in England when the rate for those payable 4 months after sight is 1*s.* $8\frac{1}{4}$*d.* per rupee?

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337. A merchant buys 60 yards of cloth; he sells half of it at a gain of 3 annas per yard, and the remainder at a gain of 2 annas per yard, and realises R44. 1*a.* What was the cost price per yard?

338. A man buys a number of mangoes for R9, the price in pies of each mango being equal to the square root of the number purchased; find the number purchased and the price of each.

339. A train which travels at the uniform rate of 30·8 ft. a second, leaves Madras at 7 A. M.; at what distance from Madras will it meet a train which leaves Arconum for Madras at 7-20 A.M., and travels one-third faster than it does, the distance from Madras to Arconum being 42 miles?

340. If 5 men, 2 women and 3 boys, or 6 men and 4 boys, can mow 3 acres in 5 days; how many acres would 3 men, 2 women and one boy mow in 11 days, supposing a man to do as much work as 3 boys?

341. A person loses in his first year 23 per cent. of his capital, but in the next year he gains 40 per cent. of what he had at the end of the first year, and his capital is now R720 more than it was at first; find his original capital.

342. A person invested equal sums of money in the 3 per cents. at $97\frac{1}{2}$, and in the $3\frac{1}{2}$ per cents. at $102\frac{1}{2}$; his resulting income was £259. 10*s.* How much did he invest?

343. A merchant in London receives two bills, drawn at 4 months after sight, each for R5000; one he discounts immediately,

the rate of interest being 3 per cent. per annum; the other he keeps till maturity, and then exchanges at the rate of 1*s*. 9*d*. per rupee, and finds that he has got as much as he did for the first bill. What was the rate of exchange when the first bill was discounted?

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344. A man, having bought 128 yards of cloth for R80, sells one-fourth at a loss of 2 annas per yard; by how much must he raise that selling price, in order that, by selling the rest at the increased rate, he may gain 2 annas per yard on the whole?

345. Incomes below £150 a year being subject to 5*d*. in the £ income-tax, and incomes above £150 to 7*d*. in the £; find what income above £150 a man must have, that he may be just 7$\frac{3}{4}$*d*. a year poorer than a man who has £149. 10*s*. a year.

346. *A* and *B* run a mile, and *A* wins by 160 yd.; *A* and *C* run over the same course and *A* wins by 20 min.; *B* and *C* run over it and *B* wins by 12 min. In what time can *A* run a mile?

347. If 16 darics make 17 guineas, 19 guineas, make 24 pistoles, 31 pistoles make 38 sequins, then how many sequins are there in 1581 darics?

348. What sum must be paid on the insurance of a cargo of the value of R33575. 4*a*. so that in case of loss the cargo and all expenses of insurance may be recovered? The premium is at the rate of 4·725 per cent., policy duty 3$\frac{1}{8}$ annas per cent. and agent's commission $\frac{1}{2}$ per cent.

349. A person has £26041 of a 4 per cent. stock. He saves each year $\frac{1}{4}$ of his income, which he invests at 4 per cent. What is his income in the 4th year?

350. If gold be at a premium of 5 per cent. and a person buy goods marked 300 rupees, and offer gold to the amount of 300 rupees, what change ought he to receive in notes, 5 per cent. being abated for ready payment?

PROBLEMS. 202.

1. By what number less than 1000 must 4389 be multiplied so that the last three figures (to the right) of the product may be 438?

2. If 5 cwt. 3 qr. 14 lb. cost £6 per cwt., what will be the cost per pound when the cost of the whole has been reduced by £7. 16*s*. 8*d*.?

3. On measuring a distance of 32 yards with a rod of a certain lenght it was found that the rod was contained 41 times with half an inch over; how many inches will there be over in measuring 44 yards with the same rod?

4. Find the least number above 1000, which when divided by 5 or by 6 or by 9, will leave the same remainder 3.

5. A bill of £100 was paid with guineas and half-crowns, and 48 more half-crowns than guineas were used; find how many of each were paid.

6. *A* has twice as much money as *B*. They play together, and at the end of the first game *B* wins from *A* one-third of *A*'s money; what fraction of the sum which *B* now has must *A* win back in the second game that they may have exactly equal sums?

7. What is the smallest whole number which is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{21}$ and $3\frac{1}{7}$?

8. *A* pays £9 . 3 . 4 more rates than *B*, their incomes being equal; living in different towns they are rated at 2*s.* and 1*s.* 4*d.* in the £ respectively; what is their income?

9. A pint of water weighs a pound and a quarter, and a cu. foot weighs 1000 oz.; how many gallons are there in a cu. foot? How many gallons will fill a cistern 5 ft. long, $2\frac{1}{2}$ feet wide and 2 feet deep?

10. A gallon contains 277·274 cu. in.; a cu. ft. of water weighs 1000 oz. How many gallons weigh a ton? And what is the weight of a pint?

11. If 162 gallons fill a cistern $5\frac{1}{3}$ ft. by $4\frac{1}{3}$ ft. by $1\frac{1}{8}$ ft., find the number of cu. inches in a pint.

12. If a cu. inch of water weighs 252·458 grains, which is the more accurate of the following rough statements:—a cu. ft. of water weighs 1000 oz., a cu. yd. weighs $\frac{3}{4}$ of a ton?

13. If a decilitre be ·052 gallon, find the value of a pint of liquid which is worth 2 francs the decilitre; 1200 francs being equal to £49.

14. Three men are employed on a work, working respectively 8, 9, 10 hours per day, and receiving the same daily wages. After three days each works one hour a day more, and the work is finished in three days more. If the total sum paid for wages be £2 . 7 . $6\frac{1}{2}$, how much of it should each receive?

15. The sum of two numbers is 5760, and their difference is equal to one-third of the greater; find the numbers.

16. Two casks contain equal quantities of beer; from the first 34 quarts are drawn, and from the second 80; the quantity remaining in one cask is twice that in the other. How much did each cask originally contain?

17. Shew that if the price in rupees of a cwt. of goods is divided by 7, the result is the price in annas of a lb. weight of the goods.

18. If R72 be divided among 5 men, 7 women and 13 boys so that 2 men receive as much as 5 boys, and 2 women as much as 3 boys, how much will each man, woman and boy receive?

19. How many revolutions will be made by a wheel which revolves at the rate of 329 revolutions in 3 min. while another wheel revolving 431 times in 4 min. makes 2586 revolutions?

20. If a train goes $22\frac{1}{2}$ miles an hour, how many revolutions does the driving-wheel, 11 ft. in circumference, make in a second?

21. A game licence costs 15*s.*, and a cartridge 2*d.* A sportsman kills his bird once in 5 shots. If birds are worth 2*s.* 6*d.* a brace, how many birds must be shot just to pay expenses?

22. A vulgar fraction has for its numerator 157 and its nearest approximate value in thousandths is ·370; what is the denominator?

23. A man after a tour in England finds that he had spent every day half as many rupees as the total number of days he had been from home. His tour cost R1800. How many days did it occupy?

24. A plate of metal is beaten to the thickness of $\frac{1}{8}$ of an inch, and the weight of a circular medal cut from it, whose diameter is $1\frac{1}{8}$ inches, is $1\frac{1}{4}$ oz. Troy. If the same plate be beaten to the thickness of $\frac{1}{8}$ of an inch, what will be the weight of a medal cut out of it of the diameter of $1\frac{1}{2}$ inches, (the areas of circles being proportional to the squares of their diameters)?

25. It is said that 240,000 letters are posted in Berlin daily, 16·6 per cent. of which are town letters. This gives one letter for every 3 persons in Berlin; what is its population?

26. The French unit of linear measure is a *metre* equal to 39·371 English inches; the square formed on a line of 10 metres (called an *are*) is the French unit of surface. Find the equivalent, in English square measure, of a hectare (100 ares).

27. A rectangular swimming bath is 60 ft. long and 40 ft. broad; it can be filled by a supply-pipe in 5 days, and if 6,000 cubic feet of water be thrown in, the rest can be filled in 3 days 18 hours. Find the depth of the bath.

28. The debts of a bankrupt amount to R21345. 4*a.* and his assets consist of property worth R9167. 10*a.* 8*p.* and an undiscounted bill of R5130 due 4 months hence, simple interest being reckoned at 4 p. c. per annum. How much in the rupee can he pay his creditors?

29. The diameter of the four-wheel of a carriage is $1\frac{3}{4}$ ft. and that of the hind-wheel is 3 feet; how far will the carriage have travelled when the fore-wheel has made 100 more revolutions than the hind-wheel? (The circumference of a circle : diameter :: 3·1416 : 1).

30. Tea at $4s.\ 5\frac{1}{2}d.$ per lb. is mixed with tea at $3s.\ 7\frac{1}{2}d.$ per lb. so that the mixture contains 72 per cent. of the former. Find the weight of a chest of this mixture which is worth £6. 16*s.* 10*d.*

31. A merchant buys China tea at 3*s.* 6*d.* per lb. To improve the flavour he adds 2 oz. of Assam tea to every lb. of China tea, and finds that the mixture costs him 4*s.* per lb. How much per lb. did he give for the Assam ?

32. Standard silver, of which 111 parts in 120 are pure silver being worth R31 per lb., find the value of a Sicca Rupee which weighs 7 dwt. 12 gr. and has a fineness of 979 parts in 1000.

33. A contract is to be finished in 5 months and 17 days, and 43 men are put on to work at once ; at the end of $\frac{2}{3}$ of the time it is found that only $\frac{1}{4}$ of the work is done ; what extra number of hands will be required to complete the contract in the given time, the last employed men to work 12 hours a day, whilst the first 43 men work until the contract is completed only 10 hours a day ?

34. A man can do as much work in 4 hours as a woman in 6 hours, or as a boy in 9 hours ; how long will it take a boy to complete a piece of work, one-half of which has been done by a man working 10 hours and a woman working 16 hours ?

35. If a piece of cloth, 4 yd. long and 15 in. wide, cost R3. 2*a.*, how much should you give for another piece, 19 yd. long and 12 in. wide, every sq. in. of which is worth $\frac{3}{8}$ of the value of a sq. ft. of the former ?

36. A person sets out to walk 26 miles ; for a quarter of the distance he goes at the rate of 5 miles an hour, for half the remaining distance at 4 miles an hour and 3 miles an hour for the other half. State the exact time occupied in the journey.

37. How often between 12 and 1 are the hands of a clock an integral number of minute-spaces apart ?

38. Two clocks begin striking the hour of noon together on a centain day, the interval between every two strokes being 1″ and 2″ respectively. They gain 1″ and 2″ respectively in every 24 hours. After what length of time will they end striking the hour of noon together ?

39. *A* and *B* start at the same time on a journey. *A* walks at the rate of 4 miles an hour, and *B* of 3 miles an hour. When *A* has gone half way, *B* gets a ride and goes at twice the rate of *A*, until he has ridden a distance equal to $\frac{2}{15}$ of the whole journey beyond the spot at which he passes *A*. *B* then walks the remainder of the journey, *A* having walked it all. Will *A* or *B* arrive first ? And what fraction of the whole journey will the other still have to travel ?

40. If 15 men can dig 600 cu. ft. of earth in 5 days, working

8 hours a day, how many men would be required to dig 1575 cu. ft. in 14 days, working 9 hours a day, supposing that a man who works 8 hours a day does in 25 hours the same amount of work that a man who works 9 hours does in 26?

41. If 21 horses and 217 sheep can be kept 10 days for the same sum as it would cost to keep 9 horses and 60 sheep for 27 days, find how many sheep eat as much as 3 horses.

42. In running a four-mile race on a course half a mile round, *A* overlaps *B* at the middle of the 6th round. By what distance will *A* win?

43. *A* and *B* start to run a race at 3 o'clock. The winner comes in at $6\frac{3}{4}$ minutes past 3, beating the other by 40 yards. At 4 minutes past 3 the loser was 1140 yards from the winning-post. Find the length of the course, and the speed of the winner in miles per hour.

44. Five men do $\cdot 600\dot{6}$ of a piece of work in 2·12 hours, how long will 6 boys take to finish it, it being known that 3 men, and 7 boys have done the whole of a similar piece of work in 3 hours?

45. If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working together for 10 days?

46. The distance by Railway from Madras to Salem is $206\frac{3}{4}$ miles. A Passenger Train travelling 20 miles an hour leaves Madras at 7 A. M.; and a Special Train at 10 A. M. the same day. At what rate must the latter travel, so as just to overtake the former at Jollarpett Junction (132 miles from Madras), and at what hour must a Goods Train leave Salem for Madras travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other Trains?

47. Two trains measuring 330 ft. and 264 ft. respectively, run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before the faster train passes the other in $27\frac{1}{2}$ seconds. Find the speeds of the two trains in miles per hour.

48. A man near the sea-shore sees the flash of a gun fired from a vessel, steaming directly towards him, and hears the report in 15″. He then walks towards the ship at the rate of 3 miles an hour, and sees a second flash 5 minutes after the first, and immediatly stops; the report follows in 10·5″. Find the rate of the ship, the velocity of sound being 1200 feet per second.

49. A soldier has 4 hours' leave of absence; how far may he ride on a coach which travels 8 miles an hour, so as to return to the camp in time, walking at the rate of 4 miles an hour?

50. Two trains start at the same time, the one from Calcutta to Allahabad, the otner from Allahabad to Calcutta. If they arrive at Allahabad and Calcutta respectively 5 hours and 20 hours after they passed each other, show that one travels twice as fast as the other.

51. A cistern is provided with two pipes, *A* and *B*. *A* can fill it in 20 minutes, and *B* can empty it in 30 minutes. If *A* and *B* be kept open alternately for one minute each, how soon will the cistern be filled ?

52. *A*, *B*, *C* are pipes attached to a cistern. *A* and *B* can fill the cistern in 20 and 30 minutes respectively, while *C* can empty it in 15 minutes. If *A*, *B*, *C* be kept open successively for one minute each, how soon will the cistern be filled ?

53. A train having to perform a journey of 150 miles, is obliged after 100 miles to reduce its speed by one-fifth. The result is that the train arrives at its destination half an hour behind time. What is its ordinary rate ?

54. A down Passenger Train, 176 yd. long, travelling at the rate of 20 miles an hour, meets at 7 A. M. an up Goods Train, $293\frac{1}{3}$ yd. long and passes it in 24 seconds. At 7·30 A. M. the down Passenger meets the up Mail, 88 yd. long, and passes it in 12 seconds. When will the Mail overtake the Goods ?

55. *A* and *B* start together from the same point on a walking match round a circular course. After half an hour *A* has walked 3 complete circuits, and *B* four and a half. Assuming that each walks with uniform speed, find when *B* next overtakes *A*.

56. A certain sum is to be divided among *A*, *B* and *C*. *A* is to have £30 less than the half, *B* is to have £10 less than the third part, and *C* is to have £8 more than the fourth part. What does each get ?

57. £4212 is divided among *A*, *B*, *C*, so that *A* receives $\frac{4}{9}$ as much as *B* and *C* together, and *B* $\frac{4}{9}$ of what *A* and *C* together receive. Find how much each receives.

58. Two-thirds of a certain number of persons received 18*d.* each, and one-third received 2*s.* 6*d.* each. The whole sum spent was £2. 15*s.* How many persons were there ?

59. A crew which can pull at the rate of 9 miles an hour, find that it takes twice as long to come up a river as to go down ; at what number of miles an hour does the river flow ?

60. *A*, *B*, *C* are partners ; *A* whose money has been in the business for 4 months claims $\frac{1}{8}$ of the profits ; *B* whose money has been in the business for 6 months claims $\frac{1}{3}$ of the profits ; *C* had R1560 in the business for 8 months : how much money did *A* and *B* contribute to the business ?

61. Two persons A and B rent a field, A puts on it 12 horses for $2\frac{1}{2}$ months, 20 cows for 4 months and 50 sheep for 5 months; B puts 18 horses for $3\frac{1}{2}$ months, 15 cows for 5 months and 40 sheep for $4\frac{1}{2}$ months. If in one day 3 horses eat as much as 5 cows, and 6 cows as much as 10 sheep, what part of the rent should A pay?

62. A can dig a trench in $\frac{1}{2}$ the time that B can; B can dig it in $\frac{2}{3}$ of the time that C can; all together they can dig it in 6 days. Find the time it would take each of them alone.

63. For 5 guineas can be obtained either 12 lb. of tea and 15 lb. of coffee, or 36 lb. of tea and 9 lb. of coffee; find the price of a pound of each.

64. Divide 48 into two parts such that if one part be multiplied by 3 and the other by 5, the sum of the products shall be 180.

65. Divide 20 into two parts such that three times one part may be equal to twice the other part.

66. A decimetre is equal to 3·937 inches, and a cubic decimetre of water weighs 1 kilogram. If a cubic inch of water weighs 252·45 grains, express a kilogram in pounds Avoir. correct to two decimal places.

67. Twenty gallons of a liquid contain 60 per cent. of nitric acid and the rest water. How many gallons of water should be added to the mixture to lower the proportion of nitric acid to 40 per cent.?

68. Divide R1000 among 1 man, 3 women and 36 children so that the man gets 4 times as much as each woman, and the women together get 12 times as much as each child.

69. Two men undertake to do a piece of work for R40. One could do it alone in 5 days, the other in 8 days. With the help of a boy they finish it in 3 days. How should the money be divided?

70. The sum of the ages of A and B is now 55 years, and their ages 10 years ago were as 4 is to 3; find the present ages.

71. A tradesman's prices are 20 p. c. above cost price; what profit does he make, if he allows his customers a discount of a penny in the shilling?

72. Four apples are worth as much as 5 plums, 3 pears as much a 7 apples, 8 apricots as much as 15 pears, and 5 apples sell for 2*d.* I wish to buy an equal number of each of the four fruits, and to spend an exact number of pence: find the least sum I can spend.

73. The manufacturer of an article makes a profit of 20 per cent., the whole-sale dealer, of 10 per cent., and the retail-dealer, of 5 per cent. What is the cost of the manufacture of an article which is retailed for R7. 8*a.* 9*p.*?

74. Two cogged wheels, of which one has 16 cogs and the other 20, work in each other. If the latter turns 60 times in $\frac{4}{5}$ of a minute, how often does the former turn in 16 seconds ?

75. The price of butter having risen 25 p. c., the daily allowance of each person in a family is reduced from 1 oz. to $\frac{4}{5}$ oz. If the monthly charge for butter is thenceforward 12*s.*, what was it before the changes were made ?

76. A bankrupt has book-debts equal in amount to his liabilities, but on £4000 of them he can recover only 15*s.* in the £, and the expenses of the bankruptcy are £200 ; if he pay 15*s.* $2\frac{3}{8}$*d.* in the £, what is the amount of his liabilities ?

77. A ship 40 miles from the shore springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes. 60 tons would suffice to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find the average rate of sailing so that she may reach the shore just as she begins to sink.

78. Standard silver is formed by mixing 11 parts of fine silver with one of copper. How many rupees can be coined from 1 lb. Avoir. of fine silver, if 1 lb. Troy of standard silver is coined into 32 rupees ?

79. If $2\frac{1}{4}$ tolas of gold, 22 carats fine, be worth R49. 8*a.*, of what fineness must gold be in order that $1\frac{1}{2}$ tolas of it may be worth R34. 8*a.* ?

80. A man having to work 36 miles finds that in 3 hr. 20 min. he has walked $\frac{5}{7}$ of the remaining distance ; find his speed.

81. Supposing the alloy in a rupee to be $\frac{1}{12}$ of the mass, and the coin to be worth 2 pice if it were all alloy, what could be its exact value if it were all pure silver ?

82. A mixture contains wine and water in the ratio of 3 : 2 ; if it contains 3 gallons more wine than water, what is the quantity of wine in the mixture ?

83. 3 men and 6 boys can do 4 times as much work as a man and a boy can do, in the same time. Find the ratio of the works done by a man and a boy in the same time.

84. A mixture is composed of 4 parts brandy and 1 part water ; one gallon of water is added, and the mixture contains 3 times as much brandy as water : find the quantity of brandy in the mixture.

85. *A* mixture contains wine and water in the ratio of 3 : 2, another contains wine and water in the ratio of 4 : 5 ; how many gallons of the latter must be mixed with 3 gallons of the former so that the resulting mixture may contain equal quantities of wine and water ?

86. *A*, *B* and *C* are three vessels holding 1, 2 and 4 gallons respectively. *A* is empty, *B* is full of water and *C* is full of wine. *A* is filled from *B*, *B* is replenished from *C*, and then *A* is emptied into *C*. When this operation has been performed once more, what will be the ratio of the wine in *B* to the water in *C*?

87. An alloy of silver is mixed with an alloy of gold in the ratio of ·73 to ·37; the quantity of dross in the silver alloy is 12 parts in 100, and in the gold alloy 15 parts in 100: compare the quantities of gold, silver and dross in the mixture.

88. *A* barters some sugar with *B* for flour which is worth 2*s.* 3*d.* per stone, but uses a false stone weight of $13\frac{1}{2}$ lb.; what value should *B* set upon his flour, that the exchange may be fair?

89. If the work done by a man, a woman, and a child be in the ratio of 3, 2, 1, and there be in a factory 24 men, 20 women and 16 children, whose weekly wages amount to R224, what will be the yearly wages of 27 men, 40 women and 15 children?

90. A lb. of tea and 3 lb. of sugar cost R3, but if sugar rose 50 per cent. and tea 10 per cent., they would cost R3. 8*a.*; find the prices per lb. of tea and sugar.

91. A bankrupt has goods worth R9750; and had they realised their full value, his creditors would have received 13 annas in the rupee; but $\frac{3}{8}$ths were sold at 17·5 p. c., and the remainder at 23·75 p. c., below this value. What sum did the goods fetch, and what dividend was paid?

92. Gold is sold at the Mint at £3. 17*s.* 9*d.* per oz., and is mixed with alloy, worth 5*s* 2*d.* per oz., in the ratio of 11 : 1. If sovereigns be coined of this mixture, each weighing 5 dwt. 3·47 gr., what is the Mint profit per 100 sovereigns?

93. A bag contains 160 coins consisting of half-crown, shillings, sixpences, and fourpences, and the values of the sums of money represented by each denomination of coin are the same; how many of each are there?

94. In sending 100 cheroots to England I paid freight $\frac{2}{9}$ of their prime cost; landing charges $\frac{1}{8}$ of their cost including freight; and duty $2\frac{1}{2}$ times their cost including freight and landing charges. Altogether the cheroots duty paid, in London cost me £7. What did I give for them in Madras?

95. A number of rupees is divided amongst four men. *A* receives $\frac{2}{3}$ of the whole. *B* $\frac{3}{4}$ of the remainder, *C* $\frac{1}{3}$ of what then remains, and the number of rupees given to *D* is the square root of the whole number to be divided. What sum does each receive?

96. For $\frac{2}{3}$ of the distance up a ghaut the rise is 1 foot in 24 (measured along the road) and for the remaining third the rise is 1 in 16. The top of the ghaut is 1,400 ft. above the bottom; what is its length?

97. In a company of 100 people, of whom some are rich and some are poor, the rich subscribe and give 1*a.* 3*p.* to each poor man; this costs the rich men 7*a.* 1*p.* each: how many rich and how many poor men are there?

98. Given that gold is worth £3. 17*s.* 10*d.* per oz., and silver 4*s.* 10*d.* per oz., and that the weights of equal volumes of gold and silver are as 19 : 11; find the volume of silver equal in value to a cubic inch of gold.

99. A tradesman bought a quantity of goods, and sold $\frac{3}{8}$ of them at a profit of 10 p. c.; the price rising, he got $12\frac{1}{2}$ p. c. profit on the remainder, and on the whole gained R425: wha sum did he lay out?

100. A publican buys two butts of wine, one for R1200, and one for R1100; he also buys a third and after mixing the three, retails the wine at R22. 8*a.* a dozen, making $12\frac{1}{2}$ p. c. on his outlay: supposing the number of dozens in a butt to be 52, find the price of the third butt.

101. A merchant sells 49 quarters of wheat at a profit of 7 p.c., and a certain number of quarters at a profit of 11 p. c. The cost price of a quarter of wheat being £3. 12*s.* 6*d.*, he would have lost £2. 10*s.* 9*d.*, if he had sold the whole at a profit of 9 p. c. Find the total number of quarters of wheat sold by him.

102. The shares in a banking concern are R1000 each, R426. $10\frac{2}{3}$*a.* are only paid up, and the shares are quoted in the market at R460. The dividend is $R7\frac{1}{2}$ per share quarterly. A gentleman holds 100 original shares. Find what interest he makes per cent.: and how much per cent. would he make, if he sold out and invested in 4 per cent. Govt. stock at par?

103. A person finds that if he invest a certain sum in railway shares paying £6 per share when the £100 share is at £132, he will obtain £10. 16*s.* a year more for his money than if he invest in 3 per cent. consols at 93. What sum has he to invest?

104. A person has R24,180 to invest: the $5\frac{1}{2}$ per cent. Govt. loan being at 108 and the 6 per cent. Municipal loan of R1,000 being at 1020, find how he must divide his capital between the Govt. and Municipal loans, that he may obtain the same income from each.

105. A railway proprietor receives one year a dividend of 6 per cent. on his stock, and pays an income-tax of 4*d.* in the £. The next year he receives a dividend of $6\frac{1}{2}$ per cent. and pays an income-tax of 3*d.* in the £, and finds that his net income is £249 more. How much railway stock does he hold?

106. A man sold at 48 and 95 respectively £500 ordinary stock in the *A* Railway paying a dividend at the rate of $1\frac{3}{4}$ and £800

preference stock in the *B* Railway paying a dividend of 4 per cent. He then invested $\frac{1}{4}$ of the money in the Tramway Company where the £24 share paying interest at 6 per cent. was at £6 premium; £150 in the *C* Railway which paid no interest; and the remainder in Bank shares at par: what rate of interest must he receive from the Bank in order to increase his annual income by £12. 5*s*.?

107. There are two railway engines whose rates of motion may be represented by 1 and ·75. Supposing the slower to have been 12 miles in advance of the faster train on the same line, how far would the faster train have to travel before it overtook the other?

108. The value of 1 lb. of gold is 20 times that of 1 lb. of silver and the weights of equal volumes of gold and silver are as 19 : 10; find the value of a bar of silver equal in bulk to a bar of gold of value £380.

109. A merchant owes a bill of R5,795, payable in 8 months and another of R7,822, payable in 12 months; he takes up these two bills and gives in their place one for R13,716, payable in 12 months: what is the rate of interest per cent. per annum?

110. A Calcutta merchant has to pay R10,512. 8*a*. to his agent in Bombay. What must he give for a bank draft to that amount, exchange being at 100$\frac{1}{4}$?

111. A man bequeaths his property amounting to R49,166 in such a way that $\frac{1}{3}$ of his wife's share, $\frac{2}{5}$ of his eldest son's, $\frac{3}{8}$ of his younger son's and $\frac{1}{2}$ of his daughter's share are all equal. Find the share of each.

112. *A* and *B* exchange goods; *A* gives 13 cwt. of hops, the retail price of which is 56*s*. per cwt., but in barter he rates them at £3. *B* gives 10 barrels of beer, the retail price of which is 1*s*. a gollon, but the value of which he raises in proportion to the increased price of the hops. How much must *B* give in money?

113. A person having to pay R10,572 two years hence, invests in the 4 per cent. Transfer loan to accumulate interest till the debt shall be paid, and also an equal sum the next year. Supposing the investment to be made when paper is at 86$\frac{1}{4}$, and the price to remain the same; what sum must be invested on each occasion that these may be just sufficient to pay the debt at the given time?

114. A train has been travelling 20 miles an hour: the steam power is doubled, whilst from various causes the resistance of the train is increased by one-half. (The original steam power is three times the resistance.) At what rate will the train now travel?

115. A selling vessel reaches Madras from Calcutta in 6 days; a steamer whose speed is to that of the selling vessel as 3 : 2 starts at the same time, but meets with detentions that average 6 hours daily. Which will reach Madras first? And by how much?

116. A book containing between 900 and 1000 pages is divided into four parts, each part being divided into chapters. The whole number of pages in each of the four parts is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the whole number of chapters in the book.

117. A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value is increased by one-half, so that, after reserving 20 acres for himself, he clears £200 on his purchase-money by the sale of the remainder. How many acres were there?

118. Find how much rice a family requires, monthly, when a reduction in the price from 7 to 10 measures for the rupee reduces the total monthly expenses from R$31\frac{1}{2}$ to R30.

119. *A* barters sugar with *B*, for rice which is worth $1\frac{7}{8}$ annas a measure, but in weighing his sugar uses a false maund weight. *B* discovers this, and to make the exchange fair raises the price of his rice to $2\frac{1}{8}$ annas a measure. Find the real weight of the false maund which *A* uses.

120. A person pays an income-tax of 4*d.* in the £ during the first half of the year and of 3*d.* in the £ during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income.

121. The materials of an old building were sold for R1,500 upon condition that they should be removed within 30 days under a penalty of R10 per day for every day beyond 30 days. The purchaser employed 40 men at $3\frac{1}{2}$ annas per day to do the work, and after selling the materials for R2355, he cleared R190 by his bargain. Find the number of days the men were at work.

122. *A* and *B* enter into partnership; *A* supplies the whole of the capital, amounting to R45,000 upon condition that the profits are to be equally divided, and that *B* pays *A* interest on half the capital at 10 per cent. per annum but receives R120 per mensem for carrying on the concern. Find their total yearly profits when *B*'s share is equal to $\frac{1}{2}$ of *A*'s share.

123. If the value of a rupee varies from 1*s.* 9*d.* to 1*s.* $9\frac{1}{2}$*d.* and of the francs from $9\frac{1}{2}$*d.* to 10*d.*; find the maximum number of francs which it is always safe to give for R500.

124. If the volume of a sphere $= \frac{4}{3} \times 3{\cdot}1416 \times$ the cube of the radius, find how many spherical balls each $\frac{1}{2}$ inch in diameter can be made out of a cubic inch of clay, and how much clay will remain over.

125. Paper-money is at a discount of 10 per cent. A man buys goods marked £27 (paper-money) and offers that sum in gold. How much paper-money must he receive in change, 10 per cent. abatement being allowed for cash ?

126. A reservoir is to be emptied, the rate of discharge of he contents being diminished by 100 gallons every hour. The first half will be emptied in 3 hours, the second in 4 hours. How many gallons does the reservoir contain ?

127. What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 6 or 8 deep, and also of its being formed into a solid square ?

128. *A*, *B* and *C* are partners. *A* receives $\frac{2}{3}$ of the profits, *B* and *C* dividing the remainder equally. *A*'s income is increased by ₨400 when the rate of profit rises from 5 to 7 per cent. Find the capital of *B*.

129. How many years' purchase should be given for an estate so as to get 4 per cent. for the money ?

130. An agent has to receive a rent paid in corn from a tenant, and to deliver it to the landlord. At each payment he uses, so as to benifit himself, a false balance, such that 4 seers in one scale balance 5 seers in the other. Corn being worth ₨2. 8*a*. a md., the value of his plunder is ₨4. What is the corn-rent ?

131. A zemindary is bought at 20 years' purchase for ₨27000, one-third of the purchase-money remaining at mortgage at 9 per cent. The cost of collecting rents is ₨140 per annum. What interest does the purchaser make on his investment ?

132. A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses amount to $\frac{1}{8}$ of his receipts. The price of flour falls 50 per cent. and other trade expenses are thereby reduced 25 per cent. By how much should he now reduce the price of a 5*d*. loaf to make the same amount of profit ?

133. 1000 copies of a pice newspaper weigh $\frac{1}{2}$ of a maund, and when the paper duty was removed the profits on the receipts was increased 5 per cent. What was the duty per md. on paper ?

134. A horse was sold at a loss of 10 p. c. ; if it were sold for ₨70 more there would have been a gain of 4 per cent. : for how much was the horse sold ?

135. A contractor sends in a tender of ₨7000 for a certain work ; a second sends in a tender of ₨6950, but stipulates to be

paid ₨3000 at the end of a month ; find the difference between the tenders, supposing the work to be finished in 3 months, and money to be worth $\frac{1}{2}$ per cent. per month simple interest.

136. A labourer was engaged for 20 days, on the agreement that for every day he worked he should have 4*a.*, but that for every day he absented himself he would be fined 1*a.* He received ₨2. 13*a.* at the end of the time : how many days was he absent ?

137. A man was hired to do a certain amount of work, on the condition that for every day he worked he should have 12*a.*, but that for every day he absented himself he should lose 4*a.* He worked 3 times as many days as he absented himself, and received on the whole ₨10. How long was he doing the work ?

138. A grocer buys two maunds of sugar ; he sell one maund at a profit of 10 p. c., and the other which cost ₨2. 8*a.* more, at a profit of 15 p. c. If the retail price per seer of the latter be $1\frac{8}{10}$*a.* more than that of the former, find their cost price of each maund.

139. A shop-keeper buys 2 md. of sugar, and 1 md. more of a superior kind, giving ₨1. 8*a.* a md. more for the latter than the former. He retails it, when mixed, at 4 annas a seer, and makes a profit of 25 p. c. on his outlay. What did he give per md. for each kind of sugar ?

140. Two boys begin to count two equal piles of rupees. One counts 5 while the other counts 4. When the former has just finished the latter has 6 left. What is the number of rupees in each pile ?

141. The price of a yard of jean is $\frac{3}{4}$ of the price of $2\frac{1}{4}$ yd. of longcloth ; and the weight of 5 yd. of jean is $\frac{5}{8}$ of the weight of 8 yd. of longcloth. If the price of 2 lb. of jean be ₨3, what is the price of $1\frac{1}{2}$ lb. of longcloth ?

142. Three tramps meet together for a meal : the first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 8 half-pence ; how ought they to divide the money ?

143. *A* and *B* barter : *A* has 7 md. of flour worth ₨3. 8*a.* a md., but insists on having ₨3. 12*a.* a md. : *B* has rice worth ₨1. 5*a.* a measure, which he raises in price in proportion to *A*'s demand. *A* receives 16 measures of rice ; what cash does he get besides ?

144. *A* and *B* barter : *A* has 200 lb. of tea worth 2*s.* 6*d.* a lb. but insists on 2*s.* 9*d.* a lb. : *B* has coffee worth 1*s.* 9*d.* a lb. : how much must he raise the price so that *A* gets £5. 2*s.* and 2 cwt. of coffee ?

145. A river 14 ft. deep, 182 yd. wide flows at the rate of 3 miles an hour : (i) how many tons, (ii) how many gallons of

water, pass a certain point per minute? [A cu. ft. of water weighs $62\frac{1}{2}$ lb., a gallon contains $277\frac{1}{4}$ cu. in.]

146. A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage and of the two circles of rails are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the four wheels in a complete circuit?

147. Eleven boys fired 10 shots each at a target and scored 285; 20 bull's-eyes were made and 11 misses; how many centres and outers were there? (A bull's-eye scores 4, a centre 3, an outer 2).

148. The sum of £177 is to be divided among 15 men, 20 women and 30 children, in such a manner that a man and a child may receive together as much as two women, and all the women may together receive £60; what will they each respectively receive?

149. *A* owed *B* three-fourths of what *B* owed *C*; to settle matters, *B* gave R2 to *A* who then paid *C*; what did *B* owe *C*?

150. A man for 4 years spends R500 a year more than his income. At the end of that time, he reduces his expenditure 30 per cent. and in 3 years pays off his debt and saves R1000. What is his income?

151. A tree grows 2 yards in its first year, and afterwards it grows each year 1 foot less than it did the previous year. The value of the tree at any time is equal to the number of rupees in the square of the number of yards in its height; find the value of the tree when it has done growing.

152. If standard gold worth £3. 17*s.* $10\frac{1}{2}$*d.* per ounce be so far alloyed as to be worth only £3. 16*s.* $1\frac{1}{2}$*d.* per ounce, find the least integral number of sovereigns made of the alloyed gold, which shall be equal in value to an exact number made of the standard gold.

153. Find the least integral number of ounces of pure silver, worth R2. 14*a.* $6\frac{9}{11}$*p.* per ounce, that, with the proper proportion of alloy, can be coined into an exact number of rupees.

154. Mahogany is 50 lb. to the cubic foot, water is $62\frac{1}{2}$ lb., and iron is $7\frac{1}{2}$ times as heavy as water; what thickness of iron will weigh as much as a 6-inch plank of mahogany?

155. A sum of R62 is to be divided among 10 men, 15 women, 8 boys and 12 girls. For every rupee that a man gets, a boy gets 6 annas, and for every half-rupee that a woman gets, a girl gets 2 annas. The whole money obtained by the boys is equal to that obtained by the girls. How much does each person get?

156. A wooden closed box, made of $\frac{1}{2}$-inch plank, is externally

15 in. long, 10 in. broad and 6 in. high. The box weighs 6 lb. when empty, and 86 lb. when filled with mercury. Compare the weights of equal bullks of the wood and mercury.

157. Ɍ430 is divided among 45 persons consisting of men, women and children. The sums of the men's, women's and children's shares are as 12 : 15 : 16, but the individual shares of a man, woman and a child are as 6 : 5 : 4. Find the numbers of men, women and children.

158. Bronze contains 91 per cent. of copper, 6 of zinc, and 3 of tin. A mass of bell-metal (consisting of copper and tin only) and bronze fused together is found to contain 88 per cent. of copper, 4·875 of zinc, and 7·125 of tin. Find the proportion of copper and tin in bell-metal.

159. An alloy contains 12 parts by weight of lead, 4 of antimony, and 1 of tin. How much of this alloy must be taken, and how much lead and tin added to it to make up 9 cwt. of type-metal consisting of 14 parts lead, 3 antimony and 1 tin?

160. Three persons *A*, *B*, *C*, finished a piece of work. *A* worked at it for 5 days, *B* for 7 days and *C* for 9 days. Their daily wages were as 4 : 3 : 2, and the total earnings amounted to Ɍ7. 6*a*. What were the daily wages of each?

161. Two passengers are charged for excess of luggage Ɍ1. 8*a*. and Ɍ5. 4*a*. respectively. Had the luggage all belonged to one person he would have been charged Ɍ7. 8*a*. for excess. How much is allowed free, the charge for excess being 12*a*. per md.?

162. If the cost of making bread be one rupee per bushel of wheat, what is the price of wheat when the two-anna loaf is twice as large as it is when wheat is Ɍ5 a bushel?

163. If the rate of wages vary as the price of rice, and if 57 men working for 35 days receive Ɍ405. 3*a*. 9*p*. when rice is sold at the rate of 136 measures for Ɍ39; find the price of rice per measure when 70 men working for 19 days receive Ɍ353. 4*a*. 6*p*.

164. There is a leak in the bottom of a cistern. When the cistern was in thorough repair, it would be filled in 2½ hours. It now takes half an hour longer. If the cistern is full, how long would it be in leaking itself empty?

165. *A* can do [illegible] of a piece of work in [illegible] of the time in which *B* can do [illegible] of it, and *B* can do [illegible] of it in [illegible] of the time that it would take *C* to do another piece of work one-fourth as large again as the first. If *C* can finish the former piece of work in 10 hours, how long would it take *A* and *B* together to do it?

166. *A* and *B* start on a journey at the same time. *B* travels at [illegible] of *A*'s rate, and arrives 3 hr. 15 min. after him. In what time did each complete the whole journey?

167. The expenses of a family when rice is at 20 seers for a rupee are ₨50 a month; when rice is at 25 seers for a rupee the expenses are ₨48 a month; what will they be when rice is at 30 seers for a rupee?

168. A man who can walk down a ghaut at the rate of 4½ and up it at the rate of 3¼ miles an hour, descends and returns to his starting point after walking for 2 hours 4 minutes. How far did he walk?

169. An express train owing to a defect in the engine goes at ⅝ of its proper speed, and arrives at 6-49 P.M. instead of 5-55 P.M.; at what hour did it start?

170. A person going from Pondichery to Ootacamond travels 90 miles by steamer, 330 miles by rail and 30 miles by horse-transit. The journey occupies 30 hr. 50 min., and the rate of the train is 3 times that of the horse-transit and 1½ times that of the steamer. Find the rate of the train.

171. A person walks from *A* to *B* at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to *A* by the tramway at the rate of 5 miles an hour. He then finds he has been absent 2 hours 20 minutes. Find the distance from *A* to *B*.

172. The expenses of a family, when rice is 12 seers for a rupee, are ₨50 a month; when rice is 14 seers for a rupee, the expenses are ₨48 a month (other expenses remaining unaltered); what will they be when rice is at 16 seers per rupee?

173. A bankrupt has book-debts equal in amount to his liabilities, but on ₨8640 of such debts he can recover only 8½ annas in the rupee, and on ₨6300 only 5¼ annas in the rupee. After allowing ₨1054 . 11 . 0 for the expenses of a bankruptcy, he finds that he can pay his creditors 12 annas in the rupee. Find the total amount of his debts.

174. A train starts with a certain number of passengers. At the first station it drops ⅓ of these and takes in 20 more. At the next it drops ½ of the new total and takes 10 more. On reaching the third station there are 60 left. What number started?

175. One pound Troy of standard silver which contains 37 parts in 40 of fine silver is coined into 66 shillings. If the value of pure silver rises 10 per cent., what must be the reduction of pure silver in a shilling?

176. A landlord has an estate worth ₨40000 a year, but has to pay ½ anna in the rupee on the gross income for taxes. He sells it at 20 years' purchase on the gross income, and invests the proceeds in the 4 per cents. at 95. What is the difference in his income?

177. In firing at a mark *A* hits in 2 out of 4 shots, *B* in 3 out of 5, and *C* in 4 out of 7. The mark was hit 468 times. Supposing each to have fired the same number of shots, find how many hits each made and the total number of shots fired.

178. A shop-keer buys sugar at R12. 8*a*. a md.; at what price must he sell it to gain 8 per cent., and allow a purchaser 10 per cent. discount?

179. In a manufactory 100 coolies work for 4 days a week, but on the remaining 3 days some are absent; the weekly wages of the coolies are thus reduced in the ratio of 32 : 35. Find the number of absentees.

180. The manager of a boarding house having already 50 boarders, find that an addition of a 10 increases the gross monthly expenditure by R20, but diminishes the average cost per head by R1. What did the monthly expenses originally amount to?

181. If 9 oz. of gold, 10 carats fine, and 5 oz., 11 carats fine, be mixed with 6 oz. of unknown fineness, and the fineness of the resulting mixture be 12 carats, what was the unknown fineness?

182. A tradesman's stock in trade is valued on January 1st, 1868, at £8,000, he has also £350 in cash and owes £1,870; during the year his personal expenses, £300, are paid out of the proceeds of his business, and on January 1st, 1869, his stock is valued at £7,950, he has £570 in cash and owes £1,510. What is the whole profit on the year's transactions after deducting 5 per cent. interest on the capital with which he began the year?

183. If 20 English navvies, each earning 3*s*. 6*d*. a day, can do the same piece of work in 15 days that it takes 28 foreign workmen, each earning 3 francs a day, to complete in 20 days; taking the value of the franc at 10*d*., determine which class of workmen it is most profitable to employ. If a piece of work done by the navvies cost £3,000, what would be the cost of the same work done by foreign workmen?

184. A merchant in New York wishes to remit to London 5110 dollars, a dollar being equal to 4*s*. 6*d*. English: for what sum in English money must he draw his bill when bills *on* London are at a premium of 9½ per cent.?

185. A person borrows £100, and at the end of each year pays £25 to reduce the principal and to pay interest at 4 per cent. on the sum which has been standing against him through that year. How much will remain of the debt at the end of 3 years?

186. If a metric system of area were adopted wherein 1 acre 1 rood 3 perches is represented by 5·12, express the unit of measurement in sq. yards and decimal parts of a sq. yd.

187. If gold weighs 19 times as much as water, and silver 12 times as much, find how many times heavier than water is a coin which contains 10 parts of gold and 1 of silver.

188. A certain reef of quartz when crushed yields ·0011 per cent. of gold. If the working expenses amount to 62·5 per cent. of the gross receipts, and the net profit on each 100 tons is £52. 10*s.*; find the number of grains in a sovereign.

189. A certain article of consumption is subject to a duty of 6*s.* per cwt.; in consequence of a reduction in the duty the consumption increases one-half, but the revenue falls one-third. Find the duty per cwt. after the reduction.

190. If the duty on a certain commodity were reduced 25 per cent, by how much per cent. must the consumption be increased that the same revenue may be derived from it?

191. If 2 cu. in. of gold together with 3 cu. in. of silver are equal in weight to 74 cu. in. of water, and the weights of equal volumes of gold and water be represented by the numbers 19 and 1, what number represents the weight of an equal volume of silver?

192. A farmer bought equal numbers of two kinds of sheep, one at £3 each, the other at £4 each. If he had expended his money equally in the two kinds he would have had 2 sheep more than he did; find how many he bought.

193. A man travels 150 miles in 13 hours, partly by rail and partly by steamer; if he had gone all the way by rail, he would have ended his journey 8 hours sooner, and saved $\frac{4}{5}$ of the time he was on steamer; how far did he go by rail?

194. In a distilling operation, during 3 hours the fluid contained 70 per cent. of alcohol, during $2\frac{1}{2}$ hours 60 per cent., and during the remaining $1\frac{1}{2}$ hours 40 per cent. What is the average strength of the whole fluid distilled over, assuming that it came over at a uniform rate during the whole time?

195. During a distillation the fluid that comes over in 3 consecutive hours contains 47, 35 and 20 per cent. of alcohol respectively. The rates at which it comes over during these 3 hours are in the ratios of 2, 3 and 4. What is the percentage of alcohol in the whole mixture?

196. I bought a number of mangoes at 35 for R2. I divided the whole into two equal parts, one of which I sold at 17, and the other at 18 mangoes per R1. I spent and received an integral number of rupees, but bought the least possible number of mangoes. How many did I buy?

197. Find the cost in rupees of one mile of railway, which consists of two rails, each weighing 40 lb. per yard, on wooden sleepers, weighing 70 lb. each, placed 2 ft. 8 in. apart. The rails

cost in England £6 . 13 . 0 per ton, and the sleepers 2*s*. 4½*d*. each. The rate of freight is £1 . 5 . 0 per ton, and landing charges amount to R2. 8*a*. per ton. Rate of exchange 1*s*. 8*d*. per rupee.

198. The length of a certain Railway being 110 miles and the capital employed in its construction 1500000*l*., what must be the gross annual traffic receipts per mile in order that a dividend of 5 per cent. may be paid to the share-holders after allowing 45 per cent. of the gross receipts for current expenditure ?

199. A person in India sells a bill on London for 358*l*., payable at 3 months' sight at the rate of 1*s*. 10⅜*d*. per rupee. The purchaser requires payment on presentation ; what amount does he receive after discount at 5 per cent. has been deducted ?

200. The Guernsey pound contains 18 oz. Avoir., and the Guernsey shilling contains 13 English pence. If a Guernsey pound of butter cost 1*s*. 6*d*., Guernsey money, what will be the price in English money of 2½ lb. Avoir. ?

201. A contractor employs a fixed number of men to complete a work. He may employ either of two kinds of workmen : the first at 26*s*. 6*d*. per week each, the second at 18*s*. 6*d*. per week each : the work of the one of the former being to that of one of the latter as 5 to 4. If he finishes it as quickly as possible, he spends £270 more than he would have done if he had finished it as cheaply as possible, but takes 4 weeks less time. What would it have cost if he had employed equal numbers of the two kinds of workmen ?

202. A manufactory turns out 50 tons of iron goods weekly, using up for that purpose 51 tons of iron at £6. 15*s*. per ton, 100 tons of coal at 11*s*. 6*d*. per ton, and £45 worth of other materials ; rent, rates and taxes amount to £219 annually ; wages and incidental expenses to £75 per week. At what price per cwt. must the iron be sold in order that the works may gain 8 per cent. per annum on a capital of £35000 ? [Reckon 52 weeks to the year.]

203. Two lumps, composed of gold, silver and copper, together weigh 10 oz. ; one lump contains gold 75 p. c. and silver 15 grains per oz., the other contains gold 85 p. c. and silver 12 grains per oz. The total quantity of silver in the two lumps is 141 grains. If the two lumps are melted and formed into one, what per cent. of gold will it contain ?

204. The only three creditors of an insolvent [whose assets amount to £100 and who can pay only 5*d*. in the £, agree among themselves to take dividends in the proportion of the nnmbers of £. *s*. and *d*. respectively, contained in the amounts due to them. The dividends thus taken are in the proportion of 12 : 7 : 6. What are the amounts of their debts ?

205. At an examination $\frac{1}{8}$ of a class gains $\frac{7}{8}$ of the maximum number of marks, $\frac{1}{10}$ gains $\frac{3}{4}$, $\frac{2}{5}$ gains $\frac{1}{2}$, $\frac{1}{4}$ gains $\frac{1}{4}$, and the rest $\frac{1}{8}$. The average number of marks gained by the whole class is 166; what is the maximum?

206. A mass of gold and silver weighing 9 lb. is worth £318. 13*s*. 6*d*.; if the proportions of gold and silver in it were interchanged, it would be worth £129. 10*s*. 6*d*.; it is known that 1 oz. of gold and 2 oz. of silver are worth £4. 8*s*. $1\frac{1}{2}$*d*.; what is the price of gold and silver per oz.?

207. A person shooting at a target, 550 yards distant, hears the bullet strike the target 4 seconds after he fires. A spectator, equally distant from the target and the shooter, hears the shot strike the target $2\frac{1}{2}$ seconds after he heard the report; find the velocity of sound.

208. A boatman rows 5 mi. with the tide in the time he would take to row 3 mi. against it; but if the hourly velocity of the current were $\frac{1}{2}$ a mile, he would row twice as rapidly with the tide as against it. Find his power of rowing in still water, and the velocity of the current.

209. A messenger sets out at the rate of 30 miles a day, but falls off in his speed 4 miles daily. Four days afterwards another sets off from the same place on the same route, travelling 50 miles the first day but falling off like the first 4 miles daily. After what time will one overtake the other?

210. Six months ago *A* invested £7620 in the 3 per cents. at $95\frac{1}{4}$, and six months hence he will receive £4300 four per cents. at 127. What is the present value of his property?

211. Two boats, *A* and *B*, row a race. *A* takes 4 strokes to *B*'s 5, but 6 of *B*'s are equal to 5 of *A*'s. *A* starts in front of *B* at such a distance that *B* must take 10 strokes to row over it. How many strokes must *B* take before overtaking *A*?

212. *A*, *B* and *C* run a mile race. *A* beats *C* by $76\frac{8}{13}$ yards; *B* beats *C* by 11 seconds; the pace of *A* is to that of *B* as 45 : 44. In what time does each run the mile?

213. Three boys begin to fill a cistern; one brings a seer every minute, another 2 seers every 2 minutes, and the third 3 seers every 3 minutes. If the cistern holds 40 seers, in what time will it be filled?

214. *A* sells his goods 10 per cent. cheaper than *B*, and 10 per cent. dearer than *C*; how much would a customer of *B* save by taking R100 worth of goods from *C*?

215. Cannons are fired at intervals of 10 minutes in a town towards which a passenger train is approaching at the rate of 35 miles an hour; if sound travels 1142 feet per second, find at what intervals the reports will be heard by the passengers.

216. A man bought a horse and a carriage for R500, and sold the horse at a gain of 20 p. c. and the carriage at a loss of 10 p. c., thus gaining 2 p. c. on his whole outlay; for how much was the horse bought?

217. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 6 children, or 5 women and 3 children can do in 12 days; find the relative strength of men, women and children.

218. Three round balls revolve with equal velocities in three concentric circular grooves. They start from a position in which they are all in the same radius of the outermost circle. The innermost ball occupies 10 seconds in traversing its groove once. After what time will they all be again on a radius of the outermost circle, the radii of the grooves being proportional to the numbers, 1, 3, 5?

219. Two guns are fired at the same place after an interval of 21 minutes, but a person approaching the place observes that 20 min. 14 sec. elapse between the reports; what was his rate of progress, sound travelling 1125 feet per second?

220. Ash saplings after 5 years' growth are worth 1*s.* 3*d.*, and increase in value 1*s.* 3*d.* each year afterwards. For their growth they require each twice as many square yards as the number of years they are intended to grow before cutting. A plantation is arranged so that each year the same number may be ready for cutting. Find the greatest annual income which can be obtained per acre, allowing 20 per cent. for expenses.

EXAMINATION PAPERS.

I. UNIVERSITY OF CALCUTTA. MATRICULATION PAPERS.

1920.

COMPULSORY PAPER.

1. *Either*, (1) Multiply 80070430 by 34070080.

(2) Find the G. C. M. of 47821 and 68191.

Or, (1) The dividend being 545322774 and the quotient 89706, find the divisor.

(2) Find the least number which is exactly divisible by the first nine integers.

2. (1) Simplify $\dfrac{6\frac{5}{8}+3\frac{4}{5}}{6\frac{7}{8}-3\frac{4}{5}} \div 10\frac{4}{11}$ of $\frac{1}{3}$.

(2) Express 0·16 of 2 cwt. 2 qrs. + 0·16 of 2·6 cwt. as the fraction of one ton. Convert the fraction into a recurring decimal.

3. (1) Find the rent of 19 acres 3 roods 20 square poles of land at £4. 5*s*. per acre.

(2) What sum will amount to R6375 in 5 years at $5\frac{1}{4}$ per cent. per annum simple interest ?

ADDITIONAL PAPER.

1. *Either*, Find by a contracted method the value of

0·53209853 × 0·43429448

correct to seven places of decimals.

Or, Find correct to four places of decimals the value of $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$.

2. *Either*, A clock in the kitchen loses at the rate of 6·5 seconds an hour when the fire is alight, and gains at the rate of 3·9 seconds an hour when the fire is not burning ; but in the whole day, it neither gains nor loses. How long in the twenty-four hours is the fire burning ?

Or, 40 per cent. of the gross receipts of a tramway company is taken up in meeting the working expenses, 40 per cent. of the remainder goes to the reserve fund, and the balance is paid away as dividends to shareholders at the rate of $3\frac{1}{2}$ per cent. on their shares, the total value of which is R864000 ; find the amount of the gross receipts.

1921.

COMPULSORY PAPER.

1. *Either*, (1) The divisor being 102003 and the quotient 45067, find the dividend.

(2) Find the G. C. M. of 65569 and 94829.

Or, (1) Divide 6579820764 by 98076.

(2) Four bells begin tolling at the same time, and they toll at intervals of 12, 18, 24 and 30 seconds respectively. After what interval of time will they next toll at the same time ?

2. (1) Simplify $\dfrac{2\frac{2}{3}-3\frac{4}{5}+4\frac{5}{6}}{7\frac{17}{30}\div 1\frac{1}{9} \text{ of } 1\frac{1}{8}}$ of $\frac{1}{12}$,

expressing the answer (i) as a fraction, and (ii) as a recurring decimal.

(2) A post has half of its length in mud, one-third of its length in water, and ten feet above water. Find the whole length of the post.

3 (1) Find the price of 25 maunds 15 seers 5 chataks of rice at R12. 8*a.* per maund.

(2) At what rate per cent. per annum simple interest will R6000 amount to R7650 in 5 years?

ADDITIONAL PAPER.

1. *Either*, Find the diagonal of a rectangle whose sides are 2·56 cm. and 4·73 cm.

Or, The third class railway fare in France is 5 centimes per Kilometre and in England 1*d.* per mile. Given that 1 yard = 0·9144 metre and £1 = 25·17 francs, find (in English money) the difference of the fares for a journey of 100 miles in the two countries, correct within a farthing.

2. *Either*, Find the product of 2·73065 and 0·0094738, preferably by a *contracted method*, correct to four places of decimals.

Or, Divide £1852 between *A*, *B*, and *C*, so that *A* may have 0·615 of the whole and *B* 0·615 of what is left, and *C* the remainder to the nearest pound.

1922.

COMPULSORY PAPER.

1. *Either*, (1) Multiply [illegible]0050230 by 5200780.

(2) Find the G. C. M. of 34465 and 54900.

Or, (1) What number multiplied by 238 gives the same result as 408 multiplied by 350?

(2) Find the least number that can be divided by all the even numbers up to 20 inclusive.

2. (1) Simplify $\dfrac{2\frac{1}{4}\frac{3}{8} \text{ of } \frac{1}{3}}{\frac{3}{8} \text{ of } \frac{6}{15} \div 5\frac{1}{5}} \div (\frac{1}{5} \text{ of } 1\frac{1}{4}\frac{3}{8})$.

(2) Find the value of $0{\cdot}2$ of $3{\cdot}\dot{6}$ of $81 - 1{\cdot}6$ of $3{\cdot}4 - 3{\cdot}96$.

3. (1) *Either*, (i) Find the price of 3 tons 3 cwt. 3 qrs. 14 lb. at £1. 3*s.* 4*d.* per ton.

Or, (ii) Find the price of 1448 articles at 10*a.* 8*p* each.

(2) What sum will amount to R915 in 4 years at $5\frac{1}{4}$ per cent. per annum simple interest?

ADDITIONAL PAPER.

1. *Either*, Extract the square root of 1522756.

Or, Extract the square root of 0·225 correct to 3 places of decimals.

2. *Either*, Find the sum, correct to 3 places of decimals, of the series

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \ldots\ldots$$

Or, The price of cloth having been raised 75 per cent., how much per cent. must a householder reduce his consumption of that article so as not to increase his expenditure?

1923.

COMPULSORY PAPER.

1. *Either*, (1) Multiply 3806800 by 708009.
(2) Find the G. C. M. of 34465 and 54900.

Or, (1) How often can you subtract 46 from 533006, and what is the final remainder?

(2) Find the least number which will leave a remainder 4 when divided by both 15 and 18.

2. (1) Simplify $\frac{2\frac{3}{8}+5\frac{7}{9}}{1\frac{1}{2}-\frac{4}{9}} \div \left([illegible] \text{ of } \frac{3\frac{1}{2}}{4} \right) \times \frac{3\frac{3}{11}}{32}$.

(2) Find the value of $0\cdot\dot{4} \times 2\cdot5 \div 0\cdot\dot{3} + 0\cdot75$ of $4 - 2\cdot4 \div 0\cdot4$.

3. (1) Find the price of 12 mds. 8 srs. 4 ch. at ₨36. 4*a*. per md.

(2) In what time will a sum of money double itself at 5 per cent. per annum, simple interest being charged?

ADDITIONAL PAPER.

1, *Either*, Extract the square root of 2819041.

Or, Extract the square root of 0 051 correct to 3 places of decimals.

2. *Either*, Find the sum, correct to 3 places of decimals, of the series

$$1 - \frac{1}{1} + \frac{1}{1 \times 2} - \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} - \ldots\ldots$$

Or, By selling a house for £2,576 a man gains 12 per cent. on his original outlay. How much per cent. would he have gained had the house cost him £100 less?

1924.

COMPULSORY PAPER.

1. *Either*, (1) What number multiplied by 9706 will give 5513008 as product?

(2) Find the G. C. M. of 11044 and 13464.

Or, (1) How often is the difference of 1325 and 1590 contained in their sum?

(2) Find the least number of rupees that can be divided among 8, 12 or 15 men.

2. (1) Simplify $\frac{2\frac{3}{8}-1\frac{3}{4}}{2\frac{1}{8}+\frac{3}{4}} + \frac{1\frac{1}{11}}{14\frac{2}{3}} \div \frac{1\frac{1}{?}}{1\frac{6}{7}}$.

(2) By what must 1·0035 be divided to get 3?

3. *Either*, (1) Find by practice or otherwise, the cost of 8 mds. 15 srs. 10 ch. of rice at ₨5. 5*a*. 4*p*. per md.

(2) How much carpet 2 ft. wide will be required for a room 7 yds. long and 16 ft. wide?

Or, The sum of ₨425 was lent at simple interest. At the end of 9 months the debt was cancelled by the payment of ₨437. 12*a*. What was the rate of interest?

ADDITIONAL PAPER.

1. *Either*, Extract the square root of 184389241.

Or, A house was sold for ₨4,500 at a profit of $12\frac{1}{2}$ per cent. What per cent. would have been lost if it had been sold for ₨3,800 ?

2. *Either*, Find the sum, correct to 5 decimal places, of the series

$$\frac{1}{2}\times\frac{1}{10}+\frac{1}{3}\times\frac{1}{10^2}+\frac{1}{4}\times\frac{1}{10^3}+\frac{1}{5}+\frac{1}{10^4}+\ldots\ldots$$

Or, ₨49. 12*a*. was divided amongst 150 children so that each boy received 8*a*. and each girl 4*a*. How many boys were there ?

1925.

COMPULSORY PAPER.

1. *Either*, Multiply 987604321 by 105790, and find the G. C. M. of 9498 and 21426.

Or, The product of two numbers is 864 and their L. C. M. is 72. Find their G. C. M.

2. Simplify $\dfrac{3\frac{5}{8}+2\frac{5}{12}-1\frac{5}{24}}{5\frac{4}{5}+2\frac{9}{10}-1\frac{9}{20}}$.

Find the number which multiplied by 0·225 will produce 126.

3. *Either*, Find the cost of 12 maunds 16 seers and 10 chhataks of rice at ₨6. 8*a*. per maund.

Find what sum of money laid out at 5 per cent. per annum will give one rupee as interest per day. (1 year consists of 365 days).

Or, A man rides at the rate of 352 yards per minute and stops 6 minutes to change horses at the end of every sixth mile ; how long will he take to go a distance of 108 miles ?

ADDITIONAL PAPER.

1. *Either*, Find the square root of 1522756.

Or, Simplify $\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$ to 2 places of decimals.

2. *Either*, Find correct to 4 decimal places the value of

$$1+1+\frac{1}{1\times2}+\frac{1}{1\times2\times3}+\frac{1}{1\times2\times3\times4}+\ldots\ldots$$

Or, If by selling a house for £490 there be a loss of $12\frac{1}{2}$ per cent., what per cent. is gained or lost by selling it for £596. 8*s*. ?

1926.*

COMPULSORY PAPER.

1. *Either*, (1) A certain sum was divided among 1,024 men. Each got ₨937. What was the sum ?

(2) Find the L. C. M. of 959, 3973, 2329.

Or, (1) The product of two numbers is 47608946. One of them is 2149. Find the other.

(2) What is the greatest number that will divide 7289 and 8917 without a remainder?

2 *Either*, (1) Reduce to the simplest form

$$\frac{15\frac{2}{[illegible]}-4\frac{2}{11}\times\frac{[illegible]}{[illegible]}}{\frac{1}{[illegible]}\times 23\frac{1}{[illegible]}+3\frac{1}{10}}-\frac{107}{1[illegible]}.$$

(2) Find the value of $\frac{0\cdot\dot{2}\dot{3}}{0\cdot 2875}$ giving the result in the simplest form of a vulgar fraction.

Or, If 5 men and 9 boys could do a piece of work in 17 days, in how many days could 9 men 12 boys do it, the work of 2 men being equal to that of 3 boys?

3. (1) Find, by Practice or otherwise, the price of 425 mds. 30 srs. of potatoes at R3. 13*a*. 4*p*. per md.

(2) *Either*, A certain number of plantains was bought at four for one anna, and an equal number at three for an anna. The whole was sold at seven for two annas. Find the loss or gain per cent.

Or, Two pipes can fill a cistern in 20 and 30 minutes respectively. Both the pipes being opened, find when the first pipe must be turned off so that the cistern may be filled in 10 minutes more.

ADDITIONAL PAPER.

1. *Either*, A certain number of boys spent R81, each spending twice as many two-anna pieces as there were boys altogether. How many boys were there?

Or, Find the square root of $1-(0\cdot 00135)^2$ to 5 decimal places.

2. Find the value of

$$\frac{1}{1\times 5}+\frac{1}{3\times 5^3}+\frac{1}{5\times 5^5}+\ldots \text{ to 5 places of decimals.}$$

1927.

COMPULSORY PAPER.

1. *Either*, Find the greatest number that will divide 2300 and 3500, and leave the remainders 32 and 56 respectively.

Or, If a number when divided by 56 leaves a remainder 29, what remainder will it leave when divided by 8?

2. *Either*, (1) Reduce to the simplest form

$$\frac{1\frac{[illegible]}{5}\text{ of }1\frac{1}{[illegible]}}{3\frac{[illegible]}{[illegible]}-2\frac{17}{[illegible]}}-\frac{2-1\frac{[illegible]}{[illegible]}}{5\frac{[illegible]}{[illegible]}+\frac{[illegible]}{[illegible]}-6}.$$

(2) The product of two decimals is 0·033372; one of them is 2·7; find the other.

Or, If a garrison of 750 men have provisions for 20 weeks, how long will the provisions last if at the end of 4 weeks they are reinforced by 450 men?

3. (1) Find the price of 2 cwt. 1 qr. $10\frac{1}{2}$ lb. of tea at ₨232. 10*a*. 8*p*. per cwt.

(2) *Either*, If a watch is sold at ₨60, the loss is 15 per cent.? For how much should it be sold to make a profit of 10 per cent.?

Or, At what rate per cent. will the interest on ₨800 in 4 years be the same as the interest on ₨625 for 8 years at 4 per cent.?

ADDITIONAL PAPER.

1. Find, to five decimal places, the value of

$$1+\frac{1}{1\cdot3}+\frac{1}{1\cdot3\cdot5}+\frac{1}{1\cdot3\cdot5\cdot7}+\ldots\ldots$$

1. *Either*, A general having formed his men, numbering 15,400, into a solid square, found he had 24 men left over; how many men did the front consist?

Or, Simplify $\sqrt{3\frac{3}{4}}\div\sqrt{9\frac{1}{7}}\times2\sqrt{21\cdot\dot{7}}$.

1928.

COMPULSORY PAPER.

1. (1) The sum of two numbers is 32459 and their difference is 2637. Find the numbers.

(2) Find the *least* number that must be subtracted from 347157 to make the result exactly divisible by 125.

2. *Either*, (1) Simplify

$$\frac{5\frac{3}{8}-3\frac{2}{8}\times\frac{9}{17}}{\frac{1}{3}\text{ of }21\frac{5}{8}-2\frac{9}{17}}-\frac{4}{75}.$$

(2) Reduce $\frac{3}{7}$ to a decimal fraction.

Or, If 8 men or 12 women can do a piece of work in 25 days, in how many days can the work be done by 6 men and 11 women working together?

3. (1) Find by practice, or otherwise, the price of 4 bags of rice each containing 2 mds. 5 srs. 8 ch. at ₨9. 7*a*. 8*p*. per maund.

(2) *Either*, In what time will the simple interest on ₨900 at 6 per cent. be equal to the simple interest on ₨540 for 8 years at 5 per cent.?

Or, A cistern can be filled by a tap in 5 hours, while it can be emptied by another tap in 6 hours. If both the taps are turned on together when the cistern is empty, in what time will it be filled?

ADDITIONAL PAPER.

1. *Either*, Find the value of $\sqrt{32}-\sqrt{128}+\sqrt{50}$ to three places of decimals.

Or, Find the square root of $1+(0\cdot046)^2$ to four places of decimals.

2. Find the value of

$$1+\frac{1}{1.3}+\frac{1}{1.3.5.7}+\frac{1}{1.3.5.7}+\ldots$$

correct to three places of decimals.

1929.

COMPULSORY PAPER.

1. (i) The quotient arising from the division of 9264 by a certain number is 17, and the remainder is 373. Find the divisor.

(ii) *Either*, how many times is the G. C. M. of 48, 36, 72, and 24 contained in their L. C. M. ?

Or, At a game of cricket *A*, *B* and *C* together score 108 runs; *B* and *C* together score 90 runs and *A* and *C* together score 51 runs. Find the number of runs scored by each of them.

2. *Either*, (i) Simplify $\frac{\frac{3}{4}\div\frac{3}{4}\text{ of }\frac{5}{6}}{\frac{2}{3}\div\frac{3}{4}\times\frac{5}{6}}-\frac{7\cdot7\times0\cdot12}{2\cdot1}$.

(ii) Reduce $\frac{4}{27}$ to a recurring decimal.

Or, Three equal glasses are filled with a mixture of spirits and water. The proportion of spirit to water in each glass is as follows :—in the first glass 2 : 3, in the second 3 : 4, and in the third 4 : 5. The contents of the three glasses are poured into a single vessel ; what is the proportion of spirit to water in it ?

3. (i) Find, by practice or otherwise, the price of 20 bags of potatoes, each containing 2 mds. 1 sr. 10 ch. at R5. 5*a* 4*p*. per maund.

(ii) *Either*, *A* sells a house to *B* for R4860, thereby losing 19 per cent. *B* sells it to *C* at a price which would have given *A* a profit of 17 per cent. Find *B*'s gain.

Or, If 40 men can mow a field of 19 acres in 8¾ days of 10 hours each, how many acres can 17 men mow in 50 days of 8 hours each ?

ADDITIONAL PAPER.

1. *Either*, Extract the square root of $\frac{3}{7}$ up to five places of decimals.

Or, Calculate by the contracted method the value of 0·345207 × 0·012395 up to five decimal figures.

2. *Either*, Find the value of

$$\frac{1}{1.4}+\frac{1}{3.4^3}+\frac{1}{5.4^5}+\ldots$$

correct to four places of decimals.

Or, When is a decimal said to be correct to the first place ?

Express 1 g. 6 dg. as a decimal of 2 kilogrammes correct to the third places of decimals.

1930.

COMPULSORY PAPER.

1. (i) Find the greatest number which will divide 1625, 2281, and 4218, leaving remainders 8, 4, and 5 respectively.

(ii) *Either*, Four clocks are made to chime at intervals of 1 h., 1 h. 20 min., 1 h. 30 min., and 1 h. 40 min. Having chimed together at 10 A. M., when will they next do so ?

Or, What sum of principal money, lent out at 5 per cent. per annum, simple interest, will produce in 4 years the same amount of interest as R250, lent out at 3 per cent. per anum, will produce in 6 years ?

2. (i) *Either*, Find the value of

$$0{\cdot}05 \text{ of R}10+3{\cdot}6\dot{3} \text{ of R}2.\ 8a.+2{\cdot}375 \text{ of } 1a.\ 4p.$$

Or, Simplify

$$\frac{\frac{3}{11}(\frac{3}{10}\text{ of }2\frac{4}{7}+\frac{1}{4}\text{ of }1\frac{2}{7})}{\frac{4}{13}\times1\frac{5}{8}\times1\frac{1}{7}-\frac{1}{11}}\div\frac{\frac{1}{8}}{\frac{3}{8}\text{ of }3-\frac{1}{17}\times5\frac{2}{3}}.$$

(ii) *A* can do a piece of work in 6 days and *B* in 8 days, each working 7 hours a day. In what time will they finish it together, working 8 hours a day ?

3. (i) Find, by practice or otherwise, the price of 4 tons 2 cwt. 2 qrs. 14 lb. of goods at £2. 6*s*. 8*d*. per cwt.

(ii) *Either*, The fore wheel of a carriage is 10 ft. in circumference and the hind wheel is 16 ft. How many revolutions will one make more than the other in 100 miles ?

Or, I mix tea purchased at 4*s*. per lb. with tea at 3*s*. 6*d*. per lb. in equal quantities. At what price per lb. should I sell the mixture to make a profit of 20 per cent. on my outlay ?

ADDITIONAL PAPER.

1. *Either*, Find the value of

$$\frac{\sqrt{5}+1}{\sqrt{5}-1}$$

correct to four places of decimals.

Or, Find by the contracted method the value of $0{\cdot}530785\times1{\cdot}0023549$ to five decimal places.

2. *Either*, Find the value of

$$1+\frac{1}{2}.\ \frac{1}{7}+\frac{1}{2.3}.\ \frac{1}{7^2}+\frac{1}{2.3.4}.\ \frac{1}{7^3}+\ldots$$

correct to four decimal places.

Or, Two sets of telegraph wires are carried on opposite sides of a railway on posts whose distances apart are 275 ft. in one case and 135 ft. in the other. An engine starts from a point where two posts are exactly opposite one another, runs an exact number of quarter-miles, and stops at a point where two posts are again exactly opposite one another. Find the least distance which the engine can have travelled.

1931.

COMPULSORY PAPER.

1. (i) *Either*, 12912 bottles have to be packed into boxes. Each box will hold 269 bottles. How many boxes would be required ?

Or, What number multiplied by 37 will give the same product as 296 multiplied by 309 ?

(ii) *Either*, Find the greatest number that will divide 5191 and 5854, leaving the remainder 4 in each case.

Or, Find the least number which being increased by 1 will be exactly divisible by 22, 17, 33, 102.

2. (i) *Either*, Simplify $\frac{[illegible] \div [illegible] \text{ of } [illegible]}{[illegible] \div [illegible] \times \frac{5}{6}} \div \frac{3\frac{1}{6}}{.3[illegible]}$ of 0·3.

Or, Express 0·725 of R9. 6*a*. + 3·9 of R11. 9*a*. in rupees, annas and pies.

(ii) Find, by practice or otherwise, the price of 7 mds. 13 srs. 9 chs. at R6. 10*a*. 8*p*. per maund.

3. (i) At what rate per cent. per annum, simple interest, will R737. 1*a*. 9*p*. amount to R825. 9*a*. in 3 years ?

(iii) *A* alone can do a piece of work in 12 days, and *B* alone can do it in 6 days ; they work together for 2 days, after which *B* leaves. In how many days more will *A* finish the work ?

ADDITIONAL PAPER.

1. *Either*, Find the square root of 0·01117249.

Or, Find the value of

$$1 + \frac{1}{[illegible]} \cdot \frac{1}{9} + \frac{1}{3} \cdot \left(\frac{1}{9}\right)^3 + \frac{1}{5} \cdot \left(\frac{1}{9}\right)^5 + \frac{1}{7} \cdot \left(\frac{1}{9}\right)^7 + \ldots$$

correct to four places of decimals.

2. *Either*, The monthly expenditure of a family of 40 persons on rice is R177. 8*a*., when it is selling at R4. 7*a*. per maund. Calculate what this expenditure will be in the case of 50 persons, when rice sells at R4. 13*a*. per maund, supposing the quantity of it per head is raised by one-fourth.

Or, A manufacturer sells goods to a dealer, and the latter to his customers, each at the same rate of profit, *viz*. 10 per cent. How much does a customer pay above the original cost of goods purchased by him for £605 ?

1932.

COMPULSORY PAPER.

1. (i) Reduce to the simplest vulgar fraction :—

$$\frac{5[illegible] - 1[illegible]}{4[illegible] + 6[illegible]} \div \frac{3\cdot2 - 2\cdot88}{0\cdot97 + 0\cdot83}.$$

(ii) *Either*, The population of India being 315 millions and the total yearly income being 11340 million rupees, find the yearly income of an Indian on the average.

Or, Find the least number which is exactly divisible by 24, 32, 45 and 52.

2. (i) Find the cost of 4 mds. 31 srs. 4 chs. of rice at R2. 11*a*. 4*p*. per md.

(ii) *Either*, The number of literates in India was 116 out of every thousand persons in 191[illegible] and increased to 140 per thousand by 1921. In how many years more will this number be 992 per thousand, if the rate of increase continues to be the same ?

Or, A man pays income-tax at the rate of 9 pies in the rupee and also contributes to the Provident Fund at the rate of one anna in the rupee, of his salary. If he draws a balance of ₨445. 5*a.*, find his salary.

3. (i) Express a pie as the decimal of a rupee.

(ii) *Either,* Find what sum of money will amount to ₨100 in five years at 6¾ per cent. per annum simple interest.

Or, A man buys milk at a certain rate per seer and after mixing it with water sells again at the same rate. Find how many chhataks of water there are in every seer if the man makes a profit of 20 per cent.

ADDITIONAL PAPER.

1. (i) Find to four places of decimals the square root of 0·000647.

(ii) *Either,* Find to four places of decimals the value of :—

$$1+\frac{1}{2}\cdot\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^2}+\frac{1}{4}\cdot\frac{1}{5^3}\ldots\ldots\ldots\text{ad. inf.}$$

Or, Evaluate to the nearest integer $\frac{27\cdot18282\times3\cdot14195}{0\cdot43429}$

2. (i) Find the cost of fencing a square field of 2·5 acres at 3 annas 11·244 pies per metre.

(1 acre = 4840 sq. yds., 1 metre = 39·37 inches.)

(ii) *Either,* *A* and *B* are at a distance of 95 miles and start at 7 A.M. cycling towards each other at the rate of 8 miles per hour and 10 miles per hour, respectively. After an hour *A* has an accident, whice detains him for half an hour, after which he continues as before. Find when they meet.

Or, *A, B, C* go into business as partners and collect a profit of ₨1000. If *A*'s capital : *B*'s capital = 2 : 3 and *B*'s capital : *C*'s capital = 2 : 5, find the shares of the profit which go to each.

1933.

COMPULSORY PAPER.

1. *Either,* Simplify $\frac{3\frac{1}{8}+2\frac{7}{11}}{4\frac{7}{10}-1\frac{1}{9}}\div\frac{5}{11+\frac{7}{8+\frac{5}{9}}}-4\frac{5}{7\frac{3}{8}}$

Or, Find the value of $\frac{0\cdot\dot{5}\dot{2}}{0\cdot\dot{1}5\dot{4}}\div\frac{26\cdot\dot{2}6}{4\cdot\dot{9}0\dot{4}}+\frac{2}{1+\frac{3}{1+0\cdot\dot{3}}}$

2. *Either,* Find the G. C. M. of 253512 and 568512.

Or, Find the L. C. M. of 24, 35, 52, 60, 91, 108.

3. *Either,* (i) Divide 0·2605 by 0·714285 and obtain the result correct to four decimal places.

(ii) A Grey-hound pursues a hare and takes 4 leaps for every 5 leaps of the hare, but 3 leaps of the hound are equal to 4 of the hare ; compare the speeds of hound and hare.

Or, (i) Find, by Practice, or otherwise the value of 7 mds. 5 srs. 2 chs. of clarified butter at R56. 14*a*. per md.

(ii) A person finds that a fall of interest from 4 to $3\frac{3}{4}$ per cent. per annum diminishes his yearly income by R60. What is his capital ?

ADDITIONAL PAPER.

1. (i) *Either*, Find the nearest value of $\sqrt{\cdot 3}$ to three places of decimals.

Or, A number of boys raised R9 by subscription among themselves. Each boy contributed the same number of annas as the number of boys. Find each boy's contribution.

(ii) A room is 12 ft. long, 8 ft. broad, and 10 ft. high. Find the cost of white washing the four walls of the room, leaving out two doors each measuring 6 ft. high and 4 ft. wide, and four windows each measuring 5 ft. high and 3 ft. wide, if the rate is 3 pies per sq. ft.

2. (i) *Either*, Find the nearest value of

$$1+\frac{1}{1\times 7}+\frac{1}{2\times 7^2}+\frac{1}{3\times 7^3}+\ldots\ldots \text{ to four places of decimals.}$$

Or, The distance between two places is given as 18 kilometres. Express the distance in miles and yards. (1 metre = 39·37 inches.)

(ii) A cistern has a supply-pipe which can fill it in 3 hours and also a waste-pipe which can empty it in 4 hours. If both pipes are opened when the cistern is empty. In what time will the cistern be filled ?

3. A rupee weighs 1 tola and contains 11 parts by weight of silver and 1 part of alloy. If silver may be bought in the market at the rate of R55 per 100 tolas and if the cost of a quantity of the alloy is $\frac{1}{11}$ of the cost of an equal quantity of silver, find the exact cost of the metal of the rupee in annas and pies.

1934.

COMPULSORY PAPER.

1. *Either*, Find the least number of rupees that should be added to 149250 rupees to make the sum equally divisible among 4744 persons.

Or, Find the least number of five digits which has 53 for a factor.

2. *Either*, Simplify

$$\frac{1}{1+\frac{2}{1+\frac{3}{4\frac{5}{6}}}}+\frac{?}{?}+ [illegible] \text{ of } \frac{?}{?}\times 1\frac{?}{?}-\frac{1}{11}(10+[illegible]).$$

Or, Simplify $\frac{15\cdot\dot{6}+7-0\cdot\dot{3}}{3\times 7\cdot\dot{4}\times 0\cdot 25}$.

3. *Either*, Find the cost of 45 chests of tea, each 1 md. 17 srs. 9 ch., at R80. 8a. 8p. per md.

Or, (i) Find the G. C. M. of 30906 and 41814.

(ii) Find the least number which when diminished by 39 is exactly divisible by 32, 40, 48, 56, 64.

4. *Either*, If the principal and interest for 5 years together amount to R1100 and the interest is $\frac{3}{8}$ of the principal, find the principal and the rate per cent. per annum.

Or, *A* can do a piece of work in 9 days and *B* in 18 days. They begin together but *A* goes away 3 days before the work is finished. How long does the work last?

ADDITIONAL PAPER.

1. (i) *Either*, Find the value of

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}},$$

correct to three decimal places.

Or, A rectangular field of area 160 acres is twice as long as it is broad. Find the perimeter of the field to the nearest foot. [1 acre = 4,840 sq. yd.]

(ii) Find the value of

$$1+\frac{1}{1.2^2}+\frac{1}{1.2^2.3^3}+\frac{1}{1.2^2.3^3.4^4}+\ldots$$

correct to four decimal places.

2. (i) *Either*, If glass is 2·5 times as heavy as water, what is the weight in kilograms of a cubic metre of glass? [1 c.c. of water weighs 1 gram.]

Or, A tax of R1,750 is raised from three villages whose populations are in the proportion of 10, 9, 13. Find the tax paid by each village.

(ii) A man sells a cow at $2\frac{1}{2}$ per cent. below cost price. Had he received R6 more than he did, he would have made a profit of 5 per cent. What did the cow cost?

3. A circular track is 984 yds. in circumference. Two men start to run round in opposite directions from the same point; one runs at the rate of 10 miles an hour, and the other at $10\frac{1}{2}$ miles an hour. Find when and where they will meet (i) for the first time, (ii) for the second time.

II. THE PUNJAB UNIVERSITY. ENTRANCE PAPERS.

1900.

1. Find the square root of 4001204·0906021.

2. Find the present worth of R10000 due 5 years hence at $4\frac{1}{2}$ per cent.

3. A rectangular courtyard, the sides of which are as 5 : 11, costs R144. 6a. for paving at 10a. 6p. per square yard. Find the length of its sides.

4. Show that compound interest reckoned quarterly at R1. 3a. 7½p. per cent. is nearly equal to interest reckoned yearly at 5 per cent.

1901.

1. Find the true discount on a bill for £721. 13s. 8d. paid 73 days before due, the rate of interest being 3¼ per cent. per annum.

2. Divide each of the numbers 4001250 and 2572125 by 125, and express the ratio of the quotients correctly to three places of decimals.

3. A man buys eggs at 1s. 3d. per dozen and sells them at 11s. 8d. per hundred. Find his gain per cent.

4. There are four vessels of equal capacity; ⅘ of the first is filled with spirit, ⅔ of the second, ½ of the third, and ⅓ of the last. The first is then filled with water and from this mixture the second is filled up, again from this second mixture the third is filled up and likewise the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

1902.

1. Define a prime number. Find the prime factors of 555,555.

2. A railway truck is 29 ft. 4 in. in length; how many such trucks will be required to fill up the entire length of the line between Lahore and Amritsar, a distance of 32 miles?

3. The difference between the simple and compound interest on a sum of money for 2 years and 5 per cent. per annum is R12. Find the sum.

4. If 3 fowls and 4 pigeons cost R2. 3a. 6p., and 5 fowls and 2 pigeons cost R2. 12a., find what must be paid for 4 fowls and 3 pigeons.

5. A person sold 60 yards of cloth for R28. 2a. gaining thereby the cost price of 9 yards. Find his gain per cent.

1903.

1. Show whether 983 is a prime number or not.

The greatest common measure of two numbers is 373, and their least common multiple is 28721. Find the product of the two numbers.

2. *A* does ⅓ of a piece of work in 3⅓ hours, *B* does ¾ of the remainder in 1⅓ hours, and *C* finishes it in 5¼ hours. How long would it have taken the three working together to do the work?

3. Find the simple interest on R2,541. 8a. for 2 years 8 months at 7a. per cent. per month.

4. Divide a sum of R345. 12½a. between *A*, *B*, *C*, so that *B* may receive 25 per cent. more than *A*, and 20 per cent. more than *C*.

5. *A* bought 100 maunds of wheat for R276. 9a., and sold it to *B* at a gain of 20 per cent.; *B* sold it to *C* at a loss of 20 per cent. What price per maund did *C* pay for the wheat?

1904.

1. Resolve 451584 into prime factors, and hence write down its square root.

Find the G. C. M. of the product of the first seven odd numbers and the product of the first eight even numbers.

2. Divide 3·14159 by 72, using factors, and finding the quotient correct to 3 decimal places.

Find the product of 36·827 and 401·59 correct to 2 decimal places.

3. Find, by Practice, the price of 623 feet of piping at $5\frac{1}{4}a.$ per foot.

4. *A* bought a bicycle for R275 and sold it to *B* at a gain of 2 annas in the rupee; *B* sold it to *C* at a loss of $2\frac{1}{4}$ annas in the rupee. How much did *C* pay for it?

5. Which of the fractions $\frac{17}{12}$ and $\frac{41}{29}$ is nearer the exact value of $\sqrt{2}$? Give reasons.

1905.

1. Find the value of

(i) $(2\frac{3}{8} \text{ of } \frac{7}{11}) \div (2\cdot59 \times \cdot3\dot{1}4\dot{8})$.

(ii) The square root of 8103060289.

2. The sum of R2,840. 1*a*. is to be divided between 7 men, 11 women, 5 boys, and 6 girls, so that for every R3. 12*a*. a man received a woman may get R2. 3*a*., and for every R2. 10*a*. a woman received a boy may get R1. 14*a*., and a girl R1. 2*a*. Find how much each person receives.

3. Find the difference between the interest and the discount on R25,078. 2*a*., the time being 21 months and the rate 4 per cent.

4. What will it cost to make a gravel walk 10 feet wide round the inside of the edge of a square field whose area is 10 acres, at $4\frac{1}{2}a.$ per square yard?

5. (i) The massacre at Cawnpore took place on the 28th June, 1857. What day of the week was it?

(ii) How many times in the course of the day do the hands of a watch cross each other?

1907.

1. Find the greatest number which will divide 16652, 10735 and 1968, and leave remainders 2, 5 and 7 respectively.

2. Find, by Practice, the value of 52 acres 3 roods 22 sq. poles at £115 . 12 . 6 per acre.

3. What sum lent at compound interest will amount to R16143-12-0 in $2\frac{1}{2}$ years at 5 per cent. per annum?

4. If 4 per cent. paper be at 110, what sum must I invest in order to secure a monthly income of R374, after paying an income-tax of 5 pies in the rupee?

5. Simplify the expression :—

$$(3\cdot\dot{5}\dot{6} - \cdot\dot{6}4\dot{8} + \cdot0\dot{1}6) \times \cdot\dot{1}4285\dot{7}.$$

1908.

1. The circumference of the front wheel of a carriage is $6\frac{3}{5}$ feet, and of the hind wheel $12\frac{3}{4}$ feet. How many feet must the carriage pass over so that each wheel may make an exact number of complete revolutions?

2. Find the difference between $3\cdot\dot{1}415\dot{9}$ and $3+\frac{1}{7+\frac{1}{16}}$.

Also find the difference between their squares.

3. A dealer bought a horse for £110, and sold it the same day for £121. 15s., allowing the buyer 5 months' credit. Money being worth $3\frac{1}{2}$ per cent. per annum, what was his gain per cent.?

4. The total population of India is 294 millions, out of which 150 millions are males. Out of every 1,000 males 98 can read and write, but only 5·3 per cent. of the total population can do so. Find the percentage of the women of India who can read and write.

5. Prove that the L. C. M. of two given expressions may be found by dividing their product by their H. C. F.

1909.

1. What part of R1. 14a. is $\frac{1}{7}$ of $\frac{2\frac{1}{4}-\frac{5}{8}\text{ of }1\frac{3}{4}}{\frac{1}{3}\times3\frac{1}{2}+\frac{11}{18}}$ of $\frac{8}{11}$ of R1. 5a.?

Divide the difference between 5·5225 and the square of ·075 by 126·1.

2. When $2\frac{1}{2}$ tolas of gold can be purchased for R58. 6a. 6p. what should be paid for a tola of silver if its value is fixed in the ratio of 1 to $15\frac{1}{2}$ to that of gold?

3. *A*, *B*, and *C* could reap a field in 18 days; *B*, *C*, and *D* in 20 days; *C*, *D*, and *A* in 24 days; and *D*, *A*, and *B* in 27 days. In what time would it be reaped by them all together?

4. A bookseller began business on 1st January, 1908, with a capital of R8,000. On 15th September he was joined by a partner, who brought R11,500 to the business. At the end of December the profits were found to be R1,654. Find, to the nearest anna, the share of each.

1910.

1. What is the least number which when divided by 36, by 40, by 42, gives in each case 5 as remainder?

2. Simplify $(\frac{7}{8}-\frac{3}{8})$ of $(6\frac{1}{2}-3\frac{1}{4})+\{5-(2\frac{3}{4}-1\frac{1}{4})\}$.

Express $\frac{1}{3125}$ as a decimal fraction.

3. Find, by Practice, the price of 37 cubic yards 3 cubic feet 280 cubic inches at R45. 8a. 6p. per cubic yard.

4. Explain what is meant by discount and present worth of a bill.

Find the present worth and discount on a bill of £1,036. 4s. due in $7\frac{1}{2}$ months, interest at $5\frac{1}{2}$ per cent.

5. *A*, *B* and *C* are partners in a business and their shares are in the proportion of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. *A* withdraws half his capital at the end of 4 months, and after 8 months more a profit of R2,024 is divided. What is *A*'s share?

1911.

1. Define the following terms, and give examples to illustrate your definitions :—

Notation ; Numeration ; absolute and local values of digits.

Write in words 2384751690.

What is the local value of each of the significant digits in the following numbers ?

92375, 24·7835.

2. Find the value of

$$\frac{12 + \sqrt{\cdot 009}}{1 - \sqrt{\cdot 4}}$$ to 3 places of decimals.

Express $\frac{7\text{[illegible]} - 3\text{[illegible]}}{18\text{[illegible]} \div \text{[illegible]}}$ of £33. 14*s*. 5[illegible]*d*. as the fraction of £157. 17*s*. 8[illegible]*d*.

3. What is meant by an aliquot part of a quantity ? Is 5*a*. 4*p*. an aliquot part of a Rupee ?

Find, by Practice, the price of 256479 articles at £4. 12*s*. 6¾*d*. per 100.

4. Define Present Worth and Discount

If the interest on R1187. 8*a*. at 3 per cent. is equal to the discount on R1193. 7*a*. for the same time at the same rate, when is the latter sum due ?

5. A contractor undertook to build a house in 21 days and engaged 15 men to do the work. But after 10 days he found it necessary to engage 10 men more, and then he accomplished the work one day too soon. How many days behindhand would he have been if he had not engaged the 10 additional men ?

1912.

1. Find the sum of 79368 added to itself 65937 times, and write the result in words.

Find the number which will divide 5970 and 5260 and leave remainders 7 and 9 respectively.

2. Simplify $\frac{\cdot 727 \times \cdot 727 - \cdot 273 \times \cdot 273}{\cdot 727 - \cdot 273}$.

Which is the greater of 27·84 × ·1481 and $\sqrt{17}$?

3. State and illustrate the difference between *direct* proportion and *inverse* proportion.

If 8 men and 12 boys can finish a piece of work in 12 days, in what time will 40 men and 45 boys finish another piece of work 3 times as great, supposing that 16 men can do as much work in 8 hours as 12 boys do in 24 hours ?

4. A boy buys eggs at 9 for 4*d*. and sells them at 11 for 5*d*. What does he gain or lose per cent. ?

The difference between the Simple Interest and the Compound Interest on a certain sum of money for 2 years at 4 per cent. is R20. What is the sum?

5. It is between 2 and 3 o'clock; but a person looking at the clock, and mistaking the hour-hand for the minute-hand fancies that the time of the day is 57 minutes earlier than the reality. What is the true time?

1913.

1. Explain what is meant by a *prime* number. Write down all the numbers between 108 and 120 which are prime.

What is the least number which when divided by 12, 15, 20 or 54 leaves in each case a remainder of 4?

2. (*a*) Explain the meaning of $\frac{4}{5}$ and $\frac{12}{15}$, and show by a diagram that they are equal to one another.

(*b*) Find the value of $3\cdot14159 \times \cdot45078$ correct to 4 places of decimals (contracted method preferred).

3. Two men undertake to do a piece of work for R7. One can do it alone in 7 days, the other in 8 days. With the assistance of a boy they finish the work in 3 days. How should the money be divided?

4. Exactly three years ago a man borrowed R3750 from a bank at 6 per cent. per annum. At the end of a year he paid the interest of that year and part of the loan, altogether R1200. Similarly he paid R800 at the end of the second year. What sum must he now pay to clear off the debt?

5. The area of a square is 11370·32 square inches. Find the length of its diagonal.

1921.

1. Find the least number which when divided by 33, 171, and 1900 will always leave the same remainder 21.

Simplify $\dfrac{2\frac{1}{3}+5\frac{1}{2}}{2\frac{1}{3}-1\frac{1}{4}}+\dfrac{2\frac{2}{3}+3\frac{1}{2}\text{ of }1\frac{3}{11}}{4\frac{1}{2}\text{ of }3\frac{1}{3}-1\frac{1}{3}}$.

2. Add together $\frac{7}{16}$ of R12. 13*a*. 10*p*., $\frac{11}{14}$ of R35. 10*a*. 11*p*., and ·31 of R42. 11*a*. 4*p*.; express the result as a fraction of £45. 6*s*. 8*d*. being given that 1 Rupee = 1*s*. 8*d*.

3. A room is 27½ ft. long, 23½ ft. wide, and 4 yds. high. Find the cost of papering the walls at 9 pies per square yard.

Find, by Practice, the wages of a man for 3 weeks 2 days and 4 hours at R3 a week, reckoning 6 days to a week and 12 hours to a day.

4. In what time will R1300 amount to R1403. 8*a*. at 3½ per cent. per annum simple interest? State this as a "present worth" sum.

5. A lady wishing to relieve a number of poor people, finds that if she gives them a shilling each she will have 3*s*. 4*d*. left, and that in order to enable her to give them 1*s*. 4*d*. each, she would require 2*s*. 4*d*. more than what she has; how many are there to be relieved, and how much money has she to distribute?

1922.

1. Find the greatest number by which 2500 and 3300 can be divided so as to leave remainders 4 and 36 respectively.

Simplify $\dfrac{\dfrac{2\frac{3}{8}}{3\frac{1}{4}}+\dfrac{2\frac{1}{7}}{7\frac{1}{2}}+\frac{3}{7}}{5\frac{3}{16}-4\frac{1}{8}}+4\frac{3}{11}\times 2\frac{1}{4}+1\frac{1}{3}.$

2. Find the value of $\frac{3}{11}$ of 3*s*. 6*d*. + $\frac{4}{7}$ of £1. 7*s*. 6*d*. + $\frac{3}{8}$ of £4. 17*s*. 4*d*.

Express the result as a fraction of R29. 8*a*., counting 1*s*. 8*d*. for a rupee.

3. Find, by Practice, the value of 10 tons 4 cwt. 1 qr. 12 lbs. at £1. 3*s*. 4*d*. per ton.

4. Explain the meaning of the terms "interest" and "rate per cent." If R1160 amount in 7 months to R1210. 12*a*., find the rate per cent. per annum.

If oranges be bought at the rate of 16 for a rupee, how many must be sold for a rupee to gain 25 per cent. ?

5. Divide R12540 among *A*, *B*, *C* so that *A* shall receive $\frac{3}{8}$ as much as *B* and *C* together, and *B* $\frac{3}{8}$ of what *A* and *C* together receive.

1923.

1. Simplify $\dfrac{\sqrt{25\cdot4016}-\sqrt{1\cdot0609}}{\sqrt{25\cdot4016}+\sqrt{1\cdot0609}}$ expressing the result correct to four decimal places.

2. Find the value of

$\frac{6}{11}$ of £23. 16*s*. 8*d*. + $\frac{9}{17}$ of £54. 16*s*. 6*d*. − $\frac{8}{11}$ of £2. 0*s*. 3*d*.

Express the result as a fraction of R560, counting 1*s*. 9*d*. for a rupee.

3. Find, by Practice, the price of 37 maunds 15 seers 12 chataks at R16. 10*a*. 8*p*. per maund.

4. A person borrowed R1460 from the Punjab National Bank on the 1st of January and repaid the amount on the 6th May of the same year. He had to pay R45 as interest. Find the rate of interest per cent. per annum.

A merchant sells out sugar to a customer, using false weights, and thereby gains 11$\frac{1}{9}$ per cent. on his outlay. What weight does he substitute for one seer ?

5. A path 9 feet wide, running all round a square park, has an area of exactly 3 acres. Find the area of that part of the park enclosed by the path, and the cost of covering this part with grass at 2*a*. 6*p*. per square yard.

III. UNIVERSITY OF ALLAHABAD. ENTRANCE PAPERS.

1900.

1. State the rules for multiplication and division of decimal fractions.

Assuming that the surface of a sphere is 3·1416 times the square of its diameter and that the earth is a sphere whose diameter is 8000 miles, find what fraction of the whole surface of the earth is the area of India which is 1350000 square miles. Express your result as a decimal fraction.

2. What are circulating decimals? Distinguish between pure and mixed circulating decimals.

(*a*) Add together $\frac{1}{3}$, $\frac{1}{30}$, $\frac{1}{70}$, $\frac{1}{15}$ and express the sum as a mixed circulating decimal.

(*b*) Reduce $\cdot 041\dot{6} \times \frac{\cdot\dot{1}4285\dot{7}}{(\frac{5}{12}+\frac{3}{8}) \times 40}$ of R8. 5*a*. to the fraction of 1 anna.

3. (*a*) Find, by Practice, the price of 100 bags of Rosa sugar, each weighing 4 seers 2 powas and 3 chataks, at 6*a*. 9*p*. per seer.

(*b*) Find the square root of $10\cdot\dot{0}\dot{2}$ to three places of decimals.

4. What sum of money will amount to R3528 in two years at 5 per cent. compound interest, and what will it amouut to in two more years?

5. What monthly income will be derived from the investment of one lakh of rupees in the $3\frac{1}{2}$ per cent. Government of India paper at $100\frac{1}{8}$?

1901.

1. (*a*) What is the greatest length which is contained a whole number of times exactly in both $25\frac{13}{18}$ feet and $21\frac{9}{32}$ feet?

(*b*) Find the value of $\frac{\cdot 49}{2\cdot 1}$ of $\frac{(3\frac{1}{4}-2\frac{1}{3})\div\frac{5}{6}\text{ of }\frac{2}{3}}{2\frac{2}{3}\div(\frac{1}{3}+\frac{1}{4})}$ of £46.

2. (*a*) Express the difference between $\cdot 9\dot{4}2857\dot{1}$ and $\cdot\dot{8}5714\dot{2}$ as a vulgar fraction in its lowest terms.

(*b*) Extract the square root of $\frac{\cdot 0253 \times \cdot 365}{8\cdot 03}$ to five places of decimals.

3. In a two-mile race *A* wins, *B* being 22 yards behind and *C* 106 yards behind *B*. By how much would *B* beat *C* in a three-mile race?

4. What sum at compound interest will amount to R650 at the end of the first year and to R676 at the end of the second year?

5. How much $3\frac{1}{2}$ per cent. Government Securities at $95\frac{1}{4}$ must be sold in order to perchase enough 5 per cent. Calcutta Municipal Debentures at $119\frac{3}{8}$ to produce an annual income of R665, a brokerage of $\frac{1}{8}$ per cent. being charged on each transaction?

1902.

Find the G. C. M. and also the L. C. M. of 49·383 and ·142569.

Simplify $\frac{1\cdot 5}{\cdot 075}\times\frac{3\frac{1}{4}}{1\frac{1}{3}}+\frac{1\cdot\dot{8}75}{2\cdot 1}\times\frac{3\cdot 5}{3\cdot 75}-\cdot 16$.

3. Find, by Practice, the value of 246⅝ maunds of sugar at R13. 5*a*. 4*p*. per maund.

4. *A* and *B* have between them 132 horses; ·25 of *A*'s = ·142857 of *B*'s. How many has each of them?

5. Six men and five boys can do a piece of work in 7 days; they work at it till they have completed ⅘ of it; then two of the men leave and two more boys come. How long will the work be in hand, if a boy does half as much work as a man?

6. If I lend a friend R1,250 at 4 per cent. simple interest and tell him to keep it until principal and interest amount to R1,666. 10*a*. 8*p*., how long will he have it?

1903.

1. (*a*) How many lengths of 2¾ inches each can be cut from a rod 7⅝ feet long, and what will be the length of the portion left?

(*b*) Reduce ⅘ of R4. 7*a*. 3*p*. to the fraction of ⅔ of R7. 14*a*. 8*p*.

2. (*a*) Divide ·016085 by 3·125; and express $1\cdot458\dot{3} + 1\cdot\dot{5}$ as a decimal.

(*b*) Simplify $\frac{5\cdot5}{\cdot63} \times \frac{\cdot081}{4\cdot2} \times \frac{4\cdot9}{33}$.

3. *A* and *B* can do a piece of work in 12 days; after working 2 days they are assisted by *C*, who works at the same rate as *A*, and the work is finished in 6¼ days more: in how many days would *B* alone do the work?

4. The 4 P. M. passenger train from Delhi to Tundla stops first at Ghaziabad, 12¾ miles distant, at 4-30 P. M.; the whole journey is 127½ miles, and 20 per cent. of the time is expended in stoppages; at what time is the train due at Tundla?

5. At what rate per cent. simple interest will R833. 5*a*. 4*p*. amount to R952. 1*a*. 4*p*. in 3 years and 2 months?

1904.

1. Simplify :—

(*a*) $$\frac{£44}{11+\cfrac{1}{7+\cfrac{3}{8\frac{1}{4}}}} \div \tfrac{1}{3} \text{ of } £1.\ 13s.\ 4d.$$

(*b*) $$\frac{\cdot00281 \times \cdot0625}{1\cdot405}.$$

2. (*a*) A bankrupt's liabilities are £6,235. 10*s*. and he pays his creditors 5*s*. 6*d*. in the pound. Find, by Practice, the amount of his assets.

(*b*) Find the square root of 10·001 correct to *four* places of decimals.

3. If 3 p. c. more be gained by selling a horse for £83. 5*s*. than by selling him for £81, what is the original price of the horse?

4. What will R1,000 amount to in 3 years at 5 P. C. per annum compound interest?

5. If the 3 per cent. consols are at $92\frac{3}{8}$, what sum of money must be invested in this stock to get an annual income of £630 brokerage being $\frac{1}{8}$ per cent. ?

1905.

1. (a) Simplify

$$\frac{7[illegible]}{6[illegible]}+\frac{1[illegible]-[illegible]}{1[illegible]+[illegible]}+[illegible] \text{ of } \frac{1}{1+\frac{2}{4[illegible]}}.$$

(b) Find the value of $\frac{1}{\sqrt{8}}$ correct to *four* places of decimals.

2. (a) Add together ·175 of 1 ton, ·83 of 1 cwt. and ·93 of 1 lb. and reduce the sum to the decimal of 10 tons.

(b) Find, by Practice, the rent of 3 acres 1 rood 27 poles of land at £1. 16*s.* 8*d.* per acre.

3. By selling a horse for R50 a man lost 4 per cent. ; find what would have been his gain or loss per cent. if it had been sold for R60.

4. Find the discount on R1,000 due 3 months hence at 4 per cent. per annum.

5. A person transfers £1000 stock from the 4 per cents. at 90 to the 3 per cents. at 72 : find the alteration in his income.

1906.

1. A merchant has three kinds of wine : of the first kind 403 gallons, of the second 434 gallons, and of the third 465 gallons. What is the least number of full casks of equal size in which this can be stored without mixing ?

2. Find the sum of money that is the same fraction of 5 crowns that R1. 8*a.* is of R2. 5*a.* 4*p.*

3. A sum of money amounts in 10 years at $4\frac{1}{2}$ per cent. simple interest to R2,972. 8*a.* In how many years will it amount to R4,356. 4*a.* ?

4. Extract the square root of 15848361.

1907.

1. Is 823 a prime number ? Why is it unnecessary to try factors above 23 in answering the question ?

2. Show that to 3 figures $\pi = \frac{22}{7}$, and that to 5 figures $\pi = \frac{355}{113}$, where $\pi = 3·14159265$.

3. Find the quotient of 6893·7825 by 72·6328 correct to four figures.

4. Find to 3 decimal places the square root of 5.

1908.

1. Find the sum of the 21 odd numbers which follow 15432.

2. Reduce $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}-[illegible]-[illegible]$ to a fraction in its lowest terms.

3. Find all the prime numbers less than a hundred.

1909.

1. A metre = 39·3708 inches. Express ·325 of a metre as a decimal of a yard (to six figures).

2. What will be the gain per cent. if mangoes bought at the rate of six for 5*a*. are sold at the rate of five for 6*a*. ?

3. In the first four months of 1906 the Indian Government sold Bills amounting to R97,984,311, obtaining £6,537,578 in exchange. Find the value of a rupee in English money to the nearest tenth of a penny.

N. B. Use no more figures than are necessary to obtain a result to the degree of accuracy indicated.

4. A holder of R5500 of $3\frac{1}{2}$% Government paper sells at $91\frac{3}{4}$ and invests in 4% stock at 101. If the brokerage is $\frac{1}{8}$ for the first and $\frac{1}{4}$ for the second, find the change in his income.

1910.

1. Write in figures the number—ninety-nine billion ninety-nine million ninety-nine thousand and ninety-nine.

Simplify $\frac{·0075 \times 2·1}{·0175} + \frac{4·255 \times ·064}{·00032}$.

2. Find the least integer exactly divisible by $5\frac{1}{4}$, $7\frac{1}{2}$, and 9.

Extract the square root of 763002225.

3. What sum put out at compound interest at 5 per cent. would amount in 3 years to £810. 6*s*. 9*d*. ?

1911.

1. Write in figures the number—nine billion eighty-nine million nine thousand and ten.

Simplify $\frac{·4\dot{4}2857\dot{1} + ·5\dot{5}7142\dot{8}}{·2\dot{2}8571\dot{4} + ·7\dot{7}1428\dot{5}}$.

2. Define the terms *yard* and *metre*.

If one inch is equal to 25·4 millimetres, find the number of kilometres in a mile.

3. Extract the square root of 1157428441.

1912.

1. A hall is 10·01 metres high, 40 metres long, and 8·001 metres wide. Find the number of cubic millimetres it contains and write your answer in words.

2. Simplify (1) $\frac{6\frac{1}{12} + 3\frac{1}{7} \text{ of } 1\frac{1}{11} - 4\frac{1}{12}}{1\frac{1}{3} - 1\frac{1}{8} + \frac{11}{16} + 4\frac{1}{6}}$,

and (2) find 0·41375 of £2. 10*s*.

3. Find the price of 3 per cent. stock when an increase of income of £5. 6*s*. 3*d*. is made by transferring to them a sum of £4,375. $3\frac{1}{2}$ per cent. stock at $95\frac{1}{2}$.

1913.

1. Simplify

$$\frac{7\text{[illegible]} \times 5\text{[illegible]}}{7\text{[illegible]} - 5\text{[illegible]}} + \frac{5\text{[illegible]} \times 3\text{[illegible]}}{5\text{[illegible]} - 3\text{[illegible]}} - \frac{2\text{[illegible]} - \text{[illegible]} \times 1\text{[illegible]}}{\text{[illegible]} \times 3\text{[illegible]} + 1\text{[illegible]}} + \frac{4\text{[illegible]}}{5\text{[illegible]}}.$$

2. Extract the eighth root of 214358881.

3. A man subscribes to a provident fund 4% of his income; on the remainder he pays income-tax at 5 pies in the rupee, and after this deduction he gives [illegible] of the remainder in charity. Of the remainder he gives [illegible] to his mother, who thus receives R12 a month. Find the man's gross annual income.

4. An acre is 0·40467 hectare, and £1 is equal to 25·25 francs. An estate measuring 1927 hectares is sold for ten million one hundred thousand francs. What is this in pounds per acre?

1914.

1. (1) Simplify

$$\frac{4\text{[illegible]} - 2\text{[illegible]}}{4\text{[illegible]} - 3\text{[illegible]}} \div \frac{81}{4 - \frac{4}{2\text{[illegible]}}} \text{ of } \text{[illegible]}.$$

(2) Add together [illegible] of £1, [illegible] of 1*s*., and [illegible] of 1*d*., and express the sum as the decimal fraction (correct to two places) of one guinea.

2. Find the square root of $25+\sqrt{125}$ correct to three places of decimals.

3. *A* sells an article to *B* at a profit of 20 per cent. *B* sells it to *C* at a profit of 5 per cent. If *C* pays 70*s*., what did it cost *A*?

4. I invest equal sums in a 4 per cent. stock and in a 3 per cent. stock and get 5 per cent. for my money; the 4 per cents. are at 90; what is the price of the 3 per cents.?

1915.

1. Find the square roots of (1) 3036·01, (2) ·1 to three places of decimals.

2. Prove that the product of any two numbers is equal to the product of their H. C. F. and their L. C. M.

The L. C. M. of two numbers is 244188, and their H. C. F. is 84. If one of the numbers is 1428, find the other.

3. A rectangular lawn 51 ft. long is surrounded by a path which is 4 ft. 6 in. wide. If the path is 96 sq. yds. in area, find the breadth of the lawn.

4. If £766. 13*s*. 4*d*. is the discount on £4600 due in 2[illegible] years, what is the rate per cent., at simple interest?

1916.

1. Simplify

$$\left\{\frac{\text{[illegible]} + \text{[illegible]}}{\text{[illegible]} + \text{[illegible]}} + \frac{\text{[illegible]} + \text{[illegible]}}{\text{[illegible]} + \text{[illegible]}}\right\} \text{ of 4 tons 7 cwt. 21 lb.}$$

2. Carpet 2 ft. wide at 6*s.* 9*d.* per yd. for a room 25 ft. 4 in. wide costs £30. 8*s.*, and paper 1 ft. 8 in. wide at 4½*d.* per yd. for its walls costs £5. 5*s.* (no allowance to be made for doors or windows). What is the height of the room?

3. The manufacturer of an article makes a profit of 25 per cent., the wholesale dealer makes a profit of 20 per cent., and the retailer makes a profit of 28 per cent. What is the cost of production of an article retailed for 16 shillings?

1917.

1. (1) Simplify

$$\frac{1\text{[illegible]}}{5-\frac{1}{1\text{[illegible]}}}\times\frac{2\text{[illegible]}\times 1\text{[illegible]}}{2\text{[illegible]}-1\text{[illegible]}}\times\frac{3\text{[illegible]}}{5\text{[illegible]}}+1\frac{7}{13}.$$

(2) Find a fourth proportional to 1[illegible], 0·09, [illegible]; and express the result as a decimal.

2. (1) Reduce 0·07 of £1. 5*s.* + 0·675 of £2. 1*s.* 8*d.* + 0·1875 of 8*d.* to the decimal of £10.

(2) Find the square root of 2 to four places of decimals.

3. A man buys milk at 2½*d.* per quart, dilutes it with water and sells the mixture at 3*d.* per quart. How much water is added to each quart of milk if his profit is 60 per cent.?

4. Find the persent value of R845, due 2 years hence, compound interest being reckoned at 4 per cent. per annum.

1918.

1. (*a*) Find the number nearest to 100,000 that can be divided exactly by 2, 3, 4, 5, 6 and 7 respectively.

(*b*) A man in India wishes to send to his son in England £300 a year in monthly instalments. How much will he have to pay monthly in rupees; the value of 1 rupee in English money being £0. 1*s.* 4[illegible]*d.*?

2. One revolution of the pedal crank drives a bicycle a distance equal to the circumference of a circle of 70 ins. diameter. How many revolutions does the crank make in travelling 1 mile? If the wheels are 28 ins. in diameter how often do they revolve in the same distance? [$\pi = \frac{22}{7}$]

3. One clock gains 25 secs. a day while another loses 1 minute a day. They are both set at the right time at 8 A. M. on August 15. On what day and at what time will they differ by 1 hour?

1919.

1. (*a*) Find all the prime numbers that divide both 1287 and 1144 without remainders.

(*b*) Simplify

(i) $\dfrac{3\text{[illegible]}+(4\text{[illegible]}\times\text{[illegible]})}{6\text{[illegible]}-(1\text{[illegible]}\times\text{[illegible]})}$, (ii) $\dfrac{12\cdot32-7\cdot56}{20\cdot35+3\cdot45}$.

2. The inside measurements of a room are 42 ft. 6 ins. and 22 ft. 9 ins. ; the walls are 2 ft. 3 ins thick and there is a verandah all round 10 ft. 6 ins. wide. Find the cost of paving the verandah with tiles measuring $4\frac{1}{2}$ ins. by 3 ins. and costing 6 pies each.

3. Which is the system of payment most advantageous for the student if the rate of interest obtainable is 6% in the following case ?

"For students commencing the course the entrance fee is R30. The entrance fee is payable by all students at the commencement of the course or may be paid in three instalments of R12 each at the beginning of the first, second and third years respectively."

1921.

1. (*a*) Find the least number which when divided by 128 and 96 will leave in each case the same remainder 5.

(*b*) Simplify $\dfrac{1\cdot8\dot{3}+2\cdot041\dot{6}+\cdot\dot{3}-3\frac{1}{4}}{1\cdot0025+\cdot0625-1\frac{1}{16}}$.

2. The matting of a room, 3 times as long as broad, at 4 annas per sq. ft. cost R75 ; and the painting of the walls at 2 annas per sq. yd. cost R6. 6*a*. $2\frac{1}{2}$*p*. ; what is the height of the room ?

3. Find the true discount on R5208. 12*a*. due $3\frac{1}{2}$ years hence at $4\frac{1}{2}$ per cent. per annum.

1922.

1. (*a*) There are 316 pages in a book, which is 1·5785 inches thick ; find to four places of decimals the thickness of its paper, allowing ·0843 of an inch for each side of the cover.

(*b*) Find the square root of 1·002001.

2. Find the cost of white-washing the four walls of a room, 24 ft. 5 in. long, 20 ft. 7 in. broad and 8 ft. 10 in high, having 2 doors, 7 ft. 3 in. by 4 ft. 2 in. each, and 2 windows 3 ft. 4 in. by 2 ft. 6 in. each, the cost of white-washing being 4 annas 5 pies per square foot.

3. By selling a horse for R320, a person lost $2\frac{1}{2}$ per cent. ; how much per cent. would he have lost or gained, if he had sold it for R450 ?

4. Five copies of a book can be bought for a certain sum of money payable at the end of a year, and six copies of the same book can be bought for the same sum if it is paid at once. What is the rate of interest ?

1923.

1. (*a*) Find the length in miles of fencing required for a square enclosure containing 4774671801 sq. yards.

(*b*) Divide $\dfrac{\cdot052}{1\cdot3}$ of 1·56 by $\dfrac{624}{14\cdot4}$ of 25·92.

2. The cubic content of a room is 2304 cub. ft. and its length bears to its breadth the ratio of 4 : 3. If the carpet on its floor costs R64, at the rate of 5 annas 4 pies per sq. ft., what are the dimensions of the room ?

3. A man bought certain goods of which he sold $\frac{1}{3}$ at a profit of 14 per cent. ; $\frac{1}{4}$ at a profit of $17\frac{1}{2}$ per cent., and the remainder at a profit of 20 per cent. What was his profit per cent. on the whole ?

IV. UNIVERSITY OF PATNA. MATRICULATION PAPERS.

1918.

Compulsory Paper.

1. (*a*) Multiply 876095 by 567049.

Or,

Two numbers when divided by a certain divisor leave the remainders 4375 and 2986 respectively ; but when the sum of the two numbers is divided by the same divisor, the remainder is 2361. Find the divisor.

(*b*) Find the G. C. M. of 64176 and 119184.

Or, What is the least number which, when divided by 6, 8, 12, 15, or 20, leaves a remainder of 5 ?

2. (*a*) Simplify

$$\frac{4\frac{1}{7}-2\frac{1}{4}}{3\frac{1}{5}+1\frac{1}{7}}+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{5-\frac{2}{3}}}}.$$

Or, Find the cost of 313 articles at £2. 17*s.* 11*d.* each.

(*b*) Multiply 3·25 by 0·0133, and divide the product by 3·64.

Or, Find the value of $\frac{\text{R1. 9}a.}{\text{R6. 4}a.}$ of 3 guineas, and express the result as decimal fraction of £5.

3. (*a*) What sum will amount to R587. 8*a.* in $3\frac{1}{2}$ years at 5 per cent. per annum simple interest ?

Or, In what time will £12345. 13*s.* $9\frac{1}{2}$*d.* double itself at 4 per cent. per annum simple interest ?

(*b*) *A* does $\frac{7}{18}$ of a piece of work in 14 days ; he then calls in *B*, and they finish the work in 2 days. How long would *B* take to do the work by himself ?

Additional Paper.

1. Evaluate $\sqrt{\cdot 0\dot{7}} - \sqrt{\cdot 07}$ to 6 places of decimals.

2. How many litres of water weigh 1000 lbs., given that one cubic foot of water weighs 1000 oz., and one metre = 39·37 inches ?

3. Find the value of the following series correct to four places of decimals :—

$$\frac{1}{3.1}+\frac{1}{3^2.2}+\frac{1}{3^3.3}+\frac{1}{3^4.4}+\ldots\ldots$$

4. What must be the least number of soldiers in a regiment to admit of its being drawn up 5, 6, 7, 8, 9, or 10 deep, and also of its being formed into a solid square ?

1919.

COMPULSORY PAPER.

1. (*a*) Multiply 79094451 by 7640950.

Or, Find the greatest and least numbers of six digits which are exactly divisible by 789.

(*b*) A heap of pebbles can be made up exactly into groups of 25 ; but when made up into groups of 18, 27 and 32, there is in each case a remainder of 11 : find the least number of pebbles such a heap can contain.

Or, A grocer buys 10 cwt. 3 qrs. 21 lbs. of sugar for £30, and pays 12*s.* 6*d.* for expenses ; at what rate must he sell it per pound to clear £15. 6*s.* 3*d.* by his bargain ?

2. (*a*) Simplify

$$\left\{2\tfrac{3}{4}+\tfrac{3}{5}\text{ of }\frac{7}{3\frac{3}{5}}-\frac{1\frac{4}{5}}{2\frac{1}{4}}\right\}+1\frac{77}{228}.$$

Or, Express $\frac{5}{8}$ of 12*s.* 6*d.* + ·625 of 7*s.* 6*d.* − ·56$\dot{5}$ of 16*s.* 6*d.* as the decimal of £1

(*b*) Find the cost of 9 yds. 2 ft. 10 in. at 5*s.* 7$\frac{1}{2}$*d.* per yard.

Or, What would be the cost of painting the four walls of a room whose length is 24 ft. 3 in., breadth 15 ft. 8 in., and height 11 ft. 6 in., at 4*s.* a square foot ?

3. (*a*) What sum will amount to £425. 19*s.* 4$\frac{3}{4}$*d.* in 10 years at 3$\frac{1}{2}$ per cent. simple interest ?

Or, If the 6*d.* loaf weigh 4·35 lbs. when wheat is 5·75*s.* per bushel, what ought to be paid for 49·3 lbs. of bread when wheat is 9·2*s.* per bushel ?

(*b*) If 200 men can make an embankment 5 miles long in 25 days, how much overtime must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being a day's work ?

Or, A man walks a certain distance, and rides back in 3 hrs. 45 min. ; he could ride both ways in 2$\frac{1}{2}$ hrs. How long would it take him to walk both ways ?

ADDITIONAL PAPER.

1. Find the square root of ·0024997600576.

Or, A square field contains 40 acres. Find the cost of running a fence round it at 2*s.* 6*d.* a yard.

2. Given that a metre contains 39·37 inches, express five miles in kilometres and metres, correct to the nearest metre.

Or, Find the value of

$$1-\frac{1}{1.2}+\frac{1}{1.2.3}-\frac{1}{1.2.3.4}+\frac{1}{1.2.3.4.5}-\ldots$$

correct to 4 places of decimals.

1920.

Compulsory Paper.

1. Multiply 915625 by 961024.

2. Prove that $95785^2-94340^2=16575^2$.

3. Reduce to its lowest terms [illegible].

4. Simplify $\dfrac{\frac{3}{5}+\frac{7}{8}}{\frac{3}{5}+\frac{9}{10}}$ of $\dfrac{13s.\ 5d.}{9s.\ 10d.}\div\frac{3}{5}(\frac{3}{7}+\frac{3}{5})$ of $\dfrac{3 \text{ tons } 3 \text{ cwt.}}{4 \text{ tons } 3 \text{ cwt.}}$.

5. Express $\frac{3}{8}$ of 7*s.* 6*d.* + 1·25 of 5*s.* − 0·5$\dot{4}\dot{5}$ of 9*s.* 2*d.* as a decimal of £10.

6. Find the cost of 56375 articles at £2. 15*s.* 9*d.* per hundred.

7. A tradesman who commenced business $5\frac{3}{4}$ years ago increased his capital at the rate of 15 per cent. per annum, simple interest, and it now amounts to £5960. What sum did he start with?

Additional Paper.

1. *Either*, Find the smallest number that must be added to 153·140025 to make it a perfect square.

Or, A piece of silk cost £84. 0*s.* 4*d.*, and there were as many yards in the piece as there were pence in the price of a yard. Find the length of the piece.

2. *Either*, The great Wall of China is said to be 2400 km. long and 7625 mm. thick at the bottom. Find, to the nearest square foot, the area of the ground it stands upon. (1 metre = 39·37 inches.)

Or, Employ the contracted method to divide 2·6289475 by 306·5 correct to the sixth decimal place.

1921.

Compulsory Paper.

1. Find the continued product of 3781, 3782 and 3783.

2. Add together $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$, subtract the sum from 2, multiply the result by $\frac{3}{5}$ of $\frac{27}{8}$ of 8, and find what fraction this is of 99.

3. *Either*, Find the value, correct to 6 places of decimals, of

$$2\cdot\dot{4}1\dot{8}+1\cdot1\dot{6}+3\cdot\dot{6}0\dot{9}+0\cdot735\dot{4}+24\cdot042.$$

Or, Find the square root of 5345344.

4. *Either*, Find the value of 9 yds. 2 ft. 10 in. of cloth at 5*s.* $7\frac{1}{2}$*d.* per yard.

Or, What will be the cost of painting a room which is $20\frac{1}{2}$ ft. long, $18\frac{3}{4}$ ft. broad and 10 ft. high, containing two windows whose dimensions are 7 ft. by 2 ft. each, at the rate of 2*s*. 6*d*. per sq. yd. ?

5. *Either*, At what rate per cent. simple interest, will £936. 13*s*. 4*d*. amount to £1157. 7*s*. $4\frac{1}{2}$*d*. in $4\frac{3}{4}$ years ?

Or, What sum of money must be left, in order that after a legacy duty of 10 per cent. has been paid, the remainder being lent out at 3 per cent. simple interest may give a yearly income of 100 guineas ?

1922.

Compulsory Paper.

1. Multiply 23405 by 12084.

2. Find the greatest number which exactly divides both 13677 and 28012.

3. A man owns $\frac{4}{11}$ of a house and $(\frac{1}{3}+\frac{2}{5})$ of his portion is worth R112 ; find the price of the whole house.

4. *Either*, Find the cost of 25 bales at R9. 10*a*. 7*p*. per maund if each bale contains 13 mds. 24 sr. 12 ch.

Or, Find the cost of making a gravel path 6 ft. wide along the inner edge of a square field, whose side is 120 yds. long at 8*a*. per sq. yd.

5. *Either*, Extract the square root of 2701·9204.

Or, Find in what time a given sum of money will quadruple itself if lent out at simple interest at the rate of a pice per rupee per month.

1923.

Compulsory Paper.

1. By what number must 695 be multiplied so that when the product is subtracted from one million, the result is 507245 ?

2. Find the smallest number of which 135, 126, 432, and 255 are divisors.

3. Prove that,

$$2-\cfrac{4}{5-\cfrac{9}{7-\cfrac{16}{9}}}=\frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}.$$

4. *Either*, Add together £3·05425, $12\cdot1\dot{2}$ of £2. 5*s*. and $7\cdot\dot{2}8571\dot{4}$ of a guinea.

Or, Find to the nearest pie the value of 1234 maunds, 27 seers, 10 chataks of wheat at R3. 10*a*. 8*p*. per maund.

5. *Either*, How often must I run round a square field of 10 acres to run a mile ?

Or, A dealer bought 76 cows, and then sold 20 at a profit of 15 per cent., 40 at a profit of 19 per cent. and the remainder at a profit of 25 per cent. making a total profit of R657. What did he give for each ?

1924.

Compulsory Paper.

1. Divide 99790104 by 9987.

Or, Multiply 771214 by 216636.

2. Find the greatest number of six digits which exactly divisible by 27, 45, 60, 72, 96, and 120.

3. Find the price of 345 maunds, 27 seers, 13 chataks of rice at R7. 10*a*. 8*p*. per maund.

Or, Simplify

$$\frac{3\frac{3}{4}+4\frac{5}{8}}{5\frac{1}{16}-4\frac{1}{30}}\div\frac{4\cdot75+3\cdot8}{6\cdot1-5\cdot15}\times\frac{\cdot021\times\cdot0021\times210}{\cdot14\times\cdot007}.$$

4. A contractor undertakes to dig a canal 12 miles long in 350 days, and employs 45 men ; he finds that in 200 days he has completed 4½ miles. How many additional men must he employ to get the undertaking finished in time ?

5. The length of a rectangular field is to its breadth as 3 : 2, and its area is 11,094 sq. m. Find the cost of surrounding it with a fence at 2·25 francs per metre length of boundary.

Or, In an examination 52 per cent. of candidates failed in English and 42 per cent. in Mathematics. If 17 per cent. failed both in English and Mathematics, find the percentage of those who passed in both the subjects.

1925.

Compulsory Paper.

1. The quotient in a division is 479, the dividend is 34,76,418, and the remainder is 794 ; what is the divisor ?

Or, A boy miscopies a question and finds the value of 2978×978 instead of 2928×978. By how much is his answer too great ?

2. Find the greatest number that will divide both 719 and 930, leaving the remainders 5 and 6 respectively.

Or, Find the L. C. M. of 481 and 629.

3. A man standing on the platform of a station notices that a train whose speed is 36 miles an hour passed the platform in 20 seconds. If the length of the platform is 200 yards, what is the length of the train ?

Or, Simplify

$$\frac{2[illegible]-1[illegible]}{3[illegible]+1[illegible]}+\frac{[illegible]-[illegible]}{[illegible]+[illegible]}+\frac{\cdot05\times\cdot7}{\cdot071}\text{ of }\frac{\text{R}2.\ 7a.}{\text{R}1.\ 11a.}.$$

4. A certain sum of money at simple interest amounts to R632. 8*a*. in 3 years, and to R673. 12*a*. in 4 years 6 months. Find the sum and the rate of interest.

Or, What number multiplied by itself will give 109[illegible].

5. A rectangular court is 120 feet long and 90 feet broad, and has a path, 10 feet wide, round it. Find the cost of covering the path with flag-stones at R3. 6*a*. per square yard and turfing the court itself at R7. 2*a*. per 100 square feet.

ADDITIONAL PAPER.

1. In a division sum the divisor was 647042 and the quotient 497035; what was the dividend?

Or, A collection containing an equal number of rupees, half-rupees, quarter-rupees, two-anna pieces and one-anna pieces amounted to R968-12-0. What was the total number of coins?

2. (*a*) The forewheel of a bicycle is 8 ft. and the hindwheel is 10 ft. 6 in. round; what is the least distance in which the wheels will turn round completely an exact number of times?

(*b*) Express 10*s*. $10\frac{1}{2}$*d*. as the decimal of £1.

3. If a train maintains an average speed of 42 miles an hour, it arrives at its destination punctually; if, however, the average speed is 40 miles an hour, it arrives 15 minutes late. Find the length of the journey.

Or, Simplify

$$\frac{3\frac{1}{6}+4\frac{1}{3}-5\frac{1}{4}\text{ of }\frac{2}{7}}{3\frac{1}{9}-4\frac{1}{3}\text{ of }\frac{7}{20}+1\frac{4}{5}}+\frac{\cdot527\times\cdot527-\cdot223\times\cdot223}{\cdot527-\cdot223}.$$

4. In what time will R350 amount to R392 at 4 per cent. per annum simple interest?

5. Extract the square root of 29·192409.

Or, The four walls of a room have a total area of 660 sq. ft.; the area of the floor is 270 sq. ft.; the width of the floor is 15 ft. What is the height of the room?

1926.

COMPULSORY PAPER.

1. Find the smallest number of six digits which is exactly divisible by 567.

Or, Along a hedge 590 feet long 60 trees are planted at equal distances, one tree being planted at each end of the hedge. Find the distance between any two consecutive trees.

2. Ascertain whether 229 is a prime number or not.

Or, Express £3 . 7 . $11\frac{3}{4}$ as the decimal of £100.

3. Find, by Practice, the cost of 8 mds. 15 sr. $12\frac{1}{2}$ ch. of wheat at R12 . 5 . 4 per maund.

Or, A person spends $\frac{3}{4}$ of his money and then $\frac{5}{12}$ of the remainder. He now finds that he has left R10 more than $\frac{2}{15}$ of the original amount. How much had he at first?

4. *A* borrowed from *B* the sum of R960 for 4 years, simple interest being charged at 6 per cent. per annum. It was to be repaid by one-fourth of the principal being paid at the end of each of the first three years, and the last fourth with the total amount of interest due to be paid at the end of the fourth year. What would the last payment be?

Or, Find the square root of 2704·416016.

5. A lawn tennis ground is half as long again as it is wide. The cost of levelling it at 5*a*. per square yard is R1,470. Find the cost of enclosing it with an iron railing at R4 per yard.

Or, Messengers travelling 15 miles an hour are sent out every 10 minutes to meet a person approaching at the rate of 10 miles an hour; at what intervals of time will they meet him?

1926.

Supplementary Paper.

1. What is the least number which must be added to 2486132 in order that the sum may be exactly divisible by 4125?

Or, Two persons have R11 . 8 . 0 between them. If one had R1 more and the other 8*a*. less, the former would have twice as much as the latter. How much has each?

2. Multiply 25·05 by 4·6, and divide ·0213 by 3·75.

Or, Reduce $\frac{80025}{937893}$ to its lowest terms.

3. Find, by Practice, the cost of 237¼ maunds of rice at R8 . 10 . 8 per maund.

Or, Simplify

$$(4\frac{1}{3} - 3\frac{2}{5}) \times (7\frac{2}{3} - 5\frac{3}{14}) \div \frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{7}{9} \text{ of } 2\frac{10}{17}.$$

4. The simple interest on a certain sum for nine months at 5 per cent. per annum is R125 less than the simple interest on the same sum for fifteen months at 4 per cent. per annum. Find the principal.

Or, Find the square root of 57214096.

5. The cost of carpeting a room was R120; but if the length of the room had been 3 ft. less than it was the cost would have been R105. What was the length?

Or, A man buys a bicycle. When he has ridden a certain distance, he reckons that it has cost him 2*a*. 6*p*. a mile; he rides 240 miles more and finds that it has cost him 2*a*. a mile. How much farther must he ride to reduce the cost to 1*a*. 6*p*. per mile?

1927.

1. (*a*) Multiply 783496 by 856079.

Or, Find the number which when divided by 7538 leaves a remainder 629 and if the quotient is decreased by 289, the result is 2975.

(*b*) What is the greatest number which will divide 2000 and 2708 and leave remainders 11 and 17 respectively?

Or, How many Kilogrammes are there in ·708624 of a ton ?

(100 Kilogs. = 1·9684 cwt.)

2. Simplify

$$\frac{2\frac{3}{4}}{5\frac{1}{6}} \text{ of } \frac{3}{4}\left(\frac{7}{9}+\frac{1}{12}\right) \div \frac{5\frac{7}{8}}{7\frac{1}{4}} \text{ of } \frac{2s.\ 5d.}{3s.\ 11d.}$$

Or, What decimal of £2. 13*s*. 4*d*. is ·0625 of 2·6 of £1. 6*s*. 8*d*. ?

(*b*) Find, by Practice or otherwise, the price of 24 tons 12 cwt. 3 qr. $7\frac{1}{2}$ lbs. of coal at £6. 13*s*. 4*d*. per ton.

Or, Extract the square root of ·00826462810.

3. Find the sum of money which will secure at 4 per cent. simple interest the same income at R12,345 at $4\frac{1}{2}$ per cent. per annum.

Or, *A* can do a piece of work in 3 days, *B* can do 3 times as much in 8 days and *C*, 5 times as much in 12 days. In what time will they do it together supposing them to work at the rate of 9 hours daily ?

1928.

1. (*a*) What number must be added to 123456789 to give a sum exactly divisible by 5678 ?

Or, Divide R1656. 10*a*. among *A*, *B* and *C* so that as often as *A* gets R2, *B* shall get R3, and as often as *B* gets R4, *C* shall get R3.

(*b*) Reduce [illegible] to its lowest terms.

Or, A grocer by selling tea at 15*a*. a lb. loses 6 per cent. At what rate must he sell it in order to gain $17\frac{1}{2}$ per cent. ?

2. (*a*) Find, by Practice, the cost of $4013\frac{1}{2}$ cub. yds. of timber at £2. 16*s*. $6\frac{1}{2}d$. per cub. yard.

Or, If the simple interest on £3238. 3*s*. 6*d*. exceeds that on £2762. 13*s*. 6*d*. by £7. 0*s*. $9\frac{3}{4}d$. in three months, what is the rate of interest per cent. per annum ?

(*b*) Extract the square root of

$$\frac{1000\cdot60009}{10^3}.$$

Or, Reduce $\dfrac{2\cdot25 - \cdot6 \text{ of } 1\cdot8\dot{3}}{\cdot2 \text{ of } 3\frac{1}{3} + \cdot36\dot{1}} \times \cdot95$ of 5*s*. to the decimal of £11.

3. (*a*) In a race of half a mile *A* gives *B* 40 yards', *C* 75 yards' start. If their respective speeds be uniform and as 23 : 22 : 21, who will win and by how much ?

Or, Find Rs. 4*a*. + $\dfrac{\dfrac{\frac{25}{32}}{2\frac{1}{4}-\frac{7}{12}} \text{ of } \frac{8}{15} \text{ of } £5 - \frac{2}{3}\frac{1}{3} \text{ of a Guinea}}{£93.\ 8s.\ 2\frac{3}{16}d.}$.

(*b*) *A*, *B*, *C* send 55, 70 and 75 sheep respectively to a field the rent of which is £30. What percentage of rent should each pay ?

1929.

1. (*a*) Multiply 898765 by 504637.

Or, The divisor of a certain number is 25 times the quotient and 15 times the remainder. Find the number when the remainder is 375.

(*b*) What is the greatest number that can be subtracted from 5684398 so that the remainder is a multiple of each of 60, 75, 135, 140 and 156 ?

Or, What is the greatest sum of money of which £4. 15*s*., £6. 13*s*. and £8. 17*s*. 4*d*. are multiples ?

2. (*a*) Simplify

$$\frac{5\cdot75 - \frac{4}{7}\times 15\frac{3}{4} + 2\frac{1}{16}\div 1\cdot44}{\frac{3}{8}\text{ of }7\frac{3}{8} - 5\cdot6\div 3\cdot2\dot{6}}.$$

Or, Extract the square root of ·00005477 correct to 7 places of decimals.

(*b*) Find the cost (to the nearest penny) of 9 cwt. 3 qr. 11¾ lb. of goods at £1281. 5*s*. per 100 tons.

Or, In how many years will R5488 amount to R7002. 14*a*. 8*p*. at a simple interest of R4. 6*a*. 8*p*. per annum for R100.

3 (*a*) A man sells a horse for R1000 and losses 25⅓ per cent. How much per cent. should he have gained or lost, had it been sold for R1200 ?

Or, An empty cistern has three pipes. Two can fill it in 3 hours and 3 hrs. 45 min. respectively and the third can empty it in 1 hour. If these pipes be opened successively at 1, 2 and 3 o'clock, find when the cistern will be empty.

1931.

Supplementary Paper.

1. (*a*) *Either*, Find the number whose square is equal to the difference between the squares of 6467 and 4683.

Or, Find the greatest and least numbers of six digits that are exactly divisible by 69 and 96.

(*b*) *Either*, Find the L. C. M. of 4984 and 5488.

Or, Simplify $\frac{2\cdot4\dot{6} - 2\cdot\dot{3}\dot{0}}{\cdot\dot{3} + \cdot12\dot{7}} \div \frac{18}{3\frac{8}{9}}$,

giving your answer correct to 4 places of decimals.

2. (*a*) *Either*, Tea bought at 7*a*. 6½*p*. per lb. and tea bought at 10*a*. 4*p*. per lb. are mixed in the proportion of 8 : 7. The mixture is then sold at 10*a*. 7½*p*. per lb. Find the gain per cent.

Or, Find, by Practice, the cost of 110 tons 13 cwt. 14 lbs. of coal at R1,518. 12*a*. per 100 tons.

(*b*) What sum of money must a man invest in order to earn £896. 13*s*. 4*d*. in 5 years at 3⅓% simple interest per annum ?

3. A garrison of 3250 men is provisioned for 15 weeks allowing 12 oz. for each man daily. If it is reinforced by 1250 men and the whole force is now put on a daily ration of 10 oz. per man, how long can the same provision last ?

1932.

SUPPLEMENTRY PAPER.

1. *Either*, (*a*) The product of two numbers is ·1525446. If one of them is ·00125, find the other.

(*b*) Simplify $\frac{1\frac{1}{4} - \frac{1}{3} \text{ of } 1\frac{3}{5}}{\frac{2}{3} \text{ of } 4\frac{1}{5} + \frac{3}{5}} \div \frac{7\frac{1}{2} - 1\frac{2}{3} \div \frac{20}{63}}{3\frac{2}{5} \text{ of } 13\frac{1}{3} \div 2\frac{1}{4}}$.

Or, (*a*) Find the greastest number that must be subtracted from a million so that the remainder may be exactly divisible by 135, 140, 270 and 320.

(*b*) Extract the square root of $(5{\cdot}03)^3$ to four places of decimals.

2. *Either*, (*a*) Find, by Practice, the cost of 16 bags of sugar each weighing 4 cwt. 3 qr. 2 lb. at R8. 12*a*. $4\frac{1}{2}p$. per cwt.

(*b*) A rectangular lawn 45 yds. long and 15 yds. broad is surrounded by a gravel path 15 ft. wide. Find the cost of making the path at 6*a*. 9*p*. per sqaure yd.

Or, (*a*) If one metre = 39·37 inches, find in feet the height of a barometer which stands at 750 mm.

(*b*) If R737. 14*a*. 8*p* amounts to R885. 8*a*. in five years, how much will R966. 10*a*. 8*p*. amount to in 4 years at the same rate of simple interest ?

3. *Either*, At an examination in which full marks were 800. *A* got 20 per cent. more than *B*, *B* 20 per cent. more than *C*, and *C* 15 per cent. less than *D*. If *A* got 576, what percentage of full marks did *D* get ?

Or, Two pipes *A* and *B* can fill a cistern in 30 minutes and 45 minutes respectively. They are opened together and after 8 minutes *B* is stopped. How long will it take *A* to fill the cistern ?

V. SCHOOL-LEAVING CERTIFICATE EXAMINATION PAPERS, UNITED PROVINCES.

1910.

In all monetary calculations in this paper, fractional amounts less than a farthing, or a pie, may be neglected.

1. (*a*) What decimal fraction of a rupee is one pie ?

(*b*) Express the following as decimal fractions of a rupee :—

(1) 13 annas ; (2) 9 annas ; (3) 2 annas 1 pie.

2. The size of a brick is $9'' \times 4\frac{1}{2}'' \times 3''$.

Calculate how many bricks would be required to build a wall 10′ long, 6′ high, 18″ thick. Assume that 10 per cent. of the completed wall consists of mortar.

3. Convert $\dfrac{\text{[illegible]}+\frac{4}{5}+\frac{7}{\text{[illegible]}}}{2+\frac{1}{\text{[illegible]}}+\frac{1}{\text{[illegible]}}}$ into a decimal fraction, correct to 4 places.

4. A merchant buys 100 loads of wood at R1. 12*a*. per maund. Each load contains 8 maunds ; $2\frac{1}{2}$ per cent. of the wood was stolen from him. He sold 40 cart-loads, each containing 9 maunds, at R2 per maund, and 60 cart-loads, each containing 7 maunds, at R1. 14*a*. per maund. What was his profit or loss on the whole transaction ?

5. A man borrows 1000 rupees to build a house. He pays 5 per cent. simple interest. He lets the house and receives as rent R12. 8*a*. per month. In how many years ought he to be clear of debt ?

1911.

1. (*a*) Find the value of

(i) 140·08 × ·003501, correct to two decimal places ; and
(ii) $(\cdot0035)^2$, correct to two significant digits.

(*b*) Supply the missing quantity (without actually working it out) in the following identities :—

(i) 370 × ·014 = 140 × (·37) ;
(ii) ·17 ÷ 3·14 = ·00017 ÷ (31400).

Give reasons for your inference.

2. A French newspaper says : 'There has been an increase in the export of silk from this country of 85000 lb. in 5 years. Magnificent !... it means an increase of 51 lb. every day, almost 3 every hour, about ·02 of a lb. every minute'. If the first statement is correct, show that the latter statements are all incorrect. Find the amount of the error in each case.

3. A cup of milk contains 3 parts pure milk and 1 part water. How much of the mixture must be withdrawn and water substituted, in order that the resulting mixture may be half milk and half water ?

4. A person invests R32500 in $3\frac{1}{2}$ per cent. securities, at 94. What income will he derive from it ?

Or, A man borrows R50 for a week and pays 2 annas interest. Find the rate per cent. per annum.

1912.

1. Simplify $\dfrac{5\frac{3}{8}}{7\frac{1}{\text{[illegible]}}}$ of $\dfrac{\cdot21\cdot25}{\cdot046875}$.

2. A motor car covers a kilometre in 25 seconds. The speed indicator shows that it has been running at the rate of 90 miles per hour. From this show that 8 km. = 5 miles.

3. In a football field the part on which the game is played is 100 yards long and 60 yards wide. If a space, 10 yards wide, is roped off all round, what is the area of the space roped off ?

1913.

1. Find, correct to one decimal place, the number of revolutions made, in a minute, by a bicycle wheel 2·8 feet in diameter, when the bicycle is going at the rate of 1·04 miles per hour. (Take 3·14 as the ratio of the circumference of a circle to its diameter).

2. Find the number of cubic feet of wood used in making a closed box, the outside dimensions of which are 6 feet long, 2 feet 4 inches wide, and 2 feet 2 inches high; the wood being 2 inches thick.

3. The manufacturer of a pump sells it to a wholesale dealer, and makes a profit of 15 per cent.; the wholesale dealer sells it to a shop keeper, and makes a profit of 8 per cent.; the shop-keeper sells it for R250 to a zaminder, and makes a profit of ·12 per cent. Find, to the nearest rupee, the cost of the pump to the manufacturer. Avoid all unnecessary calculations.

1914.

[N. B. *Problems may be solved either arithmetically or algebraically.*]

1. Express 0·75 of R1084. 12*a*. in rupees, correct to one decimal place.

2. The area of a rectangular field is 6000 sq. yd., and one of its sides is 50 yd. long. Two boys, *A* and *B*, start from the same corner of the field in order to get to the opposite corner, but *A* walks along the diagonal at the rate of 3 miles an hour, whereas *B* walks along the two sides of the field at the rate of $3\frac{1}{2}$ miles an hour. Who arrives at the opposite corner first, and how much earlier?

3. A man owes R12000 to a merchant. He pays R4000 at the end of the first year, and a sum of R3500 at the end of the second year. How much should he pay at the end of the third year to clear off the debt? Simple interest at the rate of $4\frac{1}{2}$ per cent. is charged.

4. A milkman buys cow's milk at the rate of 8 seers for a rupee, and buffalo's milk at the rate of 6 seers for a rupee, and mixes them in the ratio of 2 : 1. The then sells the mixture at 3 annas per seer. What is his gain per cent.?

1915.

1. The single fare from Allahabad to Calcutta is R8. 13*a*. 6*p*., and the return fare is R16. 2*a*. How much does a man save in a year by taking return tickets instead of single tickets, if he has to make 5 visits monthly to Calcutta from Allahabad?

2. *A* and *B* run a race. *A* starts off at the rate of 8 miles an hour and 5 minutes ahead of *B*. *B* starts from a post 50 yards behind *A*, and runs at the rate of 10 miles an hour. How soon will *B* overtake *A*?

3. At what rate per cent. will R6605 amount to R7678. 5*a*. in 5 years, at simple interest?

1916.

[N. B. *Problems may be solved either arithmetically or algebraically*].

1. The floor area of a square room is 100·2001 sq. ft. and it is paved with marble all along its edges, at the rate of R10 per sq. ft., the breadth of the pavement being 1 ft. The rest of the floor inside the pavement is supplied with carpet, at the rate of R5 per sq. ft. Find the cost of paving and carpeting the room.

2. A person died and left property worth R10000. He had two sons, three daughters, a wife, four brothers, and two uncles. The share of each son is double of the share of a daughter, and that of the wife is $1\frac{1}{2}$ times the share of a daughter, and the shares of the brothers and the uncles are $\frac{2}{3}$ and $\frac{1}{2}$ the shares of the daughters. Find how much money each will get.

3. If the true discount on £2696. 13*s*. 4*d*., due 3 months hence is £30, what is the rate per cent., simple interest ?

1917.

[N. B. *Problems may be solved either arithmetically or algebraically.*]

1. (1) Prove that a number is divisible by 9, if the sum of its digits is divisible by 9.

(2) Extract the square root of 0·1 to three places of decimals.

2. Find the present worth of £6433. 3*s*. 4*d*., due 3 months hence, the rate of interest being $3\frac{1}{2}$ per cent. per annum.

3. A man bought a number of oranges at 3 for an anna, and an equal number at 2 for an anna. At what price per dozen should he sell them to make a profit of 20 per cent. ?

1918.

1. Divide 1·00625 by 132·5 to five places of decimals.

2. (*a*) A tradesman marks his goods at 25 per cent. above cost price. If he allows his customers 10 per cent. discount, how much per cent. profit does he make ?

Or, (*b*) On a certain day the exchange value of the rupee increased from 1*s*. $4\frac{3}{16}$*d*. to 1*s* $4\frac{1}{2}$*d*. How much difference would this make, if one were changing 1500000 rupees ?

2. Reckoning compound interest at 6 per cent. payable half-yearly, find the present value of £6450, due 2 years hence.

1919.

1. (*a*) Reduce $\frac{34452}{65142}$ to its lowest terms.

(*b*) What is the least number which, when divided by 12, 20, 30 or 54, leaves in each case a remainder of 4 ?

(*a*) Find the greatest and the least of the following :—

$$\frac{1}{7}, \frac{5}{28}, \frac{6}{49}, \frac{23}{196}.$$

(*b*) Simplify

$$\frac{2\frac{1}{2}}{\frac{8}{11} - \frac{2}{\frac{3\frac{1}{2}+1}{2\frac{1}{2}-1\frac{1}{2}}}}.$$

3. An iron chest 5 ft. long, 3 ft. broad and 2 ft. 5 in. high is made with iron sheet, 1 in. thick. Find the inside capacity of the chest and the weight of the chest, if 1 cub. ft. iron weighs 6 maunds.

4. (*a*) A man incurs a loss of 12 per cent. on his outlayby selling a horse for R550 ; find the price at which he should sell the horse in order to realise a gain of 12 per cent.

Or, (*b*) A man sitting in a train which is travelling at the rate of 50 miles an hour observes that it takes 6 seconds for a goods train travelling in the opposite direction to pass him. If the goods train is 220 yards long, find the rate at which it is travelling.

5. If one inch = 2·54 centimetres, construct a graph for finding the number of centimetres in any given number of inches and *vice versa*.

6. What number must be subtracted from each of the numbers 6, 8, 7 and 11, so that the remainders may be in proportion ?

1920.

1. (1) Divide 28937257 by $2 \times 3 \times 5 \times 7 \times 11$ by short division, and find the complete remainder.

(2) Find the value of

$$\cdot 03\dot{8}\dot{7} \text{ of } £8.\ 16s.\ 3d. + 6\tfrac{1}{7} \text{ of } \tfrac{13}{18} \text{ of } 7s.\ 8\tfrac{1}{2}d. + \tfrac{7}{11} \text{ of } 1d.$$

2. *Either* (i) Find the least number of square marble slabs required to pave a floor 276 ft. long and 204 ft. broad.

Or, (ii) The cost of painting the walls of a room at 10*a*. 8*p*. per sq. ft. is R400, and the cost of carpeting the same room at the rate of R2. 8*a*. per sq. ft. is R540. If the length of the room be to its width as 3 : 2, find the dimensions of the room.

3. At what rate per cent. (simple interest) will a given sum double itself in 30 years ?

4. A tradesman, by means of a false balance defrauds to the extent of 10 per cent. in buying goods and also defrauds 10 per cent. in selling. What per cent. does he gain on his outlay by his dishonesty ?

5. A cistern has three pipes, *A*, *B* and *C*. *A* and *B* can fill it in 4 and 5 hours respectively, and *C* can empty it in 2 hours. If these pipes be opened in order at 1, 2 and 3 o'clock A. M., when will the cistern be empty ?

6. A person rowed 12 miles down a river and back again in 8 hours and found that it took thrice as long to row against the stream as to row with it. Find the rate of the stream and of the boat in still water.

1921.

1. (*a*) Add, correct to three places of decimals :—

$$1\tfrac{1}{7},\ 3{\cdot}142859,\ 7{\cdot}\dot{6}\dot{5},\ 87{\cdot}6623,\ 45{\cdot}018 \text{ and } 3\tfrac{7}{11}.$$

(*b*) A baniya buys a crop which weighs 1995 maunds at R3. 12*a*. 9*p*. per maund. He sells it for R3. 15*a*. 6*p*. per maund. How much profit does he make and how much profit per cent. ?

2. A clerk began his work in January 1, 1918, on R25 per mensem. On June 16, 1918, he was promoted to R27. 8a. per mensem, but reverted to R25 per mensem on September 1, 1918. On January 1, 1919, he was promoted to R30 per mensem and again on November 1, 1919, to R32. 8a. per mensem. On April 1, 1920, he was promoted to R50 per mensem and on September 16, 1920, to R55 per mensem. He reverted to R50 per mensem on December 1, 1920. What was his average pay per mensem for the past three years on January 1, 1921?

3. What is the present worth of £15550 payable six months hence, if the rate of interest is $7\frac{1}{2}$ per cent. per annum?

4. Of 10000 children who passed the age of 9, 490 died at 10 years of age, 272 died at 14 years of age, 650 at 22, 617 at 25, 757 at 35, 950 at 45, 1399 at 55, 2141 at 65, 2578 at 74, 1138 at 85, and 86 at 95. Draw a graph to illustrate this and estimate from it the number of deaths among these children at the ages of 18, 40 and 80.

5. Find two numbers one of which is greater by four than three-fifths of the other so that the difference of their squares may be equal to 24.

1922.

1. (1) Find the value of

$$5\cdot\dot{3} \div \frac{2\cdot8}{0\cdot\dot{2}\dot{1}} \text{ of } \frac{\text{R}1.\ 5a.\ 4p.}{\text{R}4.\ 2a.\ 8p.}$$

(2) Multiply 0·0987321 by 23·376542 correct to three places of decimals by contracted method of multiplication.

2. If a man's debts amount to R3199. 10a. 8p., and he can pay only 1a. 3p. in the rupee, find, by Practice, the amount of money that his creditors get.

3. *A* can reap a field in 10 days, *B* can reap it in 12 days, and *C* can reap it in 15 days. How long will it take them all together to reap it, and what part of the work will be done by each?

4. *Either* (1) The simple interest on a certain sum of money for one year at a certain rate of interest is £80; and the compound interest on the same sum for two years at the same rate of interest (interest being payable yearly) is £164. Find the rate of interest.

Or, (2) One side of a rectangular lawn is $\frac{4}{7}$ times the other side. If the cost of levelling it at 6a. per square yard be R1764, find the length of the lawn.

5. In a certain hospital the temperature of an indoor patient was recorded as follows:—

Temperature at	7 A. M.	...	...	98·9°C.
,,	9 A. M.	...	...	100·4°C.
,,	11 A. M.	...	...	102·2°C.
,,	1 P. M.	...	...	103·1°C.
,,	3 P. M.	...	...	102·4°C.
,,	5 P. M.	...	...	100·2°C.

Draw a graph showing the temperature of the patient at different times of the day, and find the temperature at 8 A. M. and 4 P. M.

1923.

1. (1) Simplify

$$\frac{3\frac{1}{4}-(\frac{4}{5}-1\frac{1}{4})}{\frac{4}{5}+\frac{7}{8} \text{ of } 1\frac{3}{5}\times 1\frac{1}{4}}+\frac{\cdot 8}{1\cdot\dot{1}}.$$

(2) Find the numbers less than 1000 which, when divided by 18, 20, 15, 45, or 24, will leave the remainder 5 in every case.

2. The sum of R250 was lent at simple interest and at the end of $1\frac{1}{4}$ years the debt was cancelled by the payment of R272. 8*a*. What was the rate of interest?

3. A bicycle agent allows 25 per cent. discount on his advertised prices, and then makes a profit of 20 per cent. on his outlay. What is the advertised price of a machine on which he gains £3?

4. A train overtakes two persons who are walking at the rate of 2 miles and 4 miles an hour respectively, and completely passes them in respectively 9 seconds and 10 seconds. What is the length of the train, and its speed in miles per hour?

5. A person bought some sheep for £72, and found that if he had bought 6 more for the same money he would have paid £1 less for each. How many did he buy?

6. *A* starts from a place and walks in a given direction at the rate of 5 miles an hour. *B* starts from the same place, one hour later, and moves in the same direction at the rate of 7 miles an hour. Draw the motion-graphs of *A* and *B*, and find when and where *B* overtakes *A*?

1924.

1 Simplify

(i) $$\frac{3}{4\frac{1}{2} \text{ of } 1\frac{1}{3}+4\frac{1}{4}}\times\left(\frac{5}{12}+\frac{4}{15}\right).$$

(ii) $$\frac{\cdot 0016\times\cdot 025}{\cdot 325 \text{ of } \cdot 05}\times\frac{\cdot 1216\times\cdot 105\times\cdot 002}{\cdot 8512\times\cdot 625\times\cdot 039}.$$

2. Two men with seven boys can do a piece of work in fourteen days; three men with eight boys can do the same in eleven days. In how many days can eight men with six boys do three times the amount of work?

3. Two trains start simultaneously from *A* and *B* and travel towards each other at the rates of 45 and $27\frac{1}{2}$ miles per hour respectively. When they meet one has travelled 28 miles more than the other. Find the distance from *A* to *B*.

4. Distinguish between *true* and *banker's* discount. The true discount on a sum of money for 4 months is R100 and the banker's discount is R102. Find the sum and the rate per cent. of the interest.

5. The sum of the squares of two consecutive even numbers is 100. Find the numbers.

VI. HIGH SCHOOL EXAMINATION PAPERS OF THE BOARD OF HIGH SCHOOL AND INTERMEDIATE EDUCATION, UNITED PROVINCES.

1925.

1. (*a*) Simplify

$$5\frac{1}{3} \text{ of } \frac{3{\cdot}15 \times 2{\cdot}41}{8\frac{4}{5} - 6{\cdot}8}.$$

(*b*) One rupee is worth £0 . 1 . $5\frac{5}{16}$; calculate in rupees the value of £1.

2. Find the cost of 435 mds. 38 seers and 12 chataks of rice at R6. 8*a*. per md.

3. A man sells a horse for R600 gaining thereby $\frac{1}{5}$ of the selling price; at what price would he have sold it to lose $\frac{1}{4}$ of the selling price?

4. A person deposited R5600 in a bank at $3\frac{3}{4}$% per annum; after 6 months he withdrew R3200 and after 12 months the remainder. How much did he receive altogether as interest?

1926.

1. Simplify the following:—

(*a*) $$\frac{2\frac{1}{3} \times 1\frac{7}{12} - 5\frac{1}{2} \div 16\frac{1}{2}}{16\frac{1}{2} - (5\frac{1}{3} - 1\frac{8}{12}) + 2\frac{1}{3}}.$$

(*b*) $$\frac{{\cdot}0016 \times {\cdot}025}{{\cdot}325 \times {\cdot}05} \div \frac{{\cdot}1216 \times {\cdot}105 \times {\cdot}002}{{\cdot}08512 \times {\cdot}625 \times {\cdot}039}.$$

2. If R50 be divided among 6 men, 12 women, and 17 boys, so that 2 men may receive as much as 5 boys, and 2 women as much as 3 boys; how much will each man, woman, and boy receive?

3. *A* sells goods to *B* at a gain of $22\frac{1}{2}$% and *B* sells them to *C* at a gain of $7\frac{1}{2}$%; *C* gave R5267. 8*a*. for them. How much did *A* give for them?

4. A square recreation ground is bordered by a path 9 ft. wide on the inside and this path covers 3 acres. Find the extent of the recreation ground in acres and square yards.

1927.

1. (*a*) Simplify

$$\left[\frac{(3\frac{3}{8} \times 5\frac{1}{3}) - (3\frac{1}{3} \times 4\frac{5}{8})}{3\frac{3}{8} - (5\frac{1}{4} + 3\frac{1}{2}) + 4\frac{5}{8}} \text{ of R}10\right] + \left[\frac{1{\cdot}405}{2{\cdot}1} \times \frac{6{\cdot}3}{28{\cdot}1} \text{ of R}7\frac{1}{2}\right]$$
$$- [{\cdot}02\tfrac{3}{7} \text{ of R}125.\ 7a.\ 3p.]$$

(*b*) The numbers 4·512, 19·353, 3·569 and 4·213 are known correct to three decimal places. Determine the limits of error in their sum.

2. (*a*) The average of ten results is 1·015102, that of the first six is 1·01267, and that of the last five is 1·01688. Find the sixth result.

(*b*) If oranges are bought at 11 for ten annas and sold at 10 for eleven annas, what is the gain per cent. ?

3. In the Matriculation Examination, a candidate has to offer five subjects namely, English, Mathematics, History with Geography, Vernacular, and an Optional. The maximum marks in each subject is 100, and to secure a first class in any subject or in the aggregate 60% of the maximum or the total marks must be obtained. A certain candidate who *just* secures a first class in aggregate obtains marks in the above subjects in the proportion of 3 : 6 : 4 : 5 : 7. Find in which of the subjects he secured first class marks.

4. The compound interest on a certain sum of money for two years is R25 . 10 . 0. The simple interest for the same period is R25 only. Find the rate per cent. and the sum of money.

1928.

1. (*a*) Simplify

$$\frac{2\cdot5-6\cdot08+4\cdot7}{(\cdot2\times8\cdot3)-(2\times\cdot55)}+\frac{(3\frac{3}{8}\text{ of }\frac{1}{3})-\frac{5}{18}}{(\frac{1}{3}\text{ of }\frac{1}{18})+(\frac{3}{18}-\frac{1}{3})}.$$

(*b*) An examination paper is set to 2500 pupils of whom one-fifth are girls and the rest boys ; 5 p. c. of the boys and 40 p. c. of the girls fail. What percentage of the whole passed ?

2. (*a*) Find the square root of 2 correct to three places of decimals.

(*b*) By selling oranges at 10 annas per dozen, a woman loses 10 p. c. of her outlay. What would she gain or lose per cent. if she sold them at 10 for 12 annas ?

3. A man borrows R200 at 5 p. c. per annum compound interest. Principal and interest of the loan are to be repaid by yearly instalment of R50. How much is outstanding at the end of three years ?

4. Find the cost of white-washing the walls and ceiling of a room $22\frac{1}{2}$ ft. long, 12 ft. broad, and 11 ft. high at one anna per square yd., making allowance for four windows each 4 ft. × $2\frac{1}{2}$ ft. and two doors each $6\frac{1}{2}$ ft. × 4 ft.

1929.

1. (*a*) Express as a decimal

$$\left\{\frac{1}{13}\times1\frac{5}{7}+\frac{2}{49}\times1\frac{2}{5}\right\}\text{ of }\frac{2\frac{3}{8}}{36\frac{1}{4}}.$$

(*b*) If 4% be lost by selling silk at R5 per yard, at what price should it be sold in order to gain 5% ?

2. (a) Simplify ·125 × ·9152 ÷ ·0715 without using vulgar fractions.

(b) A, B and C play cricket, A's runs are to B's and also B's runs are to C's as 3 : 2. They score altogether 342 runs. How many did each score?

3. A rectangular field is 330 yds. in length and 188 yds. in breadth; find the number of acres in the field, and the money which would be obtained by selling half the field for R17. 4a. 6p. an acre and the other half for R21. [illegible]a. 6p. an acre. [4840 sq. yds. make one acre.]

4. A man borrows the sum of R16,58,775 on the understanding that at the end of each year a portion of the principal is to be paid off, with interest at [illegible] per cent per annum on the amount standing unpaid during that year. Shew that the debt can be cleared off in four years by an annual payment of R4,56,976.

1930.

1. (a) Simplify

$$\frac{\frac{1}{3} \text{ of } 1\frac{1}{4} \text{ of } 4\frac{1}{2}}{\frac{5}{9} \text{ of } 1\frac{1}{4} \text{ of } 3\frac{1}{[illegible]}} - \frac{3\frac{1}{4}+4\frac{1}{2}}{6\frac{1}{[illegible]}+1\frac{1}{16}} + \frac{5.013 \times .075}{1.002}$$

correct to 3 places of decimals.

(b) By what must 152207 be multiplied so that product may consist of 8 digits, each digit being 1?

2. (a) A batsman has a certain average of runs for 16 innings. In the 17th inning he makes a score of 85 runs thereby increasing his average by 3. What is his average after 17th inning?

(b) If 11 yards of cloth is bought for R10 and sold at the rate of 10 yd. for R11. What is the gain per cent.?

3. (a) If R50 be divided among 8 men 12 women and 17 boys so that 2 men receive as much as 5 boys and 2 women as much as 3 boys, how much each man, woman and boy receive?

(b) A rectangular plot of grass, 90 ft. by 60 ft. has a gravel walk $3\frac{1}{2}$ ft. wide all round it on the outside. Find the area of the walk in square yards.

4. (a) The difference between banker's and true discount on a sum of money for 3 months at 5 per cent. is R4. 2a. 8p. Find the sum.

(b) On what sum does the difference between the simple and compound interest for 2 years at 5 per cent. amount to R15?

1931.

1. (a) Simplify the following and express the answer in decimal:

$$\frac{5.75 - \frac{3}{4} \times 1\frac{[illegible]}{[illegible]} + 2\frac{[illegible]}{[illegible]} \div 1.44}{\frac{4}{[illegible]} \text{ of } 7\frac{[illegible]}{[illegible]} - 5\frac{[illegible]}{6} \div 3\frac{[illegible]}{[illegible]}}$$

(b) The national debt of Italy on 31 March 1921, amounted to six hundred and ninety million six hundred and sixty-four thousand pounds, and has since then been gradually reduced at the average rate of seven million nine hundred and sixty-five thousand pounds a year. If this rate be maintained since 31 March 1931, state in words the amount of the debt at that date.

2. (a) Multiply 298·765 by ·0336567, correct to six places of decimals.

(b) *A* lends *B* R320. *B* is to pay interest on whatever amount he has not paid back, at the rate of 5 per cent. for the first year, 6 per cent. for the second year, and 7 per cent. for the third year. *B* pays *A* R100 at the end of the first year, R100 at the end of the second year, and enough to pay off completely the debt and interest at the end of the third. How much is the last payment? [Correct to the nearest pie.]

3. (a) If a man walks 252 miles at the rate of $3\frac{1}{2}$ miles per hour for 8 hours a day, find how many hours a day another man must walk at the rate of $4\frac{1}{2}$ miles an hour in order to walk 243 miles in the sam time.

(b) A man sells three motor cars for R5,400, R3,300, R5,250, respectively. He makes 20 per cent. profit on the first and 10 per cent. on the second but on the whole he loses $3\frac{1}{2}$ per cent. What did the third motor car cost him?

4. (a) The difference between the interest and the discount on a certain sum of money for 6 months at 4 per cent. is R2. What is the sum?

(b) A room is 18 ft. long and 12 ft. wide. The two long walls and one short wall are boarded to a height of 3 ft. 6 inches with board 7 in. wide. How many feet of boarding is used?

1932.

1. (a) What fraction of £21. 5*s*. 6*d*. is $\frac{0{\cdot}04255 \times 0{\cdot}32}{0{\cdot}0016}$ of £1. 11*s*. 3*d*.?

(b) The average daily number of persons passing a certain point on Sunday, Monday, Friday and Saturday is 1765. The average daily number passing on Tuesday, Wednesday and Thursday is 1541. What is the daily average for the whole week?

2. (a) A boat moves down stream at the rate of a mile in 6 minutes and up stream at the rate of 6 miles an hour. What is the velocity of the current?

(b) A rectangle 120 ft. by 100 ft. has a grass-plot 60 ft. by 50 ft. in the centre. Find the cost of gravelling the rest of it to a depth of 6 inches at R4. 8*a*. per cubic yard.

3. (*a*) A tradesman borrowed a certain sum of money at 5 per cent. compound interest and at the end of 4 years he paid to the lender as principal and interest £607. 15*s*. $0\frac{3}{4}$*d*. What was the sum borrowed?

(*b*) A trader asked the price of a watch which was 40 per cent. above cost and gave the purchaser 10 per cent. discount on the price asked, gaining thereby R5. 6*a*. 8*p*. Find its cost price.

ANSWERS TO EXAMPLES.

Examples. I.

1. Ten; sixteen; forty-eight; ninety-nine; seventy-six; forty-three; fifty; thirty-one; sixty two.

2. One hundred; one hundred and eleven; nine hundred and two; six hundred and twenty; three hundred; one hundred and three; two hundred and thirty-four; one hundred and thirty.

3. Nine thousand, two hundred and sixteen; five thousand, four hundred and nine; five thousand and four; one thousand and eleven; one thousand, two hundred and ten; nine thousand; nine thousand, nine hundred and ninety-nine.

4. Twelve thousand, three hundred and forty-five; twenty thousand, one hundred and three; forty thousand and forty; fifty thousand and one; ninety thousand, six hundred; eighty-nine thousand, three hundred and forty-six.

5. Five hundred thousand; seven hundred and eight thousand, nine hundred; one hundred and two thousand and thirty; three hundred and nine thousand, eight hundred and nine; three hundred and seventy-nine thousand, five hundred and eighty-six.

6. Seven million, two hundred and thirty-four thousand, six hundred and fifty-one; seven million, ninety thousand, seven hundred and nine; nine million; seven million, eight hundred thousand and forty; three million, five hundred and sixty-seven thousand, eight hundred and ninety-one.

7. Thirty-two million, five hundred and sixty-seven thousand, eight hundred and ninety-two; thirty-four million, eighty-three thousand and ninety-two; ninety million, nine thousand; fifty-five million, five hundred thousand and fifty-five.

8. Seven hundred and eighty-nine million, three hundred and forty-five thousand, six hundred and twenty-one; three hundred and ninety million, eighty-five thousand, two hundred and twenty-two million.

9. Seven thousand and nine million, fifty-six thousand, seven hundred; three thousand two hundred and fifty-nine million, two hundred and eighty-seven thousand, eight hundred and ninety-one; eight thousand and seventy million, eighty-eight thousand, two hundred.

10. Thirty-two thousand and five hundred million, ninety-four thousand and one; three hundred and eight thousand five hundred and six million, eight thousand, two hundred and thirty; one billion, three hundred and fifty-seven thousand nine hundred

and eighty-six million, four hundred and twenty-eight thousand, one hundred and twenty-three.

11 70, 2 ; 300, 50, 2 ; 4000, 200, 3 ; 70000, 800, 9 ; 1000000000, 300000000, 400000, 50000, 700, 80, 9 ; 3000000000000, 70000000000, 9000000000, 4000000, 70000, 8000, 20, 3.

12. Counting from left, the zeroes respectively indicate the absence of—thousands, tens ; tens of millions, hundreds of thousands, tens of thousands, hundreds, units ; tens of thousands of millions, thousands of millions, tens of millions, thousands, tens.

13. (10,000) ten thousand ; (9,999) nine thousand, nine hundred and ninety-nine.

14. 8531, 1358 15. 87420, 20478. 16. 8991, 8001.

Examples. 2.

1. 13 ; 17 ; 19 ; 12 ; 11.
2. 23 ; 34 ; 40 ; 27.
3. 77 ; 90 ; 84 ; 63.
4. 342 ; 486 ; 504 ; 900.
5. 203 ; 430 ; 555 ; 400.
6. 892 ; 704 ; 640 ; 512.
7. 7,835 ; 9,028 ; 6,009 ; 4,000 ; 6,085.
8. 5,992 ; 8,074 ; 2,003 ; 4,040 ; 3,403.
9. 1,200 ; 80,008 ; 18,454 ; 36,012 ; 90,000.
10. 20,070 ; 30,008 ; 54,400 ; 16,004.
11. 405,000 ; 800,040 ; 702,074
12. 3,000,904 ; 9,000,400 ; 15,000,050 ; 108,003,004 ; 4,005,000.
13. 5,000,700,028 ; 315,764,009,003.
14. 3,000,000,000,050 ; 405,000,010,020,007 ; 1,000,001,001,000 ; 6,000,000,000,006.
15. 512,255,762,713,473.
16. 12,000,000,000,012 ; 700,000,000,700,700 ; 3,000,003,003,303.
17. 7,305,000,502,006,024 ; 47,000,047,047,047.
18. 1,000,000 ; 99,999.
19. The number expressed in figures is 7707 ; therefore (counting from left), the first boy's mistake consisted in writing three ciphers unnecessarily to the right of the first 7, and two ciphers instead of one to the right of the second 7 ; the second boy's mistake consisted in omitting to write a cipher to the right of the second 7.

Examples. 3.

1. Three lacs, forty-five thousand, five hundred and forty-three ; thirty lacs, twenty thousand and fifty ; seventy-nine lacs,

ninety thousand, five hundred and seventy; seventy lacs, fifty thousand, three hundred and four.

2. One crore, twenty-three lacs, forty-five thousand, six hundred and seventy-eight; thirty crores, fifty-seven lacs, fifty thousand and eighty; four crores, fifty lacs.

3. Twenty-three crores, seventy-eight thousand and one; seven hundred and eight crores, nine lacs, four thousand and eighty; three hundred and seventy-nine crores, forty-eight lacs, fifty-seven thousand, six hundred and twelve.

4. Eight hundred and twenty-seven crores, forty lacs, fifty-seven thousand and nine; three hundred and fifty crores, one thousand, two hundred and thirty; three hundred and ten crores, thirty-seven lacs, five thousand and forty.

5. One hundred and twenty-three crores, forty-five lacs, sixty-seven thousand, eight hundred and ninety; six hundred crores, seven lacs, eighty-nine thousand; five hundred and one crores, seven lacs, two thousand and nine.

6. 1,14,000; 78,00,000; 15,04,030; 7,00,007.

7. 1,00,00,500; 28,03,00,004; 20,00,00,000; 1,01,01,001.

8. 300,05,04,000; 101,01,00,101.

9. 328,17,45,715.

10. 705,17,24,738.

11. One hundred thousand; ten lacs; ten million.

12. 103,028,401 = 10,30,28,401 which is read—ten crores, thirty lacs, twenty-eight thousand, four hundred and one.

13. 103,07,00,704 = 1,030,700,704 which is read—one thousand and thirty million, seven hundred thousand, seven hundred and four.

Examples. 4.

1.	6.	2.	9.	3.	49.	4.	99.	5.	75.
6.	264.	7.	609.	8.	664.	9.	1990.	10.	60010.
11.	2764.	12.	XLIV.	13.	LXVI.	14.	LXXIX.		
15.	LXXXIII.			16.	CXLIX.	17.	CDXXXVI.		
18.	CMXC.			19.	MCCCLI.	20.	$\overline{V}$DCLXX.		
21.	MMMCXLIX.			22.	$\overline{XLV}$CMLXXVIII.	23.	$\overline{M}$.		

Examples. 5.

1.	21.	2.	30.	3.	31.	4.	29.	5.	34.
6.	93.	7.	99.	8.	77.	9.	140.	10.	163.

11. 1323. 12. 1151. 13. 792. 14. 2727. 15. 2000.
16. 14129. 17. 9993. 18. 3674. 19. 5620. 20. 4696.
21. 146175. 22. 59038. 23. 234671. 24. 379462.
25. 45271. 26. 2262514. 27. 920114. 28. 982255.
29. 7474095. 30. 39679341. 31. 42450564. 32. 496651.
33. 92439. 34. 8082862. 35. 931979. 36. 531284.
37. 5691685. 38. 311989. 39. 9925098. 40. 984610763.
41. 74307. 42. 10246451. 43. 765168567. 44. 3129223218.
45. 46451330. 46. 3936. 47. 1890. 48. 365.
49. 741. 50. 2040. 51. 138187. 52. 42004 rupees.
53. 7193165 maunds. 54. 1468. 55. 163554.
56. 32. 57. 12, 15. 58. Since the sums of the rows, columns and diagonals are in each case 15, it is a magic square.

Examples. 6.

1. 43. 2. 52. 3. 222. 4. 543. 5. 4321.
6. 25. 7. 49. 8. 8. 9. 9. 10. 33.
11. 189. 12. 90. 13. 178. 14. 459. 15. 315.
16. 4641. 17. 47017. 18. 30532. 19. 27273. 20. 41976.
21. 2679. 22. 689357. 23. 687590. 24. 735347. 25. 6499247.
26. 5546. 27. 85416. 28. 707467. 29. 3562.
30. 1. 31. 688881. 32. 390794. 33. 61059.

34.	35.	36.	37.
789356	708092	805400	7000203
99999	20503	70053	500956
689357	687589	735347	6499247

38. The missing line is 55545. 39. The missing line is 89303.
40. „ „ „ „ 1. 41. „ „ „ „ 1819990.
42. 999981 ; 999695 ; 990525 ; 900554 ; 956500.
43. 92964. 44. 99971. 45. 9998999. 46. 9921.
47. 83 years. 48. In 1642. 49. 923. 50. 117681 rupees.
51. 325 rupees. 52. 9460 rupees. 53. 16516.
54. 777101. 55. 6390. 56. 2000.
57. 35242 rupees. 58. 30000600. 59. 4503600.

Examples. 7.

1. 458. 2. 62784. 3. 2740. 4. 288. 5. 19335.
6. 970. 7. 9960. 8. 14006. 9. 92788. 10. 99803.

11. 2106. 12 From left to right and top to bottom the digits are (1) 1, 4, 4, 5, 6. (2) 1 ; 1, 5, 6, 1, 7. (3) 6, 7 ; 6 ; 2 ; 1.
13. 20477.

Examples. 8.

1. 46. 2 96. 3. 84. 4. 195. 5. 282.
6. 522. 7. 784. 8 684. 9 765. 10. 937.
11. 2835. 12. 7911. 13. 19170. 14. 35445.
15 73648 16. 315824. 17. 623245. 18. 769527.
19. 68158 ; 102237 ; 136316 ; 170395 ; 204474 ; 238553 ; 272632 ; 306711. 20. 3625

21. 41273
9
371457

22. 32469
7
227283

Examples. 9.

1. 10770. 2. 281400. 3. 195250. 4. 421800. 5. 35100.
6. 5760300. 7. 24040000. 8. 81036000. 9. 183018000.
10. 656550 ; 5836000 ; 51065000 ; 437700000 ; 3647500000.

Examples. 10.

1. 20250. 2. 88592. 3. 51060. 4. 1715340.
5. 7920843. 6. 7845984. 7. 501264. 8. 2877420.
9. 41269151. 10. 712823175. 11. 546962350.
12. 8741795904. 13. 609360400000. 14. 73866065616.
15. 4278833730. 16. 771645339059. 17. 222372622250000.
18. 389341782447. 19. 2993392500000. 20. 8784920736579.
21. 2247882292480. 22. 27706959000. 23. 62834211900.
24. 581199247904. 25. 10612283522500. 26. 234916991512.
27. 83779349418000. 28. 47619. 29. 45708.
30. 93652. 31. 99148. 32. 73350. 33. 140624.
34. 230690. 35. 505260. 36. 82764. 37. 711360.
38. 2170671. 39. 316875 rupees. 40. 10727350.
41. 20692 maunds. 42. 33114. 43. 3744.
44. (i) 823
85
6584
4115
69955

(ii) 2524
32
7572
5048
80768

Examples. 11.

1. 2771928. 2. 7386918. 3. 3747321.
4. 94876320. 5. 627399162. 6. 222013980.
7. 153660000. 8. 313199250. 9. 6783119793.

Examples. 12.

1. 432. 2. 4720645. 3. 16905000. 4. 1905700.
5. 1153800. 6. 44274384. 7. 1314. 8. 86400.
9. 3200. 10. 399735. 11. 9425. 12. 2208.

Examples. 13.

1. See the Multiplication Tables.
2. 576. 3. 2500. 4. 4624. 5. 10000.
6. 12544. 7. 61504. 8. 531441. 9. 763876.
10. 1 ; 8 ; 27 ; 64 ; 125 ; 216 ; 343 ; 512 ; 729 ; 1000 ; 1331 ; 1728 ; 2197 ; 2744 ; 3375 ; 4096 ; 4913 ; 5832 ; 6859 ; 8000.
11. 804357. 12. 1000000. 13. 679151439.
14. 170953875. 15. 19503629. 16. 62913.

Examples. 14.

1. 2195. 2. 75582. 3. 871882. 4. 304166.
5. 18776 6. 85040. 7. 1595. 8. 8832.
9. 92080. 10. 45138.

Examples. 15.

1. 188. 2. 4617. 3. 3542, rem. 1.
4. 2333, rem. 1. 5. 2675. 6. 30042.
7. 20511, rem. 1. 8. 8203, rem. 1. 9. 11419, rem. 2.
10. 2469. 11. 20040. 12. 15555, rem 2.
13. 15067, rem. 1. 14. 14557, rem. 3. 15. 13155, rem. 4.
16. 541, rem. 2. 17. 6569, rem. 3. 18. 4640.
19. 4809, rem. 2. 20 4313, rem. 5. 21. 2005, rem. 2.
22. 8013, rem. 7. 23. 10000, rem. 1. 24. 8666, rem. 6.
25. 3897, rem. 2. 26. (1) 2456. (2) 3200.
27. (1) 4. (2) 3. (3) 1. (4) 1 or 5. (5) 3.

28. 7070, rem. 7. 29. 2440, rem. 2. 30. 3004, rem. 8.
31. 1917, rem. 4. 32. 169, rem. 29. 33. 11404, rem. 22.
34. 407, rem. 80. 35. 87, rem. 300. 36. 48, rem. 101.
37. 160, rem. 289. 38. 453, rem. 219. 39. 706, rem. 354.
40. 112, rem. 4543. 41. 3263, rem. 931. 42. 1017, rem. 2556.
43. 2559, rem. 2316 44. 6652, rem. 5423.
45 114285, rem. 3351. 46. 1250, rem. 539.
47. 15200, rem. 10321. 48. 15005, rem. 54720.
49. 1338, rem. 110590. 50. 423297, rem. 37608.
51. 240100, rem. 117400. 52. 420, rem. 114903.
53. 63261, rem. 6731383. 54. 8425323113, rem. 75.
55. 9386426883, rem. 672. 56. 1498, rem. 8.
57. 2002, rem 4. 58. 135, rem. 30. 59. 521, rem. 89
60. 694, rem. 2. 61. 45, rem. 254. 62. 58, rem. 356.
63. 44, rem. 357. 64. 234, rem. 641. 65. 381, rem. 1664.
66 507. 67. 36 68. 525 times. 69. 13.
70. 229 times. 71. 30115. 72. 7674.
73 375 rupees. 74. 256 days. 75 22.

Examples. 16

1 17480, rem. 1. 2. 26310. 3. 20089, rem. 2.
4. 2558, rem. 2. 5. 3842, rem. 5. 6. 14057, rem. 1.
7. 4320, rem. 7. 8. 2207, rem. 7. 9. 3456, rem. 7.
10. 52731, rem. 5. 11. 67253, rem. 4. 12. 10437, rem. 8.
13. 32198, rem. 10. 14. 49538, rem. 10. 15 58491, rem. 6.
16. 228850, rem. 7. 17. 455961, rem. 7. 18. 649772, rem. 10.
19. (i) 1728394, rem 1 ; 1152263 ; 864197, rem. 1 ; 691357, rem. 4 ; 576131, rem. 3 ; 493827 ; 432098, rem. 5 ; 384087, rem. 6 ; 345678, rem. 9 ; 314253, rem. 6 ; 288065, rem. 9 ; 265906, rem. 11 ; 246913, rem. 7 ; 230452, rem. 9 ; 216049, rem. 5 ; 203340, rem. 9 ; 192043, rem. 15 ; 181936, rem. 5 ; 172839, rem. 9.

(ii) 40352015, ; 26901343, rem. 1 ; 20176007, rem. 2 ; 16140806 ; 13450671, rem. 4 ; 11529147, rem. 1 ; 10088003, rem. 6 ; 8967114, rem. 4 ; 8070403 ; 7336730 ; 6725335, rem. 10 ; 6208002, rem. 4 ; 5764573, rem. 8 ; 5380268, rem. 10 ; 5044001, rem. 14 ; 4747295, rem. 15 ; 4483557, rem. 4 ; 4247580, rem. 10 ; 4035201, rem. 10.

(iii) 493827160, rem 1 ; 329218107 ; 246913580, rem. 1 ; 197530864, rem. 1 ; 164609053, rem. 3 ; 141093474, rem. 3 ; 123456790, rem. 1 ; 109739369 ; 98765432, rem. 1 ; 89786756, rem. 5 ; 82304526, rem. 9 ; 75973409, rem. 4 ; 70546737, rem. 3 ; 65843621, rem. 6 ; 61728395, rem. 1 ; 58097313 ; 54869684, rem 9 ; 51981806, rem. 7 ; 49382716, rem. 1.

Examples. 17.

1. 210.	2. 465	3. 1035.	4. 2850.
5. 2050.	6. 1254.	7. 3315.	8. 15150.
9. 245.	10. 44818.	11. 4568.	12. 37951.
13. 4628 and 3899.		14. 5444 and 4556.	

Examples. 18.

1. 17472.	2. 337050.	3. 672840.	4. 132624.
5. 244160.	6. 94976.	7. 2599400.	8. 601425.
9. 1233282.	10. 143472.	11 446048.	12. 3532008.
13. 295100780.	14. 1220242681.	15. 3625.	16. 1645.
17. 4060.	18 2100.	19. 18225.	20. 2300.
21. 12250.	22. 15625.	23. 25875.	24. 11088.
25. 281718.	26. 2039798.	27. 420158.	28. 4182640.
29. 8267519.	30. 36950.	31. 5565.	32. 31220.
33 53175.	34. 4560.	35. 59175.	36. 1225.
37. 3025.	38. 7396.	39. 9409.	40. 105625.
41. 216225.	42. 606841.	43. 802816.	

Examples. 19.

1. 39.	2. 23.	3. 42.	4. 63.	5. 23.
6. 330, rem. 24.		7. 540, rem. 40.		8. 372, rem. 20.

9. 755, rem. 84. 10. 677, rem. 117. 11. 2935, rem. 168.
12. 12882, rem. 58. 13 359, rem. 319. 14. 2057, rem. 294.
15 1422, rem. 138. 16. 389, rem. 4. 17. 34, rem. 56.
18. 89, rem 345. 19. 827, rem. 46. 20. 89, rem. 346.
21. 18, rem. 3456. 22. 129, rem 22. 23. 157, rem. 42.
24. 123, rem. 67. 25. 38, rem. 1368. 26. 46, rem. 894.
27. 783, rem. 10743. 28. 122, rem. 893. 29. 9733, rem. 176.
30. 2716, rem. 187. 31. 75, rem. 3. 32. 937, rem. 4.
33. 255, rem. 1. 34. 313, rem. 20. 35. 3310, rem. 19.
36. 5515, rem. 17. 37. 670, rem. 14. 38. 1103, rem. 16.
39. 30, rem. 42. 40. 24, rem. 14. 41. 22, rem. 19.
42. 20, rem. 21. 43. 16, rem. 34. 44. 21, rem. 29.
45. 108, rem. 66. 46. 57327, rem. 90. 47. 65500, rem. 36.
48. 460932, rem. 72. 49. 17297, rem. 936. 50. 8941, rem. 453.
51. 3082, rem. 5997.

Examples. 20.

1. 14. 2. 6. 3. 2. 4. 3. 5. 20.
6. 4. 7. 31. 8. 2. 9. 2. 10. 28.
11 4. 12. 14. 13. 0. 14. 10. 15. 450.
16. 14. 17. 83. 18. 65. 19. 200. 20. 0.
21. (i) 4567. (ii) 56748.
22. (i) $4\times 10^2+5\times 10+6$. (ii) $5\times 10^3+6\times 10^2+7\times 10+8$.
(iii) $6\times 10^3+8\times 10^2+9\times 10+1$ (iv) $7\times 10^4+8\times 10^3+9\times 10+1$.

Examples. 21.

1. 2. 2. 3. 3. 12. 4. 24. 5. 8.
6. 8. 7. 1. 8. 18. 9. 10. 10. 5.
11. 31. 12. 9. 13. 15. 14. 19. 15. 4.
16. 16. 17. 1. 18. 9. 19. 1. 20. 2.
21. 4. 22. 5.

Miscellaneous Examples. 22.

1. 2548. 2. 2022. 3. 8611. 4. 621. 5. 788.
6. 9001. 7. 316. 8. 11. 9. 3791. 10. 17.
11. 1477. 12. 6354. 13. 33794. 14. 459801.

15. 40023 times, rem. 21. 16. 532. 17. 176. 18. 3
19. 150; 83. 20. 7 times. 21. 1545. 22. 159943.
23. 89. 24. 362. 25. 514590. 26. 99 and 106.
27. 23 years. 28. 176913. 29. 189461. 30. 71265.
31. 615. 32. 134807. 33. 545 pice. 34. 812168964.
35. 313289352. 36. 475 rupees. 37. *A*, 58; *B*, 34; *C*, 42.
38. *A*, 40 rupees; *B*, 39 rupees; *C*, 30 rupees. 39. 135 rupees.
40. 18 per rupee. 41. 60 seers; 100 seers. 42. 1800 rupees.
43. 5 years. 44. 10 years; 70 years. 45. 60. 46. 3 P. M.

51.

82396
27465 rem. 1
3923 rem. 4

52. (i) The missing figures from the left in the dividend and quotient are respectively

0, 2 / 4, 0; 0, 9 / 4, 1; 7, 2 / 5, 0 or 7, 9 / 5, 1.

(ii) 6 | 25537 / 4256 rem. 1 or 6 | 25507 / 4251 rem. 1

53. He copied 125 instead of 123. 54. Divisor 23, quotient 215.

Examples. 23.

1. 624*a*. 2. 1664*a*. 3. 115328*a*. 4. 59168*a*.
5. 121*a*. 6. 372*a*. 7. 604*a* 8. 830*a*.
9. 59328*p* 10. 142080*p*. 11. 653184*p*. 12. 38700*p*.
13. 21624*p*. 14. 135324*p* 15. 5187*p*. 16. 7641*p*.
17. 13055*p*. 18. 194 pice; 582*p* 19. 501 pice; 1503*p*.
20. 635 pice; 1905*p*. 21. 7410. 22. 1632. 23. 631.
24. 100 25. 3896. 26. 482. 27. 14400*s*.
28. 4800*s*. 29. 14180*s*. 30. 6100*s*. 31. 405*s*.
32. 532*s*. 33. 617*s*. 34. 719*s*. 35. 8400*d*.
36. 160800*d* 37. 1684800*d* 38. 10932*d* 39. 12156*d*.
40. 18420*d*. 41. 870*d*. 42. 2170*d*. 43. 1883*d*. 44. 960000*q*.
45. 293616*q*. 46. 7332*q*. 47. 3229*q*. 48. 6758*q*.
49. 2691*q* 50. 37 crowns; 370 sixpences; 555 fourpences
51. 42 crowns; 420 sixpences; 630 fourpences.
52. 63 crowns; 630 sixpences; 945 fourpences.

53.	19 half-crowns.	54.	255 three-pences.	55.	36000*q*.
56.	28224 half-pence.	57.	100 oranges.	58.	2286 farthings.
59.	125 books.	60.	55 children.	61.	396 beggars.

Examples. 24.

1.	R52. 1*a*. 4*p*.	2.	R160. 6*a*. 1*p*	3.	R405. 1*a*. 5*p*.
4.	R20. 9*a*.	5.	R40. 11*a*. 11*p*.	6.	R57. 13*a*. 11*p*.
7.	R157. 13*a*. 3*p*.	8.	R247. 4*a*. 2*p*.	9.	R52. 1*a*. 5*p*.
10.	R15. 10*a*.	11.	R59. 2*a*. 3*p*.	12	R48. 2*a*. 6*p*.
13.	R55. 5*a*. 3*p*.	14.	R69. 13*a*.	15.	R120.
16.	£1. 11*s*. 4*d*.	17.	£29. 5*s*. 3*d*.	18.	£37. 3*s*. 4*d*
19.	£1. 0*s*. 10*d*.	20.	£10. 8*s*. 6*d*.	21.	£3. 9*s*. 5½*d*.
22.	£8. 7*s*. 6*d*.	23.	£8. 5*s*. 2½*d*.	24.	£4. 11*s*. 10*d*.
25.	15*s*. 9½*d*.	26.	£49. 5*s*.	27.	£28. 7*s*.
28.	£48. 15*s*.	29.	£9. 18*s*.	30.	£40. 10*s*
31.	R15.	32.	R9 6*a*.	33.	15*s*.

Examples 25.

1.	R1. 11*a*. 2 pice.	2.	R2. 14*a*. 1 pice.	3.	R3 1*a*. 1 pice.
4.	R2. 9*a*. 2 pice.	5.	R2. 9*a*.	6.	R2 15*a*.
7.	R3. 6*a* 3*p*.	8.	R2. 14*a*. 6*p*.	9.	R52 12*a*. 9*p*.
10.	R85. 12*a* 10*p*.	11.	R82. 9*a*.	12.	R518. 5*a*.
13.	R1888.	14.	R1380. 11*a*. 4*p*.	15.	R1973. 14*a*. 7*p*.
16.	R4657. 1*a*. 5*p*.	17.	R17776. 6*a*. 10*p*.	18.	£509. 1*s*. 5*d*.
19.	£470. 19*s*.	20.	£1010. 5*s*. 9*d*.	21.	£10103. 0*s*. 6*d*.
22.	£5746. 19*s*. 6*d*	23.	£466. 12*s*.	24.	£877. 17*s*. 5½*d*.
25.	£850. 6*s*. 4½*d*.	26.	£1758. 17*s*. 2¾*d*.		

Examples. 26.

1.	R6. 3*a*. 1 pice.	2.	R1. 12*a*. 3 pice.	3.	R9. 10*a*. 3 pice.
4.	R3. 11*a*. 9*p*.	5.	R39. 14*a*. 9*p*.	6.	R9. 8*a*. 4*p*.
7.	R15. 3*a*. 5*p*.	8.	13*a*. 9*p*.	9.	R10. 8*a*. 10*p*.
10.	R58. 3*a*. 8*p*.	11.	R273. 13*a*. 11*p*.	12.	6*a*. 6*p*.
13.	£5. 9*s*. 7*d*.	14.	£13. 15*s*. 9*d*.	15.	£20. 18*s*. 9*d*.
16.	£2. 12*s*. 5*d*.	17.	£2. 3*s*. 3¾*d*.	18.	£11. 12*s*. 8¾*d*.
19.	R3. 6*a*. 8*p*.	20.	R3. 6*a*. 7*p*.	21.	R3. 11*a*. 11*p*.

Examples. 27.

1. R2. 1a. ; R50 ; R24. 5a. ; R99 ; R108. 11a. ; R118.
2. 2s. ; £2. 5s. ; £72. 8s. ; £19 ; £118 ; £143.
3. £4 ; £9 ; R8 ; R7. 8a. 4. R20. 4a. ; R114 ; R104.

Examples. 28.

1. R10. 10a. 1 pice ; R17. 11a. 3 pice ; R24. 13a. 1 pice.
2. R48. 14a. 6p. ; R63. 7a. 6p. ; R88. 0a. 6p.
3. R439. 4a. 1p ; R519. 1a. 11p. ; R638. 14a. 8p.
4. £89 16s. 3d. ; £209. 11s. 3d. ; £269 8s. 9d.
5. £226. 12s. ; £302. 2s. 8d. ; £755. 6s. 8d.
6. £201. 19s. 4½d. ; £363. 10s. 10½d. ; £484. 14s. 6d.
7. R47. 14a. 2 pice ; R73 ; R57. 0a. 2 pice.
8. R2228. 10a. ; R3939. 14a. 3p. ; R3979. 11a.
9. R6106. 12a. 4p. ; R5911. 5a. 8p. ; R7035.
10. £2819. 16s. 3d. ; £2228 ; £27850.
11. £4816. 13s. 2½d. ; £3503. 0s. 6d. ; £20134. 6s. 3d.
12. R1. 14a. 13. R126. 14. £10. 2s. 6d. 15. £37. 14s. 2d.
16. R5468. 12a. 17. £266. 17s. 6d. 18. R10031. 4a.

Examples. 29.

1. R75. 7a. 2 pice ; R121. 6a. 2 pice.
2. R288. 7a. 9p. ; R366. 7a. 3p. 3. R1618. 3a. 6p. ; R2706.
4. R6015. 3a. 9p. ; R8490. 7a. 6p 5. £2235. 12s. 6d. ; £490.
6. £12755. 17s. ; £4283. 2s. 3d
7. £4934. 10s. 0¾d. ; £5432. 10s. 9¾d.
8. £7783. 18s. 10½d. ; £8624. 13s. 10½d.
9. R2754. 9a. 9p. 10. R1799. 12a. 9p.

Examples. 30.

1. R3. 2a. 1 pice 2. R4. 13a. 3 pice. 3. R7. 7a. 7p.
4. R10. 12a. 4p. 5. R12. 13a. 1p. 6. R5. 15a. 3p.
7. R15. 5a. 3p. 8. R10. 1a. 11p. 9. £3. 7s. 3d.
10. 11s. 3d. 11. £55. 12s. 9d 12. £53. 18s. 7d.
13. £3. 7s. 10d. 14. £2. 7s. 1d. 15. R6. 15a. 10p.
16. R56. 7a. 5p. 17. R145. 12a. 6p. 18. R143. 15a. 2p.

19. R41. 3*a*. 5*p*. 20. R138. 2*a*. 8*p*. 21. £9. 15*s*. $10\frac{1}{2}d$.
22. £55. 13*s*. $2\frac{1}{2}d$. 23. £47. 7*s*. $1\frac{1}{4}d$. 24. £420. 2*s*. $3\frac{3}{4}d$.
25. R1. 2*a*. 5*p*. 26. R3. 4*a*. 3*p*. 27. R5. 12*a*. 4*p*.
28. R12. 10*a*. 5*p*. 29. £125. 15*s*. $9\frac{1}{2}d$. 30. £12. 18*s*. 10*d*.
31. 3*a*. 9*p*. 32. 10 annas. 33. 2*a*. 8*p*. 34. 3*s*. 6*d*.

Examples. 31.

1. R13. 9*a*. 3*p*. 2. R37. 9*a*. 10*p*. 3. R2 12*a*. 9*p*.
4. R12. 7*a*. 4*p*. 5. R40. 10*a*. 10*p*. 6. R61. 0*a*. 1*p*.
7. 3*a*. 3*p*. 8. R2. 2*a*. 2*p*. 9. £43. 16*s*. 8*d*.
10. £22. 15*s*. 8*d*. 11. £5. 2*s*. $2\frac{3}{4}d$. 12. £3. 0*s*. $1\frac{3}{4}d$.

Examples. 32.

1. R5. 1*a*. 1*p*. 2. R4. 15*a*. 7*p*. or 8*p*. 3. R1. 10*a*. 6*p*.
4. R3. 4*a*. 5*p*. 5. R7. 10*a*. 2*p*. 6. R3. 15*a*. 2*p*.
7. R10. 13*a*. 10*p*. 8. R9. 3*a*. 10*p*. 9. £5. 11*s*. 6*d*. 2*f*.
10. £4. 5*s*. 10*d*. 11. £11. 10*s*. 3*d*. 3*f*. 12. £4. 19*s*. 9*d*.
13. £2. 13*s*. 1*d*. 2*f*. 14. £2. 18*s*. 5*d*. 1*f*.
15. R204. 11*a*., rem. 8*p*. 16. R143. 8*a*. 9*p*., rem. 38*p*.
17. R65. 8*a*. 3*p*., rem. 15*p*. 18. R93. 12*a*. 2*p*., rem. 959*p*.
19. £14. 10*s*. 6*d*., rem. 6*d*. 20. £127. 16*s*. 2*d*., rem. 230*d*.

Examples. 33.

1. 9. 2. 15. 3. 24. 4. 21. 5. 56.
6. 28, rem. R2. 11*a*. 6*p*. 7. 21, rem. R3. 7*a*. 4*p*.
8. 40, rem. R3. 1*a*. 9*p*. 9. 32, rem. £18. 4*s*. 7*d*.
10. 102, rem. £8. 5*s*. 6*d*. 11. 57. 12. 184.
13. 300. 14. 3120. 15. 7 days. 16. 100.

Examples. 34.

1. 1192320 gr. 2. 170880 gr. 3. 21927 gr.
4. 165000 gr. 5. 319896 gr. 6. 41865 gr.
7. 1 lb. 4 oz. 6 dwt. 21 gr. 8. 1 lb. 6 oz. 11 dwt. 19 gr.
9. 10 lb. 0 oz. 12 dwt. 4 gr. 10. 17 lb. 4 oz. 6 dwt. 16 gr.
11. 2 lb. 3 oz. 0 dwt. 23 gr. 12. 3 lb. 0 oz. 9 dwt. 9 gr.
13. 24 lb. 6 oz. 8 dwt. 13 gr. 14. 2 oz. 16 dwt. 22 gr.
15. 2 lb. 6 oz. 14 dwt. 8 gr.

16. 1 lb. 4 oz. 8 dwt. 8 gr. ; 8 lb. 9 oz. 1 dwt. 8 gr. ; 116 lb. 9 oz. 19 dwt. 16 gr.
17. 8 oz. 6 dwt. 16 gr. ; 20. 18. 4 lb. 9 oz.
19. 3 dwt. 18 gr. 20. 34.

Examples. 35.

1. 4386816 dr. 2. 1218560 dr. 3. 2005392 dr.
4. 5361664 dr. 5. 1240064 dr. 6. 84156 dr.
7. 1 ton 14 cwt. 3 qr. 14 lb. 3 oz. 15 dr. 8. 4 cwt. 1 qr. 6 lb. 4 oz.
9. 12 lb. 6000 gr. 10. 63775 tons 10 cwt. 0 qr. 22 lb. 6000 gr.
11. 38 lb. 1 oz. 6 dr. 12. 14 cwt. 3 qr. 26 lb. 8 oz.
13. 11 tons 9 cwt. 3 qr. 4 lb. 14. 3 lb. 4 oz. 6 dr.
15. 6 tons 8 cwt. 2 qr. 18 lb.
16. 2 tons 15 cwt. 0 qr. 3 lb. 15 oz. 14 dr. ; 34 tons 11 cwt. 3 qr. 14 lb. 3 oz. ; 129 tons 6 cwt. 2 qr. 19 lb. 10 oz. 2 dr.
17. 1 cwt. 2 qr. 27 lb. 5 oz. ; 500. 18. 2 tons 1 cwt. 3 qr. 11 lb. 8 oz
19. 2 cwt. 2 qr. 2 lb. 20. 768.
21. A pound of feathers is heavier by 1240 grains.
22. 175 lb. Troy.

Examples. 36.

1. 8140 kanchas ; 10175 tolas. 2. 6148 kanchas ; 8060 tolas.
3. 4796 kanchas ; 5995 tolas. 4. 6176 kanchas ; 7720 tolas.
5. 2288 kanchas ; 2860 tolas. 6. 7040 kanchas ; 8800 tolas.
7. 1 md. 32 seers 14 ch. 8. 1 md. 12 seers 1 ch. 1 kancha.
9. 12 md. 18 seers 3 ch. 10. 31 md. 10 seers.
11. 31 md. 13 seers 13 ch. 12. 41 md. 13 seers 7 ch.
13. 81 md. 12 seers 1 ch. 1 kancha. 14. 4 md. 27 seers 13 ch.
15. 7 md. 31 seers 10 ch. 2 kanchas.
16. 1 md. 11 seers 0 ch. 3 kanchas ; 5 md. 38 seers 3 ch. 2 kanchas ; 305 md. 11 seers 8 ch. 3 kanchas.
17. 39 seers 1 ch. ; 25. 18. 595 md. 2 seers 3 ch.
19. 1 seer 2 kanchas. 20. 640. 21. 18900. 22. 75.

Examples. 37.

1. 4500 in. 2. 39600 in. 3. 190080 in. 4. 380160 in.
5. 182556 in. 6. 209380 in. 7. 612018 in. 8. 762 in.

9. 1110 in. 10. 1467 in. 11. 184878 in. 12. 431766 in.
13. 28 po. 2 yd. 14. 36 po. 4 yd. 15. 19 po. 2 yd. 1 ft. 6 in.
16. 35 po. 3 yd. 1 ft. 6 in. 17. 6 po. 1 yd. 10 in.
18. 1 mi. 36 po. 5 yd. 1 ft. 19. 1 mi. 1 fur. 9 po. 4 yd. 6 in.
20. 1 mi. 2 fur. 4 po. 2 ft. 5 in. 21. 5 po. 10 in.
22. 1 mi. 7 fur. 6 po. 1 ft. 23. 3 mi. 5 fur. 24 po. 3 yd. 2 ft. 3 in.
24. 15 mi. 4 fur. 28 po. 2 ft. 6 in.

Examples. 38.

1. 29808 sq. in. 2. 4704480 sq. in. 3. 752716800 sq. in.
4. 8028979200 sq. in. 5. 47358432 sq. in. 6. 80760240 sq. in.
7. 7880004 sq. in. 8. 127692 sq. in. 9. 200196 sq. in.
10. 300384 sq. in. 11. 17546220 sq. in. 12 22632732 sq. in.
13. 12 sq. po. 2 yd 14 24 sq. po. 14 yd. 15. 32 sq. po. 3 yd.
16. 33 sq. po. 1 yd. 6 ft. 108 in.
17. 1 ac. 2 ro. 18 po. 19 yd. 4 ft. 72 in.
18. 7 ac. 3 ro. 10 po. 8 yd. 4 ft. 72 in.
19. 2 ac. 23 po. 8 yd. 2 ft. 36 in.
20. 2 ac. 2 po 25 yd 3 ft. 72 in.
21. 5 sq. yd. 5 ft. 34 in. 22. 2 sq po. 3 ft. 94 in.
23. 25 sq. po. 5 yd. 7 ft. 62 in. 24. 1 ac. 2 ro. 11 po. 28 yd. 51 in.
25. 4390848 sq in. 26. 48400 sq. yd.

Examples. 39.

1. 139968 cu. in.; 326592 cu. in.; 559372 cu. in.; 746496 cu. in.; 933120 cu. in.; 1819584 cu. in.
2. 2 cu. yd. 17 ft. 768 in.; 21 cu. yd. 4 ft. 966 in.

Examples. 40.

1. 404 gills. 2. 2816 gills. 3. 1504 gills. 4. 1696 gills.
5. 9344 gills. 6. 18176 gills. 7. 159744 gills. 8. 50432 gills.
9. 428032 gills. 10. 31 gall. 1 qt.
11. 1 barrel 28 gall. 3 qt. 1 gill. 12. 2 barrels 34 gall. 1 qt.
13. 6 barrels 9 gall. 3 qt. 1 gill. 14. 1 qr. 3 bus. 2 pk. 1 gall. 3 qt.
15. 5 bus. 3 pk. 3 qt. 1 pt. 16. 1 last 2 qr. 1 bus. 2 pk. 1 gall. 1 qt.

17. 4 lasts 1 lb. 3 qr. 1 bus. 3 pk. 1 qt. 1 pt. 1 gill.
18. 25 lb. Avoir. 19. 3500 lb. Avoir.

Examples. 41.

1. 259223 sec. 2. 637800 sec. 3. 1512000 sec.
4. 1 hr. 23 min. 20 sec. 5. 1 da. 3 hr. 26 min. 5 sec.
6. 1 da. 3 hr. 46 min. 40 sec. 7. 1 wk. 4 da. 13 hr. 46 min. 40 sec.
8. 94. 9. 121. 10. 244. 11. 577.
12. 289. 13. 821. 14. Thursday. 15. Wednesday.

Examples. 42.

1. 26247″. 2. 865535″. 3. 1296000″.
4. 1°. 6′. 40″. 5. 10°. 32′. 36″. 6. 1 rt. gle. 26°. 40′.
7. 1 rt. gle. 47°. 36′. 8. 3 rt. gle. 4°. 20′. 54″.

Examples. 43.

1. 24000. 2. 104 reams 3 quires 8 sheets. 3. 432.

Examples. 44.

1. 1120 gr. 2. 1632 gr. 3. 24960 m.
4. 192000 m. 5. 612309 m.

Miscellaneous Examples. 45.

1. 61200. 2. R19. 13*a*. 6*p*. 3. £569. 1*s*. 7½*d*.
4. 479 mi. 2 fur. 5. R13. 3*a*. 6. 2028.
7. 1*a*. 4*p*. 8. 1*s*. 9¾*d*. 9. 16384. 10. 105 parcels, 30 seers rem.
11. 93. 12. 1920. 13. 11. 14. R188. 11*a*. 9*p*.
15. R12. 15*a*. 6*p*. 16. R48. 14*a*. 9*p*. ; R343. 6*a*. 3*p*.
17. R2. 10*a*. 3*p*. 18. R500. 13*a*. 9*p*. 19. £1. 1*s*. 11*d*.
20. [illegible] 1*a*. [illegible] 21. R3754. 9*a*. 9*p*. 22. 6*s*. 3*d*.
23. 56 yr. 3 mo. 7 da. 24. 160. 25. 5 sec.
26. 3960. 27. 2 ft. 7 in. 28. 4196. 29. R83. 12*a*.
30. R32. 11*a*. 9*p*. 31. £66. 12*s*. 6*d*. 32. 17. 33. R687. 10*a*.
34. £30. 5*s*. 1½*d*. 35. £66. 13*s*. 4*d*. 36. 104. 37. 53.
38. 130 lb. 39. 16 yr. 4 mo. 2 da. 40. 4*s*. 2*d*. 41. 2*s*. 6*d*.
42. 63. 43. 12 seers. 44. 5 md. 45. 8 min. 18 sec.
46. 5 ft. 4 in. 47. 16th September. 48. Friday the 8th of May.

49. 53 hours. 50. 192000 miles per sec. 51. 68. 52. 19.
53. 3 yd. 54. R2. 3a. 55. 11088. 56. 4497 times.
57. 18000. 58. R2745. 59. 41 yd. 4 in. 60. 28 yr. 13 wk. 4 ds.

Examples. 46.

1. 84. 2. 44. 3. 5a.
4. Receives £13. 13s. 9d. 5. R1. 7a. 3p.

Examples. 47.

1. Gains R2. 8a. 2. R21. 1a. 6p. 3. R30. 4. R7. 12a.
5. R30. 7a. 6p. 6. R1. 10a. 3p. 7. 3p. 8. 4d.
9. £1. 1s. 10. 24 qr. 11. 8s. 4d. per yard.
12. R1. 5a. per lb. 13. Gain 12s. 6d. 14. 4d.
15. (i) R1. 2a. ; (ii) R1. 3a.

Examples. 48.

1. 4a. 2p. 2. £1. 4s. 3. 15a. 4. R9. 6a. 5. 2s. 3d.
6. 2s. 3d. 7. 2d. 8. 6 seers. 9. 9 lb. 10. 2s. 6d.

Examples. 49.

1. *A*, R23. 6a. ; *B*, R16. 1a. 9p.
2. *A*, £12. 6s. 7¼d. ; *B*, £16. 0s. 10½d.
3. The two get R34. 3a. 1p. each ; the rest R22. 4a. 4p. each.
4. Each man, R20. 4a. 6p. ; each woman, R26. 4a. 6p.
5. *A*, R16 6a. 10p. ; *B*, R13. 6a. 10p. ; *C*, R9. 6a. 10p.
6. *A*, R113. 13a. 3p. ; *B*, R106. 13a. 3p. *C*, R108. 13a. 3p.
7. £40.

Examples. 50.

1. Boy, R10. 6a. 4p. ; girl, R5. 3a. 2p.
2. *A*'s share = R15. 9a. 6p. ; *B*'s = R10. 6a. 4p. ; *C*'s = R5. 3a. 2p.
3. Each man, R12. 8a. ; each woman, R6. 4a. ; each boy, R3. 2a.
4. *A*, £6. 14s. 6d. ; *B*, £3. 7s. 3d. ; *C*, £1. 13s. 7½d.
5. One gets £5. 3s. 9d. ; and the other two, £2. 11s. 10½d. each.
6. *A*, R26. 15a. 3p. ; *B*, R12. 8a. 6p.

Examples. 51.

1. 12. 2. 10. 3. 12. 4. 16.
5. 11 rupees, 22 half-rupees, 44 quarter-rupees. 6. 32.
7. 40 shillings, 21 sixpences.
8. 23 eight-anna pieces and 27 four-anna pieces.

Examples. 52.

1. R3. 7*a*. 9*p*. 2. R10. 2*a*.
3. The price of a horse is R75. 8*a*., of a cow, R25. 8*a*. and of a sheep, R5. 8*a*.
4. A mark = $11\frac{3}{4}d.$; a gulden = $1s.\ 11\frac{3}{4}d.$; a rouble = $3s.\ 1\frac{1}{2}d.$
5. R38. 4*a*. 6*p*.

Examples. 53.

1. 2, 3. 2. 3, 5, 9. 3. 2, 3, 4, 9. 4. 2, 3, 4, 5, 10.
5. 2, 3, 4, 11. 6. 2, 11. 7. 2, 3, 5, 10. 8. 2, 4.
9. None. 10. 5. 11. 2, 3, 4, 8, 11. 12. 2, 3, 4, 8, 9, 11.
13. 3, 5. 14. 5. 15. 2, 4, 5, 8, 10. 16. 2, 4, 5, 8, 10.
17. 3, 9. 18. 3, 11. 19. 2, 3. 20. 2, 3, 5, 9, 10.
21. 7. 22. 11. 23. 13. 24. 7, 11, 13.
25. 11. 26. 7, 13. 27. None. 28. 7, 11, 13.
29. 6, 12. 30. 6, 12. 31. 6, 12, 30. 32. None.
33. 2 ; 1. 34. 1 ; 7 ; 2. 35. 2717. 36. 1, 5, 3.

Examples. 54.

1. 2^3. 2. $2^2.3$. 3. 2.3^2. 4. $2^3.3$. 5. 3^3.
6. 2^5. 7. $2^4.3$. 8. 2.5^2. 9. $3^2.7$. 10. 2^6.
11. $2^4.5$. 12. $2^3.11$. 13. $3^2.11$. 14. $2^3.5^2$. 15. $2^2.3^3$.
16. $2^4.11$. 17. $3^2.13$. 18. $2^5.3^2$. 19. $3^2.5.11$. 20. 5^4.
21. $3^3.37$. 22. $2^3.5^2.7$. 23. $2^4.3^4$. 24. $2^5.5.11$. 25. $2^4.5^3$.
26. $2.5^2.73$. 27. $2^7.3^2.5$. 28. $3^3.7.13$. 29. $2^9.3^3$.
30. $2^2.3.5^2.23.29$. 31. Prime. 32. Prime. 33. 3^4.
34. Prime. 35. Prime. 36. Prime. 37. Prime. 38. $3^2.23$.
39. Prime. 40. Prime. 41. $11^2.31$. 42. 3.13^2. 43. 17.269.
44. Prime. 45. 23.31. 46. Prime. 47. 13.503. 48. 11.163.
49. Prime. 50. 29.47. 51. 10. 52. 11. 53. 11.
54. 5, 7. 55. 5, 7. 56. 6, 8, 12, 24.

Examples. 55.

1. 2. 2. 3. 3. 6. 4. 4. 5. 4. 6. 7.
7. 21. 8. 11. 9. 8. 10. 5. 11. 13. 12. 2.
13. 5. 14. 7. 15. 3. 16. 17. 17. 18. 18. 5.
19. 12. 20. 75. 21. 4. 22. 24. 23. 5. 24. 4.
25. No common factor. 26. 56. 27. 25. 28. 28.

Examples. 56.

1. 48. 2. 2. 3. 4. 4. 12. 5. 29. 6. 124.
7. 101. 8. 143. 9. 377. 10. 7. 11. 133. 12. 25.
13. 19. 14. 15. 15. 53. 16. 28. 17. 39. 18. 113.
19. 173. 20. 147. 21. 221. 22. 3. 23. 57. 24. 287.
25. 213. 26. 221. 27. 15. 28. 1536. 29. 257. 30. 6.
31. No. 32. Yes. 33. No. 34. Yes. 35. No. 36. No.
37. Yes. 38. Yes. 39. No. 40. 37. 41. 37. 42. 23.
43. 17. 44. 3. 45. 5. 46. 3. 47. 63. 48. 17.
49. 57. 50. 2. 51. 2. 52. R1. 4*a*. 53. 3*d*.
54. 2 ft. 3 in. 55. Half crown. 56. 16.
57. 32. 58. No. 59. 180 gallons. 60. 1 tola.
61. 7, 13, 11. 62. 17, 11, 41. 65. 385 and 525.

Examples. 57.

1. 8. 2. 6. 3. 30. 4. 35. 5. 30.
6. 16. 7. 48. 8. 75. 9. 8. 10. 48.
11. 66. 12. 12. 13. 90. 14. 60. 15. 42.
16. 36. 17. 7488. 18. 259488. 19. 672. 20. 23374.
21. 87087. 22. 759655. 23. 4[illegible]77. 24. 734877.
25. 96672. 26. 159137. 27. 183645. 28. 2672700.
29. 2310. 30. 2376. 31. R5256. 12*a*.
32. 64. 33. 390.

Examples. 58.

1. 48. 2. 48. 3. 720. 4. 36. 5. 2520.
6. 1680. 7. 28050. 8. 360. 9. 1890. 10. 7560.
11. 7200. 12. 144. 13. 8415. 14. 7920. 15. 792.
16. 3570. 17. 228150. 18. 98280. 19. 49140. 20. 5481.

21. 237510. **22.** 2520. **23.** 1680. **24.** 10800.
25. 98280. **26.** 189. **27.** 389. **28.** 141.
29. 1296 sq. in. **30.** £189. **31.** 14 min. **32.** 90 miles.
33. 131 yd. 9 in. **34.** 677. **35.** 232792560. **36.** 75 yards.

Examples. 59.

1. 144 and 192. **2.** 108 and 144 ; 144 and 180, or 108 and 180.
3. 222 and 259, or 259 and 296. **4.** 420 and 480.
5. 221 and 293. **6.** 1999 and 998. **7.** 9 and 60597.
8. 997920 and 103680. **9.** 273 and 357. **10.** 21, 35, 77.
11. Any two of the following 3 numbers, 3575 and 1001.
13. 62. **14.** 20150. **16.** 35 or 7 or 5. **17.** 91 or 7 or 13
18. 84 or 3, 4, 7, 12, 21. **19.** 105. **20.** 121.
21. 319 and 377. **22.** 875. **23.** 3455.
24. 9600. **25.** 93679. **26.** 7, 140 ; 35, 2F.
27. 23704543 ; 8143.

Examples. 60.

1. [illegible] ; $\frac{1}{2}$; $\frac{3}{4}$. **2.** $\frac{1}{12}$; $\frac{1}{2}$; $\frac{1}{3}$. **3.** (*a*) $\frac{1}{3}$; $\frac{2}{3}$; (*b*) $\frac{1}{15}$. **4.** 4*a*.
5. 4*s*. **6.** 2*q*. **7.** 1 seer. **8.** 5*a*. **9.** 9*s*. **10.** 7 in.
11. 5*p*. **12.** 10 in. **13.** A*d*. **14.** 3 pice. **15.** 3 cwt.
17. 160 yd. **18.** 6 ch. **19.** 9 sq. in. **20.** 7 lb. **21.** 6*a*.
22. 9*a*. **23.** 1 ft. **24.** 4*d*. **25.** 15 min.

Examples. 61

1. 2 ; 6 ; 14 ; 1 ; 5 ; 13. **2.** 6 ; 4 ; 3 ; 2 ; 8 ; 5.
3. 18 ; 24 ; 27 ; 30 ; 21 ; 22 ; 1. **4.** 8 ; 4 ; 12 ; 10 ; 7.
5. 16 ; 20 ; 21 ; 10 ; 1. **6.** 4. **7.** 22. **8.** 2. **9.** 24.
16. 4. **17.** 5. **18.** $\frac{1}{12}$. **19.** [illegible] **20.** $\frac{1}{12}$. **21.** $\frac{1}{15}$.

Examples. 62.

1. [illegible]
2. [illegible]
3. [illegible] **4.** [illegible]
5. [illegible] **6.** [illegible] **7.** [illegible]

Examples. 63.

1. $\frac{1}{2}$. 2. $\frac{1}{3}$. 3. $\frac{1}{2}$. 4. $\frac{3}{4}$. 5. $\frac{2}{3}$.
6. $\frac{4}{5}$. 7. $\frac{3}{4}$. 8. $\frac{3}{5}$. 9. $\frac{2}{3}$. 10. $\frac{1}{5}$.
11. $\frac{4}{5}$. 12. $\frac{7}{8}$. 13. $\frac{3}{8}$. 15. $\frac{3}{4}$.
16. $\frac{1}{3}$. 17. $\frac{1}{3}$. 18. $\frac{3}{4}$. 19. $\frac{4}{25}$. 20. $\frac{2}{3}$.

Examples. 64.

1. $\frac{2}{3}$. 2. $\frac{4}{9}$. 3. $\frac{3}{10}$. 4. $\frac{2}{15}$. 5. $\frac{3}{8}$. 6. $\frac{1}{2}$.
7. $\frac{10}{21}$. 8. $\frac{7}{15}$. 9. $\frac{7}{9}$. 10. $\frac{5}{9}$. 11. $\frac{7}{8}$. 12. $\frac{5}{6}$.
13. $\frac{9}{22}$. 14. $\frac{5}{11}$ 15. $\frac{22}{43}$. 16. $\frac{110}{117}$. 17. $\frac{851}{910}$. 18. $\frac{11}{16}$.
19. $\frac{169}{195}$. 20. $\frac{60}{77}$. 21. $\frac{13}{17}$. 22. $\frac{2}{3}$. 23. $\frac{11}{13}$. 24. $\frac{17}{31}$.
25. $\frac{103}{135}$. 26. $\frac{29}{57}$. 27. $\frac{3}{4}$. 28. $\frac{603}{65}$. 29. $\frac{23}{67}$. 30. $\frac{80}{141}$.
31. $\frac{43}{54}$. 32. $\frac{2}{7}$ 33. $\frac{98}{99}$. 34. $\frac{5}{7}$. 35. $\frac{11}{13}$.

Examples. 65.

1. $\frac{1}{6}$. 2. 3. $\frac{5}{9}$. 4. $\frac{8}{9}$. 5. $\frac{8}{15}$. 6. $\frac{1}{16}$.
7. $\frac{1}{12}$. 9. $\frac{7}{8}$. 10. $\frac{34}{5}$. 11. $\frac{4}{15}$. 12. $\frac{6}{7}$.

Examples. 66.

3. $\frac{102}{11}$. 4. $\frac{87}{10}$.
7. $\frac{843}{70}$. 8. $\frac{403}{20}$. 9. $\frac{1001}{41}$. 10. $\frac{8101}{30}$.
11. $\frac{5807}{200}$. 12. $\frac{7109}{98}$. 13. $\frac{648}{25}$. 14. $\frac{12421}{111}$. 15. $\frac{9313}{99}$.
16. $\frac{743}{100}$. 17. $\frac{8329}{1000}$. 18. $\frac{2255}{101}$. 19. $\frac{8044}{201}$. 20. $\frac{400}{99}$.

Examples. 67.

1. $3\frac{1}{3}$. 2. $2\frac{1}{4}$. 3. $4\frac{1}{3}$. 4. $5\frac{1}{4}$. 5. $3\frac{3}{7}$.
6. $3\frac{3}{4}$. 7. 9. 8. $6\frac{1}{3}$. 9. $9\frac{9}{10}$. 10. 5.
11. $2\frac{11}{14}$. 12. 4. 13. $3\frac{3}{4}$. 14. $4\frac{12}{17}$. 15. $3\frac{1}{8}$.
16. $4\frac{2}{7}$. 17. $2\frac{99}{101}$. 18. $7\frac{1}{11}$. 19. $56\frac{2}{3}$. 20. 7.
21. $28\frac{21}{25}$. 22. 329. 23. $101\frac{1}{4}$. 24. 10. 25. $48\frac{7}{9}$.
26 $2\frac{1}{3}$. 27. $11\frac{201}{425}$. 28. $10\frac{4}{123}$. 29. 11. 30. $4\frac{293}{377}$.

Examples. 68.

1. $\frac{3}{15}, \frac{10}{15}$. 2. $\frac{9}{30}, \frac{8}{30}$. 3. $\frac{25}{30}, \frac{12}{30}$. 4. $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$.
5. $\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$. 6. $\frac{140}{168}, \frac{144}{168}, \frac{147}{168}$. 7. $\frac{27}{72}, \frac{56}{72}, \frac{30}{72}$. 8. $\frac{24}{60}, \frac{35}{60}, \frac{38}{60}$.
9. $\frac{75}{240}, \frac{70}{240}, \frac{66}{240}$. 10. $\frac{40}{300}, \frac{45}{300}, \frac{38}{300}$. 11. $\frac{126}{300}, \frac{85}{300}, \frac{270}{300}$.
12. $\frac{36}{300}, \frac{40}{300}, \frac{45}{300}$. 13. $\frac{40}{12}, \frac{51}{12}, \frac{74}{12}$. 14. $\frac{48}{84}, \frac{4}{84}, \frac{3}{84}$.

15. $\frac{36}{12}, \frac{60}{12}, \frac{7}{12}$. 16. $\frac{15}{105}, \frac{350}{105}, \frac{210}{105}, \frac{63}{105}$. 17. $\frac{36}{12}, \frac{4}{12}, \frac{48}{12}, \frac{9}{12}$.
18. $\frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}$. 19. $\frac{14}{154}, \frac{21}{154}, \frac{9}{154}, \frac{33}{154}, \frac{154}{154}$.
20. $\frac{1155}{3465}, \frac{2079}{3465}, \frac{2475}{3465}, \frac{2695}{3465}, \frac{2835}{3465}$. 21. $\frac{720}{600}, \frac{440}{600}, \frac{335}{600}, \frac{465}{600}, \frac{702}{600}$.
22. $\frac{288}{144}, \frac{336}{144}, \frac{84}{144}, \frac{64}{144}, \frac{99}{144}$. 23. $\frac{240}{2400}, \frac{180}{2400}, \frac{40}{2400}, \frac{75}{2400}, \frac{24}{2400}$.
24. $\frac{96}{48}, \frac{100}{48}, \frac{348}{48}, \frac{15}{48}, \frac{14}{48}$. 25. $\frac{2720}{6120}, \frac{240}{6120}, \frac{72}{6120}, \frac{714}{6120}, \frac{153}{6120}$.
26. $\frac{72}{24}, \frac{180}{24}, \frac{56}{24}, \frac{20}{24}, \frac{21}{24}$. 27. $\frac{128}{112}, \frac{80}{112}, \frac{56}{112}, \frac{112}{112}, \frac{85}{112}, \frac{232}{112}$.

Examples. 69.

1. $\frac{3}{4}$. 2. $\frac{9}{13}$. 3. $\frac{13}{20}$. 4. $\frac{19}{20}$. 5. $\frac{7}{8}$. 6. $\frac{22}{27}$.
8. $\frac{24}{35}$ greatest, $\frac{15}{16}$ least. 9. $\frac{10}{11}$ greatest, $\frac{7}{8}$ least.
10. $\frac{20}{21}$ greatest, $\frac{19}{24}$ least. 11. $\frac{15}{18}$ greatest, $\frac{2}{3}$ least.
12. $\frac{9}{20}$ greatest, $\frac{5}{18}$ least. 13. $\frac{9}{28}$ greatest, $\frac{7}{24}$ least.
14. $\frac{3}{4}, \frac{7}{12}, \frac{5}{9}$. 15. $\frac{18}{35}, \frac{7}{15}, \frac{4}{45}$. 16. $\frac{11}{13}, \frac{7}{9}, \frac{3}{5}$.
17. $\frac{15}{4}, 3\frac{2}{3}, \frac{27}{8}$. 18. $\frac{29}{30}, \frac{31}{35}, \frac{21}{25}$. 19. $\frac{118}{321}, \frac{145}{428}, \frac{27}{87}$.
20. $\frac{6}{7}, \frac{5}{8}, \frac{13}{28}, \frac{4}{9}$. 21. $\frac{113}{153}, \frac{51}{76}, \frac{2}{3}, \frac{7}{12}$. 22. $\frac{17}{12}, \frac{3}{4}, \frac{11}{15}, \frac{9}{20}$.

Examples. 70.

1. $2\frac{1}{4}$. 2. $1\frac{2}{7}$. 3. $\frac{8}{9}$. 4. $1\frac{1}{11}$. 5. $2\frac{7}{10}$. 6. $1\frac{49}{50}$.
7. 1. 8. $7\frac{3}{10}$. 9. $1\frac{2}{7}$. 10. $\frac{5}{6}$. 11. $1\frac{5}{12}$. 12. $1\frac{7}{35}$.
13. $1\frac{17}{240}$. 14. $1\frac{14}{45}$. 15. $1\frac{203}{240}$. 16. $1\frac{11}{12}$. 17. $\frac{47}{48}$.
18. $1\frac{31}{420}$. 19. $\frac{47}{180}$. 20. $4\frac{37}{105}$. 21. $2\frac{11}{60}$. 22. $\frac{103}{270}$.
23. $2\frac{41}{200}$. 24. $4\frac{8}{15}$. 25. $2\frac{175}{308}$. 26. $1\frac{301}{468}$. 27. $3\frac{43}{90}$.
28. 2. 29. $3\frac{1121}{1152}$. 30. $2\frac{541}{1020}$.

Examples. 71.

1. $7\frac{3}{4}$. 2. $14\frac{1}{6}$. 3. $12\frac{7}{80}$. 4. $15\frac{23}{30}$. 5. $23\frac{3}{5}$.
6. $29\frac{19}{56}$. 7. $5\frac{5}{8}$. 8. $41\frac{2}{15}$. 9. $10\frac{103}{462}$. 10. $11\frac{7}{51}$.
11. $14\frac{25}{56}$. 12. $11\frac{1}{2}$. 13. $160\frac{52}{77}$. 14. $34\frac{59}{72}$. 15. $13\frac{5}{8}$.
16. $31\frac{5}{16}$. 17. $976\frac{4}{21}$. 18. $154\frac{1}{8}$. 19. $17\frac{109}{147}$. 20. $6\frac{3}{10}$.
21. R29. 9*a*. $5\frac{33}{56}$*p*. 22. £7. 17*s*. $0\frac{9}{13}$*d*.
23. 15 yd. 2 ft. $6\frac{15}{16}$ in. 24. 12 lb. 1 oz. $2\frac{9}{13}$ dr.
25. 21 oz. 0 dwt. $19\frac{3}{5}$ gr. 26. 20 hr. 24 min. $33\frac{9}{13}$ sec.

Examples. 72.

1. $\frac{2}{3}$. 2. $\frac{6}{17}$. 3. $\frac{3}{19}$. 4. $\frac{7}{29}$. 5. $\frac{54}{8}$ *i.e.*, $6\frac{3}{4}$. 6. $\frac{4}{7}$.
7. $\frac{4}{19}$. 8. $\frac{6}{13}$. 9. $\frac{7}{15}$. 10. $\frac{11}{20}$. 11. $\frac{13}{37}$. 12. $\frac{12}{65}$.
13. $\frac{1}{6}$. 14. $\frac{1}{12}$. 15. $\frac{11}{15}$. 16. $\frac{7}{144}$. 17. $\frac{13}{72}$. 18. $\frac{4}{15}$.

19. $\frac{23}{192}$. 20. $1\frac{1}{4}$. 21. $\frac{51}{100}$. 22. $\frac{101}{120}$. 23. $1\frac{11}{45}$. 24. $5\frac{5}{12}$.
25. $\frac{5}{16}$. 26. $\frac{5}{18}$. 27. $\frac{1}{3}$. 28. $\frac{5}{9}$. 29. $\frac{7}{16}$. 30. $\frac{103}{315}$.

Examples. 73.

1. $3\frac{1}{6}$. 2. $2\frac{7}{30}$. 3. $3\frac{1}{3}$. 4. $5\frac{5}{8}$. 5. $5\frac{4}{15}$.
6. $5\frac{3}{17}$. 7. $6\frac{31}{84}$. 8. $8\frac{5}{13}$. 9. $2\frac{5}{6}$. 10. $3\frac{23}{30}$.
11. $\frac{43}{48}$. 12. $5\frac{19}{21}$. 13. $2\frac{41}{64}$. 14. $8\frac{49}{50}$. 15. $9\frac{5}{26}$.
16. $9\frac{64}{81}$. 17. $10\frac{13}{33}$. 18. $6\frac{19}{120}$. 19. $6\frac{7}{24}$. 20. $9\frac{19}{21}$.
21. $2\frac{1}{3}$. 22. $6\frac{5}{8}$. 23. $8\frac{1}{12}$. 24. $9\frac{7}{17}$. 25. $8\frac{1}{3}$.
26. $12\frac{8}{11}$. 27. $13\frac{17}{18}$. 28. $10\frac{1}{21}$. 29. $1\frac{7}{12}$. 30. $6\frac{7}{27}$.
31. $6\frac{27}{55}$. 32. $7\frac{1}{12}$. 33. $3\frac{15}{16}$. 34. $2\frac{9}{16}$. 35. $9\frac{1}{2}$.
36. $\frac{15}{32}$. 37. $12\frac{20}{21}$. 38. $14\frac{3}{55}$. 39. $\frac{2}{7}$. 40. $\frac{1}{3}$.
41. R10 . 12 . $1\frac{1}{2}$. 42. R2 . 12 . $9\frac{4}{7}$. 43. R4 . 4 . $3\frac{7}{8}$.
44. £10 . 9 . $5\frac{7}{15}$. 45. £5 . 12 . $10\frac{8}{15}$. 46. 6 yd. $5\frac{5}{8}$ in.

Examples. 74.

1. $4\frac{3}{8}$. 2. 7. 3. $5\frac{1}{2}$. 4. $3\frac{6}{7}$. 5. $5\frac{1}{3}$.
6. $19\frac{1}{2}$. 7. $10\frac{1}{3}$. 8. 300. 9. $2\frac{4}{5}$. 10. $10\frac{1}{2}$.
11. $22\frac{1}{8}$. 12. $23\frac{3}{4}$. 13. $18\frac{33}{35}$. 14. $60\frac{3}{7}$. 15. 195.
16. $6\frac{6}{13}$. 17. $13\frac{1}{3}$. 18. $47\frac{3}{5}$. 19. $66\frac{4}{7}$. 20. $100\frac{8}{13}$.
21. $33\frac{9}{55}$. 22. $62\frac{1}{3}$. 23. $328\frac{5}{8}$. 24. $198\frac{4}{7}$. 25. 213.
26. $1224\frac{1}{2}$. 27. $487\frac{3}{7}$. 28. $177\frac{1}{2}$. 29. $28\frac{71}{100}$. 30. $38\frac{11}{25}$.
31. $18\frac{981}{1000}$. 32. $44\frac{149}{158}$. 33. $899\frac{61}{100}$.
34. $386\frac{1}{10}$. 35. $22999\frac{977}{1000}$. 36. $3190\frac{3}{5}$.
37. $209\frac{87}{100}$. 38. $6399\frac{71}{73}$. 39. £1 . 18 . $11\frac{1}{13}$.
40. £4 . 9 . $7\frac{2}{13}$. 41. R50 . 7 . $2\frac{1}{4}$. 42. R49 . 4 . $5\frac{1}{2}$.
43. £2 . 4 . 5. 44. £36 . 7 . $2\frac{1}{4}$. 45. 20 yd. 2 ft.
46. 8 md. 4 seer 14 ch. 47. 6 ton $9\frac{1}{2}$ cwt.

Examples. 75.

1. $\frac{1}{3}$. 2. $\frac{2}{15}$. 3. $\frac{3}{25}$. 4. $\frac{1}{9}$. 5. $\frac{2}{54}$.
6. $\frac{3}{116}$. 7. $\frac{5}{147}$. 8. $\frac{9}{910}$. 9. $\frac{7}{57}$. 10. [illegible].
11. [illegible]. 12. $\frac{7}{750}$. 13. $\frac{2}{325}$. 14. $\frac{2}{385}$. 15. [illegible].
16. $\frac{5}{1095}$. 17. $1\frac{5}{6}$. 18. $\frac{4}{15}$. 19. $\frac{1}{35}$. 20. $\frac{23}{56}$.
21. $1\frac{1}{3}$. 22. $\frac{2}{27}$. 23. $\frac{21}{182}$. 24. $\frac{2}{75}$. 25. $42\frac{2}{3}$.
26. $12\frac{11}{35}$. 27. 178[illegible]. 28. $6\frac{4}{7}$. 29. 15[illegible]. 30. 10[illegible].

31. $62\frac{7}{12}$. 32. $38\frac{111}{256}$. 33. $21\frac{19}{21}$. 34. $1\frac{23}{36}$.
35. R1 . 5 . $6\frac{5}{13}$ 36. R2 . 8 . $7\frac{1}{13}$. 37. £1 . 17 . $0\frac{4}{33}$.
38. £7 . 13 . $10\frac{23}{126}$. 39. 80. 40. $192\frac{3}{7}$. 41. $43\frac{11}{28}$
42. $112\frac{3}{11}$. 43. R2 . 9 . $0\frac{3}{8}$. 44. R1 . 4 . $0\frac{6}{11}$.
45. R32 . 5 . $6\frac{3}{13}$. 46. R6 . 4 . $2\frac{5}{16}$. 47. £3 . 11 . $6\frac{1}{3}$.
48. £3 . 19 . $11\frac{4}{15}$.

Examples. 76.

1. $\frac{5}{6}$. 2. $\frac{21}{32}$. 3. $\frac{2}{3}$. 4. $\frac{8}{15}$. 5. $4\frac{1}{2}$. 6. $\frac{27}{200}$.
7. $\frac{3}{4}$. 8. $\frac{9}{64}$. 9. $\frac{3}{8}$. 10. $3\frac{1}{5}$. 11. $1\frac{2}{3}$. 12. $2\frac{4}{7}$.
13. $32\frac{1}{28}$. 14. $25\frac{5}{8}$. 15. 3. 16. $14\frac{2}{3}$. 17. 10.
18. 8. 19. $28\frac{7}{15}$. 20. $16\frac{5}{16}$. 21. $9\frac{1}{2}$. 23. 8.
24. $3\frac{3}{4}$. 25. 35. 26. 8. 27. $15\frac{39}{64}$. 28. $12\frac{3}{4}$.
29. $40\frac{1}{2}$. 30. $286\frac{22}{27}$. 31. $\frac{3}{4}$. 32. $2\frac{7}{20}$. 33. $31\frac{1}{2}$.
34. $\frac{15}{28}$. 35. $\frac{5}{24}$. 36. $\frac{1}{12}$. 37. 28. 38. 294.

Examples. 77.

1. 1386 in. 2. 2574 in. 3. 5742 in. 4. 7722 in.
5. 9702 in. 6. 39582 in. 7. 673303 in. 8. 274428 sq. in.
9. 509652 sq. in. 10. 1136916 sq. in. 11. 1528956 sq. in.
12. 1920996 sq. in. 13. 59864508 sq. in. 14. 4033699560 sq. in.

Examples. 78.

1. $1\frac{1}{8}$. 2. $\frac{27}{56}$. 3. 1. 4. $1\frac{1}{27}$. 5. $1\frac{5}{9}$.
6. 14. 7. 3. 8. $2\frac{3}{4}$. 9. $1\frac{3}{5}$. 10. $1\frac{1}{3}$.
11. $\frac{4}{5}$. 12. $\frac{10}{11}$. 13. $11\frac{2}{3}$. 14. $6\frac{2}{7}$. 15. $17\frac{11}{35}$.
16. $9\frac{9}{22}$. 17. $2\frac{113}{156}$. 18. $1\frac{1}{2}$. 19. $1\frac{1}{3}$. 20. $2\frac{2}{3}$.
21. $\frac{8}{25}$. 22. $1\frac{1}{2}$. 23. $6\frac{2}{75}$. 24. $3\frac{57}{175}$. 25. 16.
26. $\frac{35}{48}$. 27. $1\frac{1}{27}$. 28. $\frac{6}{13}$. 29. $93\frac{1}{3}$. 30. The former.

Examples. 79.

1. $\frac{1}{12}$; 1. 2. $\frac{3}{16}$; $2\frac{1}{4}$. 3. $\frac{9}{147}$; $2\frac{3}{7}$. 4. $\frac{25}{48}$; 8.
5. $\frac{1}{30}$; 20. 6. $\frac{1}{12}$; $10\frac{1}{2}$. 7. $\frac{1}{32}$; $409\frac{1}{2}$. 8. $\frac{7}{20}$; 42.
9. $\frac{1}{3}$; $157\frac{1}{2}$. 10. $1\frac{1}{3}$; 63. 11. $1\frac{1}{3}$; 8. 12. $\frac{23}{350}$; $70\frac{19}{13}$.
13. 3 in. 14. $2\frac{2}{3}$. 15. 1 min. 45 sec.

Miscellaneous Examples. 80.

1. $6\frac{1}{3}$. 2. $1\frac{1}{6}$. 3. $5\frac{1}{6}$. 4. $\frac{1}{11}$. 5. $\frac{1}{10}$. 6. 5.
7. $16\frac{1}{16}$. 8. $\frac{2}{3}$. 9. £24 . 19 . $5\frac{1}{2}$. 10. R840 . 6 . $10\frac{1}{2}$.
11. 950 lb. 12. R1520. 13. £50. 14. $4\frac{1}{2} \times 3\frac{1}{3}$. 15. $\frac{5}{8}$.
16. $\frac{1}{3}$. 17. $\frac{1}{60}$. 19. $\frac{2}{3}$. 20. $\frac{9}{40}$. 21. $\frac{1}{4}$. 22. $\frac{33}{100}$.
23. $\frac{21}{50}$. 24. $\frac{19}{180}$. 25. $\frac{8}{75}$. 26. $\frac{2}{3}$. 27. R60.
28. £720. 29. 15*s*. 30. 22 miles. 31. 400 inches.
32. 8, 6, 3, 2 ; 24 kings in all. 33. 34. 34. $1\frac{6}{51}$. 35. $\frac{14}{57}$.
36. 5 times. 37. $\frac{9}{13}$. 38. 27 hours. 39. $3\frac{1}{3}$. 40. 310.
41. 13 ; 17. 42. 36.

Examples. 81.

1. $\frac{2}{3}$. 2. $1\frac{9}{17}$. 3. $3\frac{3}{5}$. 4. 12. 5. $\frac{1}{18}$. 6. $2\frac{2}{25}$.
7. $\frac{303}{584}$. 8. $4\frac{1}{10}$. 9. $\frac{7}{18}$. 10. $\frac{3}{5}$ 11. 3. 12. 3.
13. $5\frac{1}{10}$. 14. $96\frac{4}{21}$. 15. 17. 16. $\frac{248}{327}$. 17. $4\frac{1}{2}$. 18. $\frac{3}{4}$.
19. $11\frac{5}{14}$ 20. $\frac{1}{11}$. 21. 18. 22. $8\frac{431}{500}$. 23. $\frac{3}{8}$. 24. $1\frac{17}{33}$.

Examples. 82.

1. [illegible] 2. $1\frac{2}{3}$. 3. $\frac{8}{11}$. 4. $2\frac{204}{317}$. 5. $25\frac{1}{3}$. 6. $15\frac{2}{3}$
7. $1\frac{68}{157}$. 8. $6\frac{216}{1261}$. 9. $4\frac{4}{75}$. 10. $\frac{4}{5}$. 11. $\frac{29}{70}$. 12. $\frac{9}{17}$.
13. $4\frac{3}{14}$.

Examples. 83.

1. $\frac{85}{512}$. 2. $\frac{4}{5}$. 3. $\frac{81}{125}$. 4. 6. 5. 1. 6. $1\frac{13}{32}$.
7. $1\frac{1}{4}$. 8. $\frac{1}{2}$. 9. $1\frac{1}{4}$. 10. $\frac{1}{14}$. 11. $\frac{1}{3}$. 12. $\frac{3}{17}$.
13. $3\frac{3}{16}$. 14. $\frac{1}{2}$. 15. 2. 16. $\frac{81}{128}$. 17. $22\frac{2}{3}$. 18. $\frac{1}{2}$.
19. 2. 20. $\frac{81}{128}$. 21. $22\frac{2}{3}$. 22. $\frac{24}{91}$. 23. $2\frac{11}{14}$. 24. $\frac{63}{80}$.

Examples. 84.

1. 3. 2. $3\frac{1}{2}$. 3. $\frac{97}{504}$. 4. $3\frac{5}{8}$. 5. $9\frac{17}{39}$. 6. $1\frac{5}{8}$.
7. 12. 8. $7\frac{5}{24}$. 9. $7\frac{7}{36}$. 10. $\frac{19}{32}$. 11. $4\frac{149}{345}$. 12. $1\frac{19}{259}$.
13. $4\frac{11}{35}$. 14. 1. 15. $\frac{41}{60}$. 16. $12\frac{1}{2}$. 17. $4\frac{237}{304}$. 18. $\frac{11}{14}$.

Examples. 85.

1. 1. 2. $1\frac{1}{4}$. 3. $7\frac{5}{12}$. 4. $6\frac{1}{21}$. 5. $\frac{133}{165}$.
6. $5\frac{11}{153}$. 7. $\frac{61}{2162}$. 8. $3\frac{91}{225}$. 9. $1\frac{121}{225}$. 10. $9\frac{17}{18}$.

11. $7\frac{3}{4}$. 12. $4\frac{1}{4}$. 13. $4\frac{3}{4}$. 14. $5\frac{3}{4}$. 15. $8\frac{11}{15}$.
16. $12\frac{7}{15}$. 17. 1. 18. 10. 19. $1\frac{4}{17}$. 20. $\frac{11}{13}$.
21. $\frac{1}{5}$. 22. $2\frac{5}{8}$.

Examples. 86.

1. $\frac{9}{88}$. 2. 2. 3. 2. 4. $1\frac{3741}{10794}$. 5. $3\frac{1}{8}$.
6. $2\frac{73}{455}$. 7. 1. 8. $14\frac{7}{18}$. 9. $\frac{3}{283}$. 10. $1\frac{1}{5}$.
11. $\frac{425}{2304}$. 12. $\frac{30}{37}$. 13. 3. 14. $5\frac{19}{250}$. 15. $573\frac{3}{11}$.
16. 49. 17. $8\frac{44}{117}$. 18. $1\frac{11}{24}$. 19. $5\frac{1}{2}$. 20. $5\frac{569}{5080}$.
21. $1\frac{1}{84}$. 22. $\frac{5}{7}$. 23. 1. 24. 4. 25. $\frac{7}{8}$.
26. 1. 27. $\frac{3}{35}$. 28. $\frac{545}{2268}$. 29. $84\frac{72}{77}$. 30. $10\frac{49}{128}$.
31. $\frac{85}{176}$. 32. $22\frac{3}{4}$. 33. $\frac{123}{232}$. 34. $3\frac{7}{15}$. 35. $3\frac{29}{54}$.

Examples. 87.

1. R3 . 10 . 4. 2. R1 . 10 . 8. 3. R1. 14*a*.
4. R8 . 8 . 8. 5. R1 . 3 . 6. 6. 7*a*. 6*p*.
7. £33 . 16 . 4. 8. £58. 10*s*. 9. £29. 14*s*.
10. R70 . 9 . 4. 11. R1 . 12 . 8. 12. R1 . 2 . 8.
13. £11 . 5 . $9\frac{7}{33}$. 14. £38. 8*s*. 15. 6*s*. 3*d*.
16. R52 . 6 . $10\frac{1}{2}$. 17. R19 . 9 . $9\frac{1}{2}$. 18. 19*s*. $6\frac{9}{10}$*d*.
19. £15 . 10 . $2\frac{1}{3}$. 20. R284 . 2 . $6\frac{1}{33}$. 21. £22 . 14 . $3\frac{184}{441}$.
22. 4 cwt. 2 qr. 24 lb. 12 oz.. 23. 343 yd. 1 ft. $10\frac{3}{8}$ in.
24. 25 min. $25\frac{5}{13}$ sec. 25. 2 pk. $1\frac{11}{31}$ gall. 23. R146., 11 . 11.
27. R1 . 0 . $5\frac{1}{7}$. 28. R122 . 3 . 8. 29. £7 . 19 . $10\frac{1}{4}$.
30. £22 . 1 . $9\frac{13}{33}$. 31. R31 . 8 . $6\frac{3}{23}$. 32. 10*s*. $11\frac{3}{7}\frac{5}{8}$*d*.
33. 12*a*. $9\frac{133}{338}$*p*. 34. £2 . 8 . $7\frac{1}{2}$. 35. 16*s*. $10\frac{1}{4}$*d*.
36. R8 . 5 . $1\frac{1}{4}$. 37. R14 . 6 . $0\frac{104}{153}$. 38. £3 . 18 . $5\frac{3}{8}\frac{107}{385}$.
39. $\frac{11}{18}$ of R6. 11*a*., $\frac{3}{4}$ of R7., R$\frac{5}{8}$. 40. £14 . 15 . 2.
41. R8 . 9 . $4\frac{4}{5}$. 42. R6 . 5 . $9\frac{2}{3}$. 43. R217 . 15 . 6.
44. 18*s*. $9\frac{13}{33}$*d*.

Examples. 88.

1. $3\frac{1}{4}$. 2. $9\frac{3}{4}$. 3. $5\frac{6}{15}$. 4. $7\frac{1}{3}$. 5. $7\frac{21}{40}$. 6. $7\frac{3}{8}$.
7. $7\frac{1}{3}$. 8. $3\frac{1}{3}$. 9. $\frac{185}{784}$. 10. $\frac{223}{280}$. 11. $\frac{443}{280}$. 12. $\frac{118}{1785}$.
13. $\frac{3}{85}$. 14. $\frac{3971}{7920}$. 15. $\frac{103}{1800}$. 16. $\frac{9}{17}$. 17. $\frac{131}{336}$. 18. $\frac{51}{8080}$.
19. $\frac{533}{585}$. 20. $2\frac{3441}{4746}$. 21. $\frac{4163}{14575}$. 22. $\frac{11}{80}$. 23. $\frac{1}{3}$.
24. $8\frac{111}{135}$. 25. $42\frac{10}{81}$. 26. $52\frac{10}{81}$. 27. $\frac{471}{1795}$. 28. $1\frac{319}{1075}$.

29. $\frac{3779}{9000}$. 30. $\frac{9}{64}$. 31. $\frac{77}{375}$. 32. $\frac{131}{250}$. 33. $\frac{1}{5}$. 34. $\frac{7}{9}$.
35. $\frac{57}{294}$. 36. $1\frac{161}{364}$. 37. $\frac{1474}{2333}$. 38. $\frac{32}{2375}$. 39. $\frac{507}{5400}$.
40. $\frac{27}{50}$. 41. $3\frac{47}{91}$. 42. $36\frac{3}{10}$. 43. 9. 44. $\frac{8}{231}$.
45. $2\frac{14}{33}$. 46. $\frac{8}{15}$. 47. $\frac{4}{13}$. 48. $\frac{1}{21}$. 49. $\frac{11}{12}$.
50. $\frac{92}{135}$. 51. $1\frac{8}{320}$. 52. $\frac{1}{3}$. 53. $\frac{5}{9}$.

Miscellaneous Examples. 89.

4. 1000, 9999.

5. Vertical columns :—1392, 1181, 1508, 1138.
Horizontal rows :—2197, 1245, 1777.
The Grand total is 5219.

17. (i) 427, rem. 112. (ii) 154, rem. 2. (iii) 1525, rem. 338.
Local value (i) 400, (ii) 100, (iii) 1000.

18. (i) 182, rem. 4. (ii) 175, rem. 6. (iii) 393, rem. 111.

19. (i) 324573102. (Multiply the number by 1000 and subtract the number from the product thus obtained.)
(ii) 98523558. (Multiply the number by 1000 and subtract twice the number from the product.)
(iii) 8247525. (Multiply the number by 10000 and subtract 3 times the number from the product.)

20. (i) 1883, rem. 36, (ii) 378, rem. 947.
(iii) 129, rem. 8885. [see Art. 58 (6) i.]

21. (i) 239 in the 1st line and 119 in the 2nd.
(ii) 67441 in the first line and 9634 in the 2nd.

22. 67. 23. Dividend 23, 24 and the divisor 5. 24. No.

25. £366234. 15*s.* ; £7. 9*s.* 1*d.*, rem. 5*s.* 11*d.* ;
100 times, rem. £4. 13*s.*

26. 2 lb. 12 oz. $15\frac{1}{7}\frac{2}{7}$ dr. 27. 4494 half-crowns.

29. Tuesday. 31. 316800 pice ; ₨4950.

32. *A*, £161. 14*s.* 8*d.* ; *B*, £154. 8*s.* 5*d.* ; *C*, £133. 3*s.* 1*d.*

33. *A*, ₨10, *B*, ₨40 and *C*, ₨20. 34. 5. 35. 41.

36. 80 men received 12*a.* each, 160 received 10*a.* each and 240 received $9\frac{1}{3}$*a.* each and total of men = 480. 37. Yes.

41. 343 and 5929. 42. 137, 274, 411, 548, 685, 822, 959.

43. 561 divisor, 943 quotient. 44. Each man may get a maximum wages of 8*a.* 3*p* and in that case the number of men in the 1st factory is 61 and that in the 2nd factory is 65 or each man may get 9*p.* or 11*p.* and in that case the number

of men in the 1st factory are 671 and 549 respectively and there in the 2nd factory are 715 and 585 respectively.

45. 27720. 46. £10. 10s. 48. (i) $\frac{1}{4}$; (ii) $\frac{3}{4}$; (iii) $\frac{5}{8}$.

49. (i) $\frac{3}{8}$, (ii) $\frac{3}{11}$. 50. $193\frac{7}{13}$; $38\frac{9}{11}$ and 56 respectively.

51. The fractions with the lowest common denominator are $\frac{660}{720}$, $\frac{696}{720}$, $\frac{615}{720}$, $\frac{680}{720}$, and $\frac{705}{720}$ respectively, $\frac{47}{48}$, $\frac{29}{30}$, $\frac{17}{18}$, $\frac{11}{12}$, $\frac{41}{48}$ arranged in the descending order of magnitude.

52. $1\frac{7}{15}$. 53. $\frac{19}{21}$. 57. 9. 58. $\frac{68}{187}$. 59. 1. 60. 6000.

61. The fractions are $\frac{13}{12}$; $\frac{7}{12}$ and $\frac{4}{12}$ (*i.e.* $\frac{1}{3}$) respectively.

65. £123. 3s. 9d. 66. Day $12\frac{3}{4}$; night $11\frac{1}{4}$. 67. $\frac{13}{45}$.

68. R72. 69. R8. 5a. 4p. ; R12. 8a. ; R12. 8a.

70. £7 . 2 . $1\frac{1}{2}$. 71. R3. 13a. $8\frac{2}{3}$p. 72. 19s. $11\frac{2}{5}$d.

73. £1 . 13 . $7\frac{3}{4}$. 74. $6\frac{6}{13}$ ft. 75. R122 . 13 . 9.

76. £2. 9s. 77. R1. 6a. 78. $\frac{2}{5}$. 79. $\frac{1}{6}$. 80. $\frac{31}{105}$.

81. $1\frac{1}{383}$. 82. $\frac{2}{27}$. 83. $\frac{1}{2}$p. 84. R785862. 85. R3 . 4 . 6.

86. $\frac{4}{25}$. 87. 72 oz. 88. 12 lb. Avoir. 89. $\frac{1}{40}$.

Examples. 90.

1. ·3. 2. 2·01. 3. ·07. 4. ·104. 5. ·0008. 6. ·000009.

7. 12·04006. 8. ·013005. 9. ·00010001. 10. 100·502.

11. 5·555. 12. 3·456 13. 7·0305. 14. ·01203. 15. 3 units, $\frac{8}{10}$.

16. 3 hundreds 5 units, $\frac{8}{1000}$. 17. $\frac{8}{100}$, $\frac{3}{1000}$, $\frac{5}{10000}$.

18. 5 units, $\frac{8}{1000}$. 19. $\frac{3}{10000}$, $\frac{8}{1000000}$. 20. 3 units, $\frac{5}{100}$.

21. 3 units, 5 hundreds, 8 ten thousands.

22. 3 units, 3 tens, $\frac{5}{10}$, $\frac{5}{100}$, $\frac{9}{1000}$, $\frac{8}{10000}$.

23. 70, ·7 ; 7000, ·007. 24. 290, 2·9 ; 29000, ·029.

25. 2, ·02 ; 200, ·0002. 26. ·2, ·002 ; 20, ·00002.

27. 34, ·34 ; 3400, ·0034. 28. 70·3, ·703 ; 7030, ·00703.

29. 10·03, ·1003 ; 1003, ·001003. 30. ·07, ·0007 ; 7, ·000007.

31. 392, 3·92 ; 39200, ·0392. 32. 234·5, 2·345 ; 23450, ·02345

33. 30000, 300 ; 3000000, 3. 34. 1232, 12·32 ; 123200, ·1232.

35. ·1. 36. 01. 37. 35 ; 70·5 ; 40. 38. ·25 ; ·06 ; ·3.

39. 111·11. 40. 350·709. 41. 1·00246. 42. ·001475.

Examples. 91.

1. $\frac{3}{5}$. 2. $\frac{83}{100}$. 3. $\frac{1}{25}$. 4. $\frac{3}{8}$. 5. $\frac{37}{200}$.

6. $\frac{1}{80}$. 7. $\frac{1}{400}$. 8. $\frac{3}{40}$. 9. $\frac{11}{12}$. 10. $\frac{28}{45}$.

11. $\frac{12509}{3125}$. 12. $\frac{289}{40}$. 13. $\frac{5}{8}$. 14. $\frac{1}{16}$. 15. $\frac{111}{160}$.
16. $\frac{11}{10000}$. 17. $\frac{162001}{5000}$. 18. $\frac{103}{16}$. 19. $\frac{16031}{3200}$. 20. $\frac{1760001}{250000}$.
21. $2\frac{1}{2}$. 22. $7\frac{1}{4}$. 23. $8\frac{1}{8}$. 24. $1\frac{3}{4}$. 25. $2\frac{1}{40}$.
26. $3\frac{1}{20}$. 27. $9\frac{1}{80}$. 28. $6\frac{3}{400}$. 29. $3\frac{1}{20000}$. 30. $7\frac{27}{400}$.
31. $12\frac{9}{40}$. 32. $11\frac{1}{16}$. 33. $2\frac{1}{100000}$. 34. $1\frac{711}{2500}$. 35. $1\frac{289}{4000000}$.
36. $12\frac{165}{2048}$. 37. ·7. 38. ·9. 39. 12. 40. 2·4.
41. 003. 42. ·0725. 43. ·0329. 44. ·09. 45. 1·2345.
46. ·002. 47. 200·03. 48. ·01. 49. ·0125. 50. ·00079.

Examples. 92

1. 20·163. 2. 37·479. 3. 43·31. 4. 80·33. 5. 10·36411.
6. 1. 7. 10. 8. 909 9009. 9. 14·53302. 10. 8.
11. 1000. 12. 417·11157. 13. 669·2981. 14. 657·[illegible]
15. 732·131. 16. R347·23478. 17. £747·0199.
18. 41·4819 min. 19. 332·475 ft. 20. 41·307 in.

Examples. 93

1. 7·084. 2. 1·9711. 3. 1·09922. 4. 199·70334.
5. 62·65. 6. 204·103. 7. ·000275. 8. ·0118766.
9. 7·5554623. 10. 342·817. 11. ·7. 12. 2·063.
13. R7·0001. 14. £·99949. 15. 9·88309.
16. 696·162. 17. 83·9583. 18. 1909·25218.
19. 128·471. 20. By 3·14159. 21. By 2·7183.

Examples. 94.

1. ·8. 2. 1·5. 3. ·03. 4. ·04. 5. ·001.
6. ·012. 7. ·0025. 8. 2·08. 9. ·3. 10. 10.
11. ·0001. 12. ·6.

Examples. 95.

1. 74·52. 2. 36·2. 3. ·13446. 4. 6006.
5. ·001024. 6. ·000324. 7. 28·00028. 8. 2456·8884.
9. 40·804. 10. 30·228. 11. 1·62023. 12. ·0003125.
13. 4264014. 14. 8. 15. ·58. 16. 8. 17. 216·32.
18. 589·12. 19. ·00008. 20. ·0000423. 21. ·00003738028.
22. ·819. 23. ·0001. 24. ·82008. 25. 3·5.
26. 3091·7497. 27. 1209·11. 28. ·096. 29. ·1344620025.

30. 48·6328503. 31. 15·625. 32. ·015625. 33. ·00008.
34. 2·16. 35. 1·331. 36. 1. 37. ·000000125.
38. 2401. 39. ·00081. 40. 27·5. 41. 38·9375.
42. 2·607255. 43. 7·5667. 44. 90·0025. 45. 421·36875.

Examples. 96.

1. 2·1. 2. 1·6. 3. ·9. 4. 1·32. 5. 4·2.
6. ·41. 7. ·05. 8. ·05. 9. 12. 10. 7.
11. 240. 12. 130. 13. 50. 14. 70. 15. 10.
16. 100. 17. ·01. 18. 100. 19. 1000. 20. 2.

Examples. 97.

1. 1·27. 2. 1·372. 3. 1·2. 4. ·00043.
5. 1·99. 6. ·0000479. 7. ·000026375. 8. 10·3.
9. ·000002. 10. 17·125. 11. ·0000000212. 12. ·0528.
13. 1·84782... 14. ·00009... 15. 2·49367... 16. ·00040...
17. ·00002... 18. 3·71428... 19. 1·30586... 20. ·01900...
21. ·00003... 22. 2·0625. 23. ·46625. 24. ·004857...
25. ·236. 26. 12·181818... 27. 2·29375. 28. ·000540...
29. ·659. 30. ·001666... 31. 31·25. 32. 352·25.
33. ·24. 34. 2532. 35. 1200. 36. 640.
37. ·002. 38. ·374. 39. 20. 40. 2010000.
41. 22500. 42. 58070. 43. 3596. 44. 12132.
45. 17500. 46. 1·4. 47. 750000. 48. ·007853.
49. 128·18518... 50. 5·20833... 51. 33·33333... 52. ·03366...
53. ·03320... 54. ·00650... 55. 33057851·23966...
56. 83·33325. 57. 9·58904... 58. ·01216... 59. 350.
60. 752. 61. 2·533333... 62. 6·3125. 63. ·000092...
64. 32714·285714... 65. 5628·571428... 66. 1191·75.
67. 1145·833333... 68. ·018181... 69. ·021428...
70. 377·777777... 71. ·5. 72. ·25. 73. ·75. 74. ·125.
75. ·375. 76. 1·4375. 77. 3·09375. 78. 9·875.
79. 3·28. 80. 2·68. 81. ·33333... 82. ·16666...
83. ·28571... 84. ·27272... 85. ·69230... 86. 1·44444...
87. 7·18181... 88. 8·33333... 89. 10·34482... 90. 58·41666...
91. ·8, ·75, ·6666... 92. ·5, ·4166..., ·2727...

93. ·55, ·5333..., ·525. **94.** ·375, ·3125, ·2187...
95. ·44, ·4333..., ·35. **96.** ·7777..., ·7142..., ·6.
97. ·0216. **98.** ·1125. **99.** 3·135. **100.** ·2.

Examples. 98.

1. ·25; 108·75. **2.** ·03 ; 72·12. **3.** ·004 ; ·4. **4.** ·24 ; 6.
5. ·005 ; 1·6. **6.** ·12 ; 7·2. **7.** ·0001 ; ·08. **8.** ·06 ; 11754·6.
9. ·03 ; 1·8. **10.** ·06 ; 180. **11.** ·05 ; 140. **12.** ·025 ; 1·5.

Examples. 99.

1. ·9. **2.** 8. **3.** ·27. **4.** ·000125. **5.** 13316·875.
6. 1·01. **7.** ·2. **8.** 51·06. **9.** ·0025. **10.** 2·25. **11.** 851·9.

Examples. 100.

1. Non-terminating. **2.** Terminating. **3.** N-T. **4.** T.
5. N-T. **6.** N-T. **7.** N-T. **8.** N-T. **9.** N-T.
10. N-T. **11.** T. **12.** T. **13.** T. **14.** T.
15. N-T. **16.** 3, 6, 7, 9; 11, 12, 13, 14, 15, 17, 18, 19.

Examples. 101.

1. $\cdot\dot{3}$. **2.** $\cdot\dot{2}$. **3.** $\cdot\dot{7}1428\dot{5}$. **4.** $1\cdot1\dot{6}$. **5.** $1\cdot\dot{1}\dot{8}$.
6. $1\cdot\dot{5}38461$. **7.** $\cdot4\dot{6}$. **8.** $1\cdot\dot{0}0\dot{9}$. **9.** $\cdot\dot{2}\dot{7}$. **10.** $3\cdot\dot{2}3076\dot{9}$.
11. $11\cdot\dot{9}0476\dot{1}$. **12.** $\cdot0\dot{4}\dot{5}$. **13.** $3\cdot\dot{7}8000\dot{3}$.
14. $\cdot208\dot{3}$. **15.** $3\cdot8\dot{8}4615\dot{3}$. **16.** $7\cdot\dot{4}8\dot{1}$. **17.** $5\cdot\dot{2}8571\dot{4}$.
18. $10\cdot\dot{0}7692\dot{3}$. **19.** $7\cdot1\dot{3}$. **20.** $9\cdot6\dot{4}2857\dot{1}$. **21.** $1\cdot0\dot{0}19\dot{8}$.
22. $13\cdot91\dot{2}3076\dot{9}$. **23.** $4\cdot803\dot{5}7142\dot{8}$. **24.** $3\cdot45\dot{5}609\dot{7}$. **25.** $5\cdot1\dot{2}$.
26. $\cdot\dot{6}$ **27.** $6\cdot\dot{5}7142\dot{8}$. **28.** $1\cdot7\dot{7}\dot{2}$. **29.** $\cdot\dot{1}2698\dot{4}$.
30. $4\cdot\dot{8}$. **31.** $\cdot1\dot{6}$. **32.** $\cdot0\dot{1}\dot{5}$. **33.** $\cdot0\dot{0}1\dot{5}$. **34.** $\cdot0\dot{0}01\dot{5}$.
35. $\cdot0\dot{0}001\dot{5}$. **36.** $8\cdot10\dot{6}$. **37.** $3\cdot13\dot{7}1428\dot{5}$.
38. $\cdot\dot{0}58823529411764\dot{7}$. **39.** $2\cdot\dot{1}0526315789473684\dot{2}$.
40. $\cdot\dot{0}86956521739130434782\dot{6}$. **41.** $10\cdot\dot{9}\dot{0}$. **42.** $\cdot\dot{0}9990\dot{0}$.
43. $2\cdot\dot{3}0769\dot{2}$. **44.** $2\cdot\dot{8}5714\dot{2}$. **45.** $27\cdot\dot{2}\dot{7}$. **46.** $2\cdot\dot{2}\dot{7}$.
47. $7\cdot\dot{8}69565217391304347826\dot{0}$. **48.** $16\cdot\dot{7}1428\dot{5}$. **49.** $6\cdot\dot{0}7692\dot{3}$.
50. $642\cdot\dot{8}5714\dot{2}$. **51.** $\cdot8\dot{2}$. **52.** $\cdot000\dot{7}\dot{2}$.

Examples. 102.

1. $\cdot234\dot{5}3\dot{4}$. **2.** $\cdot347\dot{6}\dot{7}$. **3.** $\cdot676\dot{7}\dot{6}$. **4.** $\cdot234\dot{5}\dot{4}$.
5. $\cdot001\dot{2}3\dot{1}$. **6.** $\cdot123\dot{4}52\dot{3}$. **7.** $\cdot123\dot{4}12\dot{3}$. **8.** $\cdot1234\dot{5}62\dot{3}$.
9. $\cdot3\dot{4}4444\dot{4}$, $\cdot\dot{2}4242\dot{4}$, $\cdot2\dot{6}7867\dot{8}$.
10. $\cdot1\dot{0}20202020\dot{2}$, $\cdot1\dot{2}3423423\dot{4}$, $\cdot\dot{3}7653765376\dot{5}$.

11. $\cdot 2\dot{3}\dot{3}$, $\cdot 7\dot{8}\dot{7}$. 12. $\cdot 3\dot{4}\dot{5}$, $\cdot 7\dot{6}\dot{7}$, $\cdot 7\dot{2}\dot{2}$. 13. $\cdot 30\dot{7}\dot{7}$, $\cdot 76\dot{7}\dot{6}$.
14. $\cdot 07\dot{6}76\dot{7}$, $\cdot 7\dot{7}777\dot{7}$, $\cdot 00\dot{0}12\dot{3}$. 15. $\cdot 23\dot{8}88\dot{8}$, $\cdot 12\dot{3}41\dot{2}$, $\cdot 02\dot{3}23\dot{2}$.
16. $\cdot \dot{3}33\dot{3}$, $\cdot \dot{7}67\dot{6}$, $\cdot \dot{7}23\dot{0}$. 17. $\cdot 77\dot{7}7777\dot{7}$, $\cdot 12\dot{4}2424\dot{2}$, $\cdot 24\dot{7}2372\dot{3}$.
18. $3\cdot 4\dot{4}4444\dot{4}$, $\cdot 2\dot{6}8686\dot{8}$, $\cdot 1\dot{2}3123\dot{1}$. 19. $3\cdot 40\dot{2}\dot{2}$, $\cdot 78\dot{2}\dot{3}$, $\cdot 31\dot{1}\dot{1}$.
20. $\cdot 4\dot{2}3232\dot{3}$, $\cdot 7\dot{2}7272\dot{7}$, $\cdot 1\dot{2}0320\dot{3}$.

Examples. 103.

1. $\frac{2}{3}$. 2. $\frac{2}{11}$. 3. $\frac{1}{7}$. 4. $\frac{10}{13}$. 5. $\frac{5}{18}$. 6. $\frac{49}{180}$. 7. $\frac{25}{66}$.
8. $\frac{29}{900}$. 9. $\frac{157}{19980}$. 10. $\frac{163}{19800}$. 11. $\frac{527}{495000}$. 12. $\frac{9}{110}$.
13. $3\frac{13}{999}$. 14. $3\frac{214}{495}$. 15. $7\frac{14}{495}$. 16. $31\frac{7}{900}$. 17. $\frac{593}{999}$.
18. $\frac{1}{18}$. 19. $2\frac{13}{21}$. 20. $10\frac{1271}{4950}$ 21. $\frac{37}{80000}$. 22. $\frac{1}{33}$.
23. $\frac{37}{3700}$. 24. $\frac{103}{270}$. 25. $\frac{1}{148}$. 26. $\frac{4}{165}$. 27. $\frac{42}{1111}$.
28. $\frac{757}{3330}$. 29. $\frac{1}{3990}$. 30. $\frac{9001}{990000}$. 31. $3\frac{7}{9990}$. 32. $\frac{29}{1332}$.
33. $\frac{11}{3}$. 34. $\frac{79}{11}$. 35. $\frac{121}{90}$. 36. $\frac{83}{30}$. 37. $\frac{103}{180}$.
38. $\frac{911}{300}$. 39. $\frac{3391}{330}$. 40. $\frac{20023}{4995}$. 41. $\frac{11753}{1650}$. 42. $\frac{1699}{225}$.
43. $\frac{45419}{9990}$. 44. $\frac{139991}{9990}$. 45. $\frac{441}{41}$. 46. $\frac{725}{52}$. 47. $\frac{30007}{3333}$.
48. $\frac{99991}{999}$. 53. $\cdot 1$. 54. $\cdot 368$. 55. $1\cdot 7$. 56. $\cdot 001$.
57. $\cdot 3$ 58. 4. 59. 4. 60. 10.

Examples. 104.

1. $3\cdot 7\dot{8}$. 2. $\cdot 79\dot{3}\dot{2}$. 3. $11\cdot 09\dot{5}$. 4. $6\cdot 48\dot{4}5\dot{3}$. 5. $4\cdot 82\dot{8}\dot{7}$.
6. $1\cdot 0313\dot{0}3290\dot{1}$. 7. $2\cdot 85\dot{7}\dot{9}$. 8. $8\cdot 98$. 9. $10\cdot 345$.
10. $8\cdot 002$. 11. $\cdot 10\dot{2}9183\dot{7}$. 12. $5\cdot 34\dot{8}065\dot{5}$. 13. $19\cdot 17230\dot{1}2\dot{7}$.
14. $\cdot 00\dot{9}3666\dot{3}$. 15. $11\cdot 1\dot{7}93\dot{7}$. 16. $1\cdot 723\dot{0}3271\dot{9}$. 17. 9.
18. $5\cdot 872\dot{0}\dot{3}$. 19. $75\cdot 01\dot{3}546433724 6\dot{5}$. 20. 4.
21. $11\cdot 59\dot{7}794\dot{2}$. 22. $2\cdot 654\dot{2}98744\dot{1}$. 23. $92\cdot 468\dot{7}5455653673\dot{4}$.
24. $3\cdot 75\dot{9}\dot{3}$. 25. $3\cdot 0\dot{7}7704\dot{9}$. 26. $\cdot 39\dot{4}8956066777\dot{8}$.
27. $\cdot \dot{9}1100\dot{1}$. 28. $3\cdot 387\dot{6}\dot{5}$. 29. $2\cdot 472\dot{8}7\dot{6}$. 30. $6\cdot 76\dot{5}2\dot{3}$.
31. $\cdot 89\dot{1}\dot{0}$. 32. $6\cdot 345\dot{8}$. 33. $2\cdot 46\dot{4}4933412260\dot{1}$.
34. $\cdot 4\dot{3}1\dot{2}$. 35. $3\cdot 89\dot{3}3629\dot{5}$. 36. $\cdot 7\dot{1}6160534972\dot{4}$.
37. $3\cdot \dot{6}44225533\dot{1}$. 38. $\cdot 123\dot{0}78\dot{6}$. 39. $771\cdot 0735\dot{1}2758\dot{2}$.
40. $29\cdot 623\dot{0}196\dot{5}$.

Examples. 105.

1. $\cdot 00\dot{2}$. 2. $1\cdot \dot{1}8\dot{5}$. 3. $1\cdot 338342$... 4. $\cdot \dot{1}\dot{6}$.
5. $\cdot 1\dot{0}8641975\dot{3}$. 6. $51\cdot \dot{9}6\dot{2}$. 7. $\cdot \dot{5}$. 8. $106\cdot 5625$.
9. $2335\cdot 882352$... 10. $1\cdot 518141$... 11. $2\cdot 794932$...
12. $\cdot 7\dot{8}5714\dot{2}$. 13. $\cdot 236232$... 14. $\cdot 08\dot{2}8185\dot{3}$. 15. $69\cdot \dot{3}95\dot{7}$.

Examples. 106.

1. $\cdot 375$. 2. $\cdot 0\dot{7}\dot{5}$. 3. 25. 4. 5.
5. $\frac{5720}{11330}$ or $\cdot 5048\ldots$ 6. 350. 7. 6. 8. $\cdot 0348\dot{3}$. 9. 20.
10. $\cdot\dot{3}8095\dot{2}$. 11. $\cdot 590625\ldots$ 12. $11344\cdot\dot{6}$. 13. 8.
14. $\frac{4676}{20998}$ or $\cdot 22269\ldots$ 15. $998\cdot 001$. 16. $32\cdot\dot{2}$.

Examples. 107.

1. 6 pies. 2. 9 pies. 3. $1\frac{1}{2}$ pies. 4. 4 pies.
5. 12 pies. 6. 4*s*. 7. 14*s*. 8. 1*s*.
9. 15*s*. 10. 51*s*. 11. $1372\cdot 8p$. 12. $4\cdot 5p$.
13. $32\frac{1}{2}d$. 14. $3\cdot 6q$. 15. $30p$. 16. $302\cdot 4q$.
17. $1580\cdot 8p$. 18. $93\cdot\dot{3}d$. 19. $1603\cdot 84$ oz. 20. $789\cdot 03$ in.
21. R7. 5*a*. $2\cdot 4p$. 22. £3. 7*s*. 23. R2. 0*a*. $3\cdot 84p$.
24. R2. 6*a*. $7\cdot 5p$. 25. £2. 15*s*. $2\cdot 4d$. 26. 12*a*. $11\cdot 52p$.
27. R34. 4*a*. $3\cdot 84p$. 28. 1 ft. $1\cdot 824$ in. 29. 4 cwt. 2 qr. $20\cdot 16$ lb
30. 12*a*. $8\cdot 5p$. 31. R6. 12*a*. 9*p*. 32. R12. 5*a*. $1\cdot 2p$.
33. R4. 9*a*. $1\cdot 2p$. 34. R15. 1*a*. 6*p*. 35. R2. 12*a*. $10\cdot 464p$.
36. 16*s*. $6\cdot 912d$. 37. 1*s*. $9\cdot 09375d$. 38. $2\cdot 7d$.
39. R2. 8*a*. $6\cdot 7p$. 40. £4. 13*s*. 9*d*. 41. 1*s*. $7\cdot 125d$.
42. 10 md. 13 seers $4\cdot 84$ ch. 43. 1 ton 8 cwt. 1 qr. 8 lb.
44. 2 po. 2 yd. 1 ft. $3\cdot 9375$ in. 45. 22 hr. 19 min. $4\cdot 275$ sec.
46. R7. 12*a*. 47 2*s*. $3\cdot 0\dot{4}\dot{5}d$. 48. R113. 7*a*. 49. R7. 13*a*.
50. £168. 7*s*. $5\cdot\dot{0}\dot{9}d$. 51. R68. 3*a*. $1\cdot 2p$. 52. R15. 2*a*. 4*p*.
53. R3. 14*a*. 54. R17. 1*a*. 8*p*. 55. R4. 15*a*. $3\cdot\dot{8}91p$.
56. £1. 3*s*. $0\frac{3}{4}d$. 57. 12*s*. $1\frac{3}{4}d$. 58. £34. 14*s*. $6\cdot 791\dot{6}d$.
59. $\frac{11}{12}$ of R3. 9*a*., $\cdot 025$ of R100. 10*a*., $\cdot 3\dot{2}$ of R5 8*a*.
60. $3\frac{1}{8}$ of 1*d*., $\cdot 256$ of 1*s*., $\cdot 003\dot{4}$ of £1. 61. R4. 12*a*. $2\cdot\dot{6}p$.
62. $2\cdot\dot{5}9\dot{2}d$. 63. 9[illegible]*d*. 64. 16*s*. 65. R68. 2*a*. $5\cdot 825536p$.
66. 1 ton 17 cwt. 2 qr. 4 lb. 67. 6 md. 68. $\frac{1}{8}d$.

Examples. 108.

1. R$17\cdot 359375$. 2. £$8\cdot 79791\dot{6}$. 3. $4\cdot 4\dot{6}4285\dot{7}$ tons.
4. $1\cdot 420\dot{4}\dot{5}$ mi. 5. $\cdot 771597\dot{2}$ da. 6. £$40\cdot 95$. 7. $7\cdot 75$.
8. $3\cdot 640625$. 9. $5\cdot 338541\dot{6}$. 10. $8\cdot 5$. 11. $1\cdot 18\dot{3}$.
12. $7\cdot 31875$. 13. $1\cdot 375$. 14. $3\cdot 95$. 15. $5\cdot\dot{7}\dot{2}$.

16. 7·23958$\dot{3}$. 17. 1·0042011... 18. 7·03$\dot{8}$. 19. ·659375.
20. ·751875. 21. ·8$\dot{2}$9$\dot{6}$.. 22. ·620543... 23. ·481283...
24. ·578481... 25. 1·06875. 26. 1·04513$\dot{8}$. 27. 1·045918...
28. ·$\dot{4}$78021$\dot{9}$. 29. 15·054375. 30. ·009$\dot{1}$4285$\dot{7}$. 31. ·26041$\dot{6}$.
32. ·$\dot{3}\dot{6}$. 33. ·208$\dot{3}$. 34. ·755$\dot{9}$5238$\dot{0}$. 35. ·0$\dot{1}$. 36. ·171$\dot{2}$9$\dot{6}$.
37. ·3$\dot{5}$. 38. ·0102339... 39. ·0$\dot{3}$8461$\dot{5}$. 40. ·328.

Miscellaneous Examples. 109.

1. The value of 2 is $\frac{2}{100}$; of 7, $\frac{7}{10000}$; of 3, $\frac{3}{100000}$.
2. ·00$\dot{7}\dot{6}$; $\frac{19}{2475}$. 3. ·7$\dot{2}$; $3\frac{443}{600}$. 4. ·000282. 5. ·362.
6. R225. 11*a*. 3*p*. 7. 1 ton 19 cwt. 3 qr. 3 lb. 8. ·50$\dot{6}$.
9. R9000. 10. ·6962. 11. 64·09, 49·3, 1·3. 12. 1520640.
13. 8000 times. 14. 29 times ; 1·576 gallons over.
15. 21 times ; rem. 2·02. 16. ·5. 17. 1508·04*d*. 18. 7·059 tons.
19. 8·571875 lb. 20. £33. 1*s*. 1½*d*. 21. 4·255. 22. ·00584... in
23. 45 yd. 2·1812 ft. 24. 1142 ; ·054 in. 25. ·8095.
26. 81·649296. 27. 448·529990016. 28. 8. 29. 8000.
30. ·15. 31. R2. 9*a*. 8*p*. 32. R81000. 33. 9·5087...
34. 4·5 lb. greater. 35. 15·$\dot{1}$ years. 36. 36 min. 24 sec.
37. 2*s*. 6*d*. 38. R20, R30. 39. *A*, £36 ; *B*, £12 ; *C*, £4. 40. ⅓.

Examples. 110.

1. R·18 ; R·1875. 2. R·31 ; R·3125. 3. R·37 ; R·3750.
4. R·68 ; R·6875. 5. R·93 ; R·9375. 6. R·020 ; R·0208.
7. R·026 ; R·0260. 8. R·041 ; R·0416. 9. R·052 ; R·0520.
10. R·057 ; R·0572. 11. R·335. 12. R·520. 13. R·718.
14. R·796. 15. R·864. 16. R·994. 17. R·6927.
18. R·9010. 19. R·8593. 20. R·8072. 21. R14·2291.

Examples. 111.

1. R·015625. 2. R·0208$\dot{3}$. 3. R·03125. 4. R·036458$\dot{3}$.
5. R·041$\dot{6}$. 6. R·046875. 7. R·05208$\dot{3}$. 8. R·072916.
9. R·1458$\dot{3}$. 10. R·369791$\dot{6}$. 11. R·53125.
12. R·661458$\dot{3}$. 13. R1·92. 14. R·984375.
15. R·994791$\dot{6}$. 16. R4·223958$\dot{3}$. 17. R12·921875

Examples. 112.

1. £·15. 2. £·25. 3. £·3. 4. £·4.
5. £·55. 6. £·6. 7. £·65. 8. £·85.
9. £·225. 10. £·375. 11. £·475. 12. £·525.
13. £·002. 14. £·003. 15. £·005. 16. £·006.
17. £·007. 18. £·009. 19. £·01. 20. £·011.
21. £·015 ; £·016. 22. £·018 ; £·019. 23. £·021 ; £·022.
24. £·023 ; £·024. 25. £·026 ; £·026. 26. £·031 ; £·032.
27. £·038 ; £·039. 28. £·043 ; £·044. 29. £·206 ; £·206.
30. £·316 ; £·316. 31. £·467 ; £·468. 32. £·675 ; £·675.
33. £·780 ; £·780. 34. £·988 ; £·989. 35. £·944 ; £·945.
36. £·997 ; £·998. 37. £5·827 ; £5·827. 38. £20·036 ; £20·036.
39. £16·039 ; £16·04. 40. £18·948 ; £18·949.

Examples. 113.

1. £·6125. 2. £·20625. 3. £·33225. 4. £·64375.
5. £·418375. 6. £·509375. 7. £·821875. 8. £·915625.
9. £·1677083. 10. £·93541$\dot{6}$. 11. £1·319791$\dot{6}$.
12. £10·155208$\dot{3}$. 13. £23·661458$\dot{3}$. 14. £30·467708$\dot{3}$.
15. £15·646875. 16. £1·25417. 17. £2·31771.
18. £4·30729. 19. £5·18125. 20. £8·43438.
21. £10·39375. 22. £12·89063. 23. £20·84792.
24. £25·00313.

Examples. 114.

1. 7*s*. 2¾*d*. 2. 11*s*. 2¾*d*. 3. 2*s*. 7*d*. 4. 9*s*. 2½*d*.
5. £4. 1*s*. 7½*d*. 6. £6. 3*s*. 10½*d*. 7. £10. 4*s*. 4½*d*.
8. £11. 17*s*. 3¾*d*. 9. £1. 18*s*. 10¼*d*. 10. £14. 12*s*. 11¼*d*.
11. £20. 16*s*. 6¼*d*. 12. £2. 14*s*. 8*d*. 13. £3. 9*s*. ½*d*.
14. £5. 3*s*. 3½*d*. 15. £10. 7*s*. 5*d*. 16. £8. 10*s*. 4¼*d*.

Examples. 115.

1. 300018*f*. 2. 456409*f*. 3. 3000599*f*. 4. 4343797*f*.
5. 936907*f*. 6. 959990*f*. 7. 2050458*f*. 8. 7345894*f*.
9. 7485021*f*. 10. 72630*d*. 11. 98170*d*. 12. 136283*d*.

Examples. 116.

1. 76000 ; 76. 2. 30000 ; 30. 3. 510000, 510.
4. 378400. 5. 736000. 6. ·5207.
7. 7·385. 8. 2·010. 9. 2·000.
10. ·03407. 11. ·009063. 12. 5·88.
13. 19·31. 14. ·03. 15. ·01.
16. 73·07. 17. 6·39. 18. 5.
19. 3. 20. 5. 21. 1·864.
22. ·024. 23. 5·401. 24. 6·317.
25. 9·333. 26. ·810. 27. 3·14 ft.
28. ·07 ft. 29. ·01 ft. 30. 3456300 ; 80057000.
31. (1) 4 ; (2) 3·9 ; (3) 3·93. 32. 89.
33. 120. 34. 249. 35. ·009. 36. ·002 ; ·2.
37. ·003 ; ·03. 38. R(5·63 ± ·005). 39. £(6·27 ± ·005).
40. (3·47 ± ·005) in. 41. (50·08 ± ·005) cm.
42. (24·80 ± ·005) lb. 43. (36·63 ± ·005) tons.
44. (i) ·0000015. 45. (i) ·17 mi.
46. R·26. 47. £15·188. 48. 6·29 yd.
49. ·143. 50. 3·14159. 51. 97·574 in. ; ·0015 in.
52. 10·01 correct to the second decimal place.
53. 2·56 and 2·58. 54. (9·95 ± ·015) in.
55. The result 414·97 is liable to an error of ± ·43.
56. Second decimal place ; 4·66 instead of 4·61 ; yes.

Examples. 117.

1. 1·0313. 2. 75·014. 3. ·3949. 4. 1·11.
5. 2·00. 6. 1·50. 7. 1·33. 8. 1·250.
9. 1·167. 10. ·26667. 11. 1·41069. 12. ·23763.
13. ·20273. 14. ·909. 15. ·632. 16. ·182.

Examples. 118.

1. 1·14286. 2. 1·02041. 3. ·85714. 4. ·95238.

Examples. 119.

(The answers given are correct to the required number of decimal places).

1. 7·306. 2. 4·234. 3. ·0077. 4. 1180·5.

5. 189·79. 6. 64·20. 7. 7·705. 8. ·393.
9. ·33800. 10. 23·91754. 11. 66·9. 12. ·1433.
13. ·8499$\dot{8}$. 14. ·0415$\dot{4}$. 15. 10·367. 16. 1·113.
17. 2·5978. 18. 231. 19. 28,632,000,000. 20. 1·618.
21. ·344. 22. 1·230. 23. 12·31. 24. ·118.
25. 193·7205. 26. 530·13237. 27. 8231·61. 28. 1072·48.
29. 1084101·71. 30. ·0066. 31. ·00785.
32. ·281. 33. 23·207. 34. ·9133$\dot{6}$. 35. ·3$\dot{7}$1.

Examples. 120.

1. ·062. 2. 1·892. 3. 20·888.
4. ·140. 5. 2·011. 6. 1·525.

Examples. 121.

1. ½ of R1. 2. ⅛ of R1. 3. ⅓ of R1. 4. ⅙ of R1.
5. $\frac{1}{12}$ of R1. 6. $\frac{1}{32}$ of R1. 7. ½ of £1. 8. ¼ of £1.
9. $\frac{1}{10}$ of £1. 10. ⅕ of £1. 11. ⅛ of £1. 12. $\frac{1}{15}$ of £1.
13. ¼ of 1 md. 14. ⅕ of 1 md. 15. $\frac{1}{10}$ of 1 md. 16. $\frac{1}{16}$ of 1 md.
17. $\frac{1}{80}$ of 1 md. 18. $\frac{1}{320}$ of 1 md. 19. ⅓ of 6*a*. 20. ¼ of 6*a*.
21. ⅛ of 6*a*. 22. $\frac{1}{18}$ of 6*a*. 23. ½ of 1*a*. 6*p*. 24. ⅙ of 1*a*. 6*p*.
25. $\frac{1}{12}$ of 1*a*. 6*p*. 26. $\frac{1}{24}$ of 1*a*. 6*p*. 27. ½ of 2*s*. 6*d*.
28. ⅕ of 2*s*. 6*d*. 29. ⅙ of 2*s*. 6*d*. 30. $\frac{1}{30}$ of 2*s*. 6*d*.

Examples. 122.

1. R1300. 2. £843. 15*s*. 3. R49. 5*a*. 4. £9. 2*s*.
5. R6. 13*a*. 9*p*. 6. £1675. 16*s*. 7. R327. 12*a*.
8. £542. 5*s*. 9. R2523. 9*a*. 10. £4. 11*s*. 8*d*.
11. R400. 12*a*. 6*p*. 12. £42. 15*s*. 13. R226. 9*a*.
14. £341. 9*s*. 6*d*. 15. R453. 14*a*. 6*p*. 16. £8. 11*s*. 5*d*.
17. R747. 5*a*. 3*p*. 18. £1730. 15*s*. 19. R2830. 12*a*. 6*p*.
20. £8002. 7*s*. 4*d*. 21. R4894. 2*a*. 8*p*. 22. £251. 15*s*. 6¾*d*.
23. R7033. 7*a* 3*p*. 24. £45531. 11*s*. 3*d*. 25. R38397. 10*a*. 6*p*.
26. £280508. 13*s*. 7½*d*. 27. R15060. 28. £11714. 18*s*. 11½*d*.
29. R191898 12*a*. 30. £2771. 19*s*. 3*d*. 31. R49514. 3*a*. 9¾*p*.
32. £39247. 4*s*. 2½*d*. 33. R644434. 11*a*. 4½*p*. 34. £78979. 3*s*. 4*d*.
35. R3003. 36. £243. 15*s*. 5$\frac{1}{13}$*d*. 37. R20994. 8*a*. 10½*p*.

38. £838. 3s. $3\frac{11}{15}d$. 39. R34075. 14a. $0\frac{1}{15}p$.
40. £33673. 9s. $10\frac{2}{75}d$. 41. R7661. 9a. $0\frac{1}{5}p$.
42. £5027. 11s. $0\frac{23}{45}d$. 43. R72. 6a. 8p. 44. £236. 4s. $9\frac{3}{4}d$.
45. R1073. 15a. $0\frac{2}{7}p$. 46. £31. 9s. $1\frac{21}{32}d$.

Examples. 123.

1. R25. 10a. $6\frac{1}{3}p$. 2. R44. 0a. 8p. 3. £93. 0s. $5\frac{5}{8}d$.
4. £68. 14s. 9d. 5. £1347. 3s. $3\frac{15}{16}d$. 6. £108. 15s. $3\frac{5}{7}d$.
7. £57. 8s. 8. R38. 2a. $10\frac{3}{4}p$. 9. R100. 7a. $10\frac{11}{125}p$.
10. R67. 7a. 2p. 11. R27. 0a. $2\frac{1}{4}p$. 12. 8s. $1\frac{1}{3}d$.
13. £2. 6s. $1\frac{1}{3}d$. 14. £150. 17s. $6\frac{275}{448}d$. 15. £59. 3s. $1\frac{1}{3}d$.
16. R1835. 11a. $9\frac{22}{35}p$. 17. R180. 2a. 3p. 18. £109. 17s. 3d.
19. R4067. 2a. $4\frac{1}{3}p$. 20. £4279. 6s. $7\frac{1}{2}d$.
21. 1 last 0 ld. 4 qr. 7 bus. $0\frac{9}{16}$ pk. 22. 19 cwt. 3 qr. $9\frac{3}{55}$ lb.
23. £11. 15s. $7\frac{1}{3}d$. 24. 30 tons 6 cwt. 1 qr. 14 lb.
25. 2529 md. 7 seers 8 ch. 26. £26. 15s. $10\frac{1}{5}d$.
27. R265. 9a. $5\frac{1}{4}p$. 28. £14. 15s. $5\frac{1}{4}d$. 29. R45. 4a. 6p.
30. £239. 7s. $9\frac{1}{4}d$. 31. R92. 1a. $5\frac{3}{5}p$. 32. R959. 7a. 7p.
33. £9. 17s. $0\frac{1581}{2560}d$. 34. R4664. 3a. $10\frac{1}{2}p$. 35. R7999. 15a. $9\frac{15}{16}p$.

Examples. 124.

1. 30. 2. 40. 3. 18. 4. 24. 5. 36.
6. 64. 7. 42. 8. 84. 9. 105. 10. 231.
11. 315. 12. 756. 13. 504. 14. 6006. 15.
16. 2. 17. 15. 18. 2. 19. 3600. 20. 900.

Examples. 125.

1. 21. 2. 24. 3. 27. 4. 31. 5. 32. 6. 81.
7. 75. 8. 96. 9. 165. 10. 234. 11. 222. 12. 135.
13. 345. 14. 440. 15. 804. 16. 847. 17. 2222. 18. 1679.
19. 1001. 20. 1234. 21. 9070. 22. 7906.
23. 9876. 24. 4607. 25. 56804. 26. 80047.
27. 15367. 28. 600098. 29. 543200. 30. 123456789.
31. 41. 32. 80. 33. 76. 34. 105. 35. 252. 36. 5.

Examples. 126.

1. ·1. 2. ·3. 3. ·7. 4. 1·1. 5. 1·3. 6. 1·5. 7. ·05.
8. ·14. 9. 3·4. 10. 2·17. 11. 6·25. 12. 9·08. 13. ·08.

14. ·073. 15. 32·9. 16. 2·403. 17. ·0231. 18. ·0045.
19. 15·367. 20. ·897. 21. ·001849. 22. 1·001.
23. 968·8669. 24. 27·6025.... 25. 1·3038... 26. 15·4147...
27. 2·2360... 28. 29·6063... 29. ·3162... 30. ·7071...
31. 4·8062... 32. ·9486... 33. 4·4721... 34. ·1264...
35. ·0252... 36. 2·6457... 37. 8·1240... 38. 3·6055...

Examples. 127.

1. $\frac{1}{2}$. 2. $\frac{2}{3}$. 3. $\frac{1}{7}$. 4. $\frac{5}{8}$. 5. $\frac{8}{11}$. 6. $\frac{9}{13}$.
7. $\frac{12}{17}$. 8. $\frac{31}{100}$. 9. $\frac{14}{75}$. 10. $74\frac{1}{4}$. 11. $5\frac{5}{7}$. 12. $10\frac{1}{20}$.
13. $1\frac{1}{3}$. 14. $1\cdot\dot{6}$. 15. $5\cdot\dot{3}$. 16. $1\cdot8\dot{3}$. 17. $2\cdot8\dot{3}$. 18. $\cdot2\dot{6}$.
19. 1·322... 20. ·845... 21. ·816... 22. ·790...
23. ·763... 24. ·577... 25. ·645... 26. 1·568...
27. ·632... 28. 20·493... 29. $7\frac{3}{7}$.

Examples. 128.

1. 2·236067... 2. 4·123105... 3. 27·602536... 4. ·019598...
5. ·774596... 6. 1·732050... 7. ·264175... 8. ·921954...
9. 87·286883... 10. ·612372... 11. 15·414765... 12. 1·303840...
13. ·845154... 14. 4·882304... 15. ·030708... 16. 3·162277...
17. 6·9280. 18. ·99999. 19. ·41421. 20. 11.
21. 3·645.

Examples. 129.

1. 11. 2. 12. 3. 14. 4. 25. 5. $\frac{6}{7}$. 6. $\frac{4}{5}$.
7. $\frac{8}{3}$ *i.e.*, $2\frac{2}{3}$. 8. $2\frac{1}{4}$. 9. 4. 10. 22. 11. 36.
12. 6·3.

Examples. 130.

1. 180 sq. ft. 2. 320 sq. ft. 3. 117 sq. ft.
4. 64 sq. ft. 106 in. 5. 78 sq. ft. $51\frac{1}{4}$ in.
6. 70 sq. yd. 8 ft. 7. 11 ft. 8. 2 ft. 4 in.
9. 99 yd. 10. 8 ft. 9 in. 11. 1067 sq. ft. 16 in.
12. 14 sq. yd. 18 in. 13. 392. 14. 18.
15. R136. 8*a*. 16. £9. 15*s*. 17. 128 sq. ft.
18. 556 sq. yd. 19. 15888. 20. R160. 15*a*.
21. $78\frac{1}{4}$ sq. yd.; £1. 6*s*. 3*d*. 22. 4800 sq. ft.
23. 15 ft. 24. $21\frac{1}{16}$ sq. ft. 25. $1\frac{1}{2}$ in.
26. $27\frac{3}{5}$ in. 27. R1112. 8*a*. 28. 26 yd. 2 ft.
29. 1024 sq. ft. 30. 300. 31. R666. 12*a*.

Examples. 131.

1. 220 yd. 2. 22 ft. 5 in. 3 280 yd. 4. 50 yd.
5. 5·656... yd. 6. 42·42... ft. 7. 18 ft. 8. 48 yd.
9. 34 yd. 10. 77 yd. 2 ft. 11 in.

Examples. 132.

1. 60 yd. 2. 37 yd. $1\frac{1}{3}$ in. 3. 60 yd. $1\frac{3}{7}$ in.
4. R44. 7*a*. $1\frac{1}{3}$*p*. 5. £33. 1*s*. 3*d*. 6. 648 sq. ft.
7. 495 sq. ft. 8. 88 sq. yd. 6 ft. 9. 288 yd.
10. 96 yd. 11. 211 yd. 12. 176 yd. 2 ft. $1\frac{1}{3}$ in.
13. R46. 4*a*. 14. £17. 15. £5. 0*s*. $4\frac{4}{5}$*d*.
16. $157\frac{1}{2}$ yd. 17. R1. 10*a*. $7\frac{1}{8}$*p*. 18. 4*s*. $8\frac{34}{175}$*d*. 19. $2\frac{1}{3}$ yd.
20. $16\frac{1}{3}$ in. 21. R3499. 3*a*. 6*p*. 22. R114. 12*a*. 23. $5\frac{3}{4}$ ft.
24. R83. 14*a*. $10\frac{2}{7}$*p*. 25. R19. 14*a*. 26. $5\frac{13}{15}$.
27. Width, $18\frac{1}{3}$ ft.; height, $14\frac{1}{2}$ ft. 28. R13. 6*a*.

Examples. 133.

1. 400 cu. ft. 2. $183\frac{3}{8}$ cu. ft. 3. $157\frac{1}{2}$ cu. ft.
4. $8\frac{3}{4}$ cu. ft. 5. $4952\frac{83}{96}$ cu. ft. 6. $42\frac{7}{8}$ cu. ft.
7. $843\frac{3}{4}$ lb. 8. 10080. 9. 3750 times. 10. 48 min.
11. 24. 12. 1 ton 16 cwt. 13. 2800 times. 14. $\cdot 02\dot{7}$.
15. $62\frac{1}{2}$. 16. $4\frac{1}{4}$. 17. 16 ft. 9 in. 18. 2 ft.
19. R1466. 10*a*. 8*p*. 20. $16407\frac{207}{224}$ tons. 21. R170.
22. $133\frac{1}{3}$. 23. 4 in. 24. 3 yd. 25. $256\frac{1}{2}$ lb. 26. 675 lb.
27. 60. 28. $15\cdot 404\dot{1}$ ft. 29. R5520. 30. R276. 5*a*. 3*p*., 31440.

Examples. 134.

1. 6*a*. 2. R2. 8*a*. 3. 4*a*. 4. 2 md. 20 seers. 5. 2 ft.
6. 7*s*. $5\frac{7}{13}$*d*. 7. 5*p*. 8. R35. 12*a*. 9. 5*s*. 10*d*. 10. $36\frac{2}{3}$.
11. $34\frac{11}{16}$ mi. 12. £2. 12*s*. 6*d*. 13. 5*a*. 14. R21.

Examples. 135.

1. 30 da. 2. 60. 3. 270 da. 4. 700 mi. 5. 91.
6. $4\frac{1}{2}$ da. 7. 7. 8. $4\frac{1}{2}$ da. 9. 11.
10. 4 md. 11. 270. 12. 270. 13. 2.

Examples. 136.

1. R2079. 2. R20. 3. R15. 12*a*. 4. R650.
5. £10. 10*s*. 6. R48. 7*a*. 7. 240. 8. 48.
9. £12. 13*s*. 10. 36 lb. 11. R8. 12*a*. 12. R9. 11*a*. $4\frac{1}{2}p$.
13. 20. 14. $8\frac{1}{3}d$. 15. £2. 6*s*. 8*d*. 16. 7*a*. 6*p*.
17. R3937. 8*a*. 18. £816. 16*s*. 19. R17640. 20. R240.
21. R472. 13*a*. $7\frac{3}{8}p$. 22. $7\frac{1}{2}$ da. 23. R31. 14*a*. 24. £1. 8*s*.
25. R168. 26. 11*s*. 3*d*. 27. £3. 12*s*. 28. 14*a*. 8*p*.
29. $94\frac{3}{8}$. 30. $21\frac{1}{3}$ md. 31. R937. 8*a*. 32. $17\frac{9}{11}$ days.
33. $16\frac{1}{14}$. 34. 4618. 35. $117\frac{1}{3}$. 36. $391\frac{1}{3}$ yd.
37. $40\frac{10}{11}$. 38. $12\frac{8}{31}$. 39. $433\frac{1}{3}$. 40. R36.
41. $190\frac{5}{8}$. 42. R7. 6*a*. $6\frac{2}{3}p$. 43. 15.
44. 12. 45. R60. 46. 100 grains. 47. $8\frac{8}{13}$.
48. R390. 49. R1. 50. 1 lb. 8 oz.

Examples. 137.

1. 6. 2. 6. 3. 8. 4. 15. 5. 10.
6. 11 md. 8 seers. 7. 4. 8. 2 hr. 40 min. 9. 12 oz.
10. 9*s*. 11. 48. 12. 180 days. 13. $46\frac{2}{3}$ days.
14. $41\frac{2}{3}$ days. 15. 4. 16. 6 months. 17. $35\frac{5}{14}$.

Examples. 138.

1. 2. 2. 5. 3. 3. 4. 7. 5. 50. 6. $67\frac{1}{2}$.
7. $22\frac{2}{7}$. 8. 32. 9. $10\frac{1}{2}$. 10. 50. 11. $8\frac{1}{4}$. 12. $53\frac{1}{3}$.
13. 75. 14. R4. 15. $23\frac{1}{3}$. 16. 60 yd. 17. $7\frac{1}{2}$ lb.
18. 2*s*. 4*d*. 19. 8. 20. 10*a*. 21. $10\frac{2}{7}$.

Examples. 139.

1. 6. 2. $3\frac{1}{2}$. 3. $11\frac{1}{3}$. 4. $30\frac{6}{85}$. 5. 24.
6. 3. 7. 16. 8. $33\frac{6}{143}$. 9. $26\frac{8}{17}$. 10. 10.
11. R12. 3*a*. 12. R80. 13. 16 days. 14. R118. 12*a*.

Examples. 140.

1. R93. 12*a*. 2. £471. 1*s*. 3. R171. 14*a*. 4. 10*a*.
5. 2*a*. 8*p*. 6. 3*d*. 7. R2967. 3*a*. 8. £4000.
9. R1920. 10. £396. 12*s*. 11. R2880. 12. £180.
13. £722 . 13 . 4. 14. 3*p*. 15. $\frac{71}{80}$. 16. £3200. 17. £3000.

Examples. 141.

1. $\frac{1}{2}$ day. 2. $\frac{3}{4}$ day. 3. $1\frac{1}{3}$ hours. 4. 1 hour.
5. 6 hours. 6. 6 hours. 7. *A* can do in 1 hour.
8. $4\frac{4}{5}$ hr. 9. $1\frac{23}{37}$ da. 10. $1\frac{47}{153}$ hr.
11. 4 da.; *A* $\frac{2}{5}$, *B* $\frac{1}{3}$, *C* $\frac{4}{15}$. 12. 12 da. 13. 1 hr.
14. $7\frac{1}{17}$ min. 15. $4\frac{1}{2}$ hr.
16. *A*, $20\frac{4}{13}$ da.; *B*, $8\frac{16}{31}$; *C*, $7\frac{13}{33}$. 17. $2\frac{74}{153}$ da. 18. 18 da.
19. $13\frac{1}{3}$ da. 20. 120 da. 21. $4\frac{4}{5}$ da. 22. Each in 60 da.
23. $7\frac{23}{31}$. 24. $5\frac{165}{221}$ hr. 25. 12 hr. 26. 16.
27. 2 min. 6 sec. 28. $6\frac{1}{4}$. 29. At 10. 30. 32. 31. 25 da.
32. 76. 33. *A* in 12 days, *B* in 6 days, *C* in 4 days.
34. $12\frac{6}{7}$ min. 35. 4 hr. 36. $56\frac{2}{3}$ da.
37. *A* in 5 days and *B* in 15 days. 38. 4 hours.
39. 28 minutes.

Examples. 142.

1. 2 h. $39\frac{47}{54}$ m. P. M. 2. 2 h. $48\frac{9}{35}$ m. P. M. 3. 9 P. M., Friday.
4. After 112 da. 12 hr. (true time); first, 7 h. $48\frac{3}{4}$ m. P. M.; second, 8 h. $18\frac{3}{4}$ m. P. M. 5. 8 h. $47\frac{481}{577}$ A. M.
6. The slower must be put on $13\frac{1}{13}$ min.; or the faster put back $13\frac{23}{31}$ min. 7. 3 P. M., Dec. 3. 8. 9 min. 9. $\frac{1}{2}$ min.
10. 4 P. M. 11. Tuesday, 4 P. M. 12. $\frac{5}{16}$ min. past 9.
13. Tuesday next, 4 h. $54\frac{1}{6}$ m. P. M. and 4 h. $32\frac{1}{2}$ m. P. M.
14. $10\frac{10}{33}$ min. past 6. 15. $\frac{1}{2}$ sec. 16. 1 h. $50\frac{70}{103}$ m. P. M.
17. On March 13, at the same hour at which it was put right.
18. 5 da. ago, at the same hour; after 235 da. at the same hour.
19. $2\frac{2}{5359}$ min.

Examples. 143.

1. (i) $10\frac{10}{11}$ min. past 2; (ii) $27\frac{3}{11}$ min.; (iii) $43\frac{7}{11}$ min.; (iv) 24 min.; (v) $34\frac{10}{11}$ min. and $52\frac{4}{11}$ min.
2. (i) $16\frac{4}{11}$ min. past 3; (ii) $32\frac{8}{11}$ min.; (iii) $49\frac{1}{11}$ min.; (iv) $3\frac{3}{11}$ min., and $29\frac{5}{11}$ min.; (v) $40\frac{4}{11}$ min., and $57\frac{9}{11}$ min.
3. (i) $32\frac{8}{11}$ min. past 6; (ii) $16\frac{4}{11}$ min., and $49\frac{1}{11}$ min.; (iii) no time; (iv) $19\frac{7}{11}$ min., and $45\frac{9}{11}$ min.; (v) $8\frac{8}{11}$ min., and $56\frac{8}{11}$ min.

4. (i) no time ; (ii) $16\frac{4}{11}$ min., and $49\frac{1}{11}$ min. past 12 ;
(iii) $32\frac{8}{11}$ min. ; (iv) $13\frac{1}{11}$ min., and $52\frac{4}{11}$ min. ;
(v) 24 min., and $41\frac{5}{11}$ min.

5. (i) $38\frac{2}{11}$ min. past 7 ; (ii) $21\frac{9}{11}$ min., and $54\frac{6}{11}$ min. ;
(iii) $5\frac{5}{11}$ min. ; (iv) $25\frac{1}{11}$ min., and $51\frac{3}{11}$ min. ; (v) $14\frac{2}{11}$ min.

6. (i) $54\frac{6}{11}$ min. past 10 ; (ii) $5\frac{5}{11}$ min., and $38\frac{2}{11}$ min. ;
(iii) $21\frac{9}{11}$ min ; (iv) $2\frac{2}{11}$ min., and $41\frac{5}{11}$ min. ;
(v) $13\frac{1}{11}$ min., and $30\frac{6}{11}$ min.

7. $22\frac{2}{319}$ min. past 2. 8. $27\frac{543}{671}$ min. past 5.

9. $41\frac{40}{451}$ past 5. 10. $4\frac{4}{11}$ min. past 12.

11. $\frac{1}{3}$ min.-div. put back. 12. Gains $56\frac{8}{77}$ min.

Examples. 144.

1. In 45 sec. 2. 417 mi. 3. At 7-30 P. M.; 300 mi. from Cal.
4. At 5 h. $34\frac{2}{7}$ m. A. M. ; $257\frac{1}{7}$ mi. from Cal. 5. $4\frac{4}{5}$ sec.
6. 36 sec. 7. $3\frac{1}{2}$ and $1\frac{1}{3}$ mi. per hr. 8. 1 hr. $26\frac{7}{13}$ min.
9. 150 yd. 10. 11 h. $38\frac{4}{7}$ m. A. M. 11. $119\frac{1}{2}$ mi.
12. 12 mi. from Cal. 13. 7 miles.
14. 5 min. $24\frac{10}{11}$ sec. after B starts. 15. 9 h. $9\frac{3}{13}$ m. A. M.
16. 240 mi. 17. 6 mi. and 5 mi. per hr. 18. 7 mi. 19. $11\frac{1}{4}$ mi.
20. 9 hr. $37\frac{8}{11}$ mi. 21. 10 hr. 46 mi. 22. 46.
23. 16 min. 42 sec. 24. $4\frac{1}{2}$ miles per hour.
25. (i) 30 miles ; 40 miles ; (ii) 55 min.
26. Second station from Sirajganj.

Examples. 145.

1. (i) 10 hr. ; (ii) $1\frac{2}{7}$ hr. 2. (i) $7\frac{1}{2}$ hr. ; (ii) $1\frac{7}{8}$ hr. 3. $31\frac{1}{2}$ da.
4. 300 da. ; 300 da. 5. 3 hr. ; 6 hr.

Examples. 146.

1. $5\frac{5}{19}$ min. 2. $79\frac{1}{11}$ yd. 3. 80 yd.
4. 9 min. 36 sec. 5. C can give B 5 points.
6. B wins by 126 yd. 2 ft. and by 1 min. 16 sec.
7. 5. 8. C wins by $60\frac{29}{78}$ yd.
9. A, 1 min. $15\frac{17}{78}$ sec. ; B, 1 min. $20\frac{1}{2}$ sec. ; C, 1 min. 23 sec.
10. A wins by $68\frac{28}{33}$ yd. 11. 9.
12. A in $16\frac{24}{133}$ sec. ; B, $17\frac{3}{5}$ sec. ; C, $18\frac{2}{3}$ sec.

13. 176 yd. 14. 5.

15. *A* in 15 min. 50 sec.; *B* in 16 min. 20 sec.; *C* in 16 min. 40 sec.

16. *C* wins by $\frac{1760}{3721}$ yd.

Examples. 147.

1. $18\frac{1}{80}$. 2. $6a.\ 10\frac{18}{31}p$. 3. 100. 4. R2. $4a.\ 6\frac{6}{7}p$. 5. $19\frac{4}{5}$.
6. 1885. 7. $10\frac{5}{24}$ da. 8. $3\frac{4}{7}$ da. 9. 32. 10. $10a$.

Examples. 148.

1. 10. 2. 45. 3. 264. 4. 75. 5. 8. 6. $10\frac{19}{27}$.
7. R37. $8a$. 8. 30. 9. R24. $4a.\ 10\frac{2}{3}p$. 10. 21 mo.
11. 8. 12. 6. 13. $43\frac{1}{3}$ da. 14. 120.
15. $6\frac{9}{11}$ oz. 16. $1s.\ 4d$. 17. $10s.\ 8d$. 18. $8\frac{1}{3}$.
19. 27. 20. 9. 21. 25. 22. 10.
23. $13\frac{1}{3}$. 24. $4\frac{1}{2}$. 25. $6\frac{2}{3}$ oz. 26. £98. $5s$.
27. 8. 28. 4. 29. 7. 30. 4.
31. 8. 32. $30\frac{2}{3}$. 33. R60. $7a.\ 9\frac{17}{27}p$. 34. 75 ac.
35. $19\frac{1}{4}$ oz. 36. 20. 37. 3.

Examples. 149.

1. R20. 2. R3; R4. 3. 180 gr.; $87\frac{3}{11}$ gr.
4. R13. 5. R5; R20. 6. 48 da.
7. 28 da. 8. $54\frac{6}{11}$ da. 9. 4 da.
10. A man in $7\frac{1}{5}$ hr.; a boy in 18 hr.; a man and a boy in $5\frac{1}{7}$ hr.
11. 6. 12. 10 hr.

Examples. 150.

1. $\frac{5}{7}$. 2. $\frac{3}{4}$. 3. $\frac{6}{11}$. 4. $\frac{4}{5}$. 5. $\frac{14}{25}$. 6. $\frac{25}{36}$.
7. $\frac{2}{3}$. 8. $\frac{3}{4}$. 9. $\frac{4}{5}$. 10. 5 : 4. 11. 1 : 4. 12. 1 : 1.
13. 1 : 4. 14. 7 : 8 is greater. 15. 18 : 29 is greater.
16. 4 : 5 greatest, 2 : 3 least. 17. 7 : 11 greatest, 3 : 7 least.
18. Yes. 19. No. 20. Yes. 21. $10\frac{2}{7}$. 22. $5\frac{1}{3}$.
23. ·0002. 24. 18 lb. 25. £1. $6s.\ 8d$. 26. 45 men.
27. £2. $5s$. 28. 30 hr. 29. $7s$. 30. 14. 31. 39.
32. 7280. 33. $\frac{8}{21}$. 34. $3\frac{1}{4}$. 35. ·06. 36. 25.
37. $4\frac{13}{22}$. 38. $12a.\ 6p$. 39. 17 : 10. 40. 27 : 64.
41. 2 : 1. 42. 192 : 240 : 280 : 315. 43. £2. $5s.\ 8\frac{4}{7}d$.

44. 18500 oz. **45.** 33 ft. **46.** 15 : 16. **47.** £32.
48. 30 gall., 20 gall. **49.** 40 gall. **50.** 16 : 15.

Examples. 151.

1. $\frac{1}{315}$. **2.** 12. **3.** 4. **4.** $\frac{5}{8}$. **5.** 1. **6.** $\frac{5}{7}$.
7. $1\frac{2}{7}$. **8.** $1\frac{1}{3}$. **9.** $\frac{39}{100}$. **10.** 1. **11.** $\frac{148}{511}$. **12.** $\frac{12}{25}$.

Examples. 152.

1. 9. **2.** 2. **3.** 7. **4.** 1. **5.** 1. **6.** 1.
7. ·01. **8.** ·125. **9.** 3·024025. **10.** ·01. **11.** 1. **12.** 1.

Examples. 153.

1. 32 and 21. **2.** *A* gets 25, *B* gets 37 and *C* gets 50.
3. *A*, R11. 4*a*., *B*, R22. 8*a*. and *C*, R67. 8*a*.
4. Boys get R31. 4*a*., women get R31. 4*a*., men get R37. 8*a*.
5. 120 shillings and 81 sixpences.
6. Boys 5 and number of R22. **7.** 20. **8.** *A* 24 and *B* 16.
9. 4460. **10.** 90 miles by steamer and 42 miles by train.
12. At 4 hr. $26\frac{12}{143}$ min. **13.** 176 yd. ; 45 min. **14.** 2 : 1.
15. Cow R30, sheep R3. **16.** 20 mi. per hour ; 100 miles.
17. $2\frac{1}{3}$ mi. **18.** $4\frac{1}{2}$ m. per hour.
19. *A* $9\frac{18}{419}$ mi. per hour ; *B* 9 miles per hour.

Examples. 154.

1. 17. **2.** R204. **3.** $3^2.5.7^2.11^2.13^2$; 5. **4.** $\frac{188}{315}$.
5. R369. 2*a*. 3*p*. **6.** 18. **7.** 9926 and 1020.
8. R65. 15*a*. 6*p*. **9.** 8. **10.** 25. **11.** 1584 lb.
12. (i) £·95 ; (ii) £ 625. **13.** 3020 men ; 2700 women.
14. R151. 2*a*. **15.** 63 times. **16.** $3\frac{3}{11}$. **17.** 123.
18. £7651·972 ; 345894*f*. **19.** 84.
20. R8. 2*a*. 6*p*. to each of 5 ; R4. 1*a*. 3*p*. to each of the others.
21. 13. **22.** 15499939. **23.** $\cdot01\dot{6}$. **24.** $14\frac{8}{37}$.
25. 6. **26.** 720. **27.** $2\frac{2}{3}$ hr. **28.** $13\frac{1}{2}$ gall.
29. 112 sq. yd. 7 ft. **30.** $4\frac{23}{53}$ hr. **31.** 50 years.
32. 10 seers. **34.** $\cdot08\dot{3}$. **35.** R110. 4*a*. ; 1 ft. **36.** 3*a*.

37. $\frac{5210}{2025}$. **38.** If 0 is inserted between 7 and 9 the diff. will be the greatest and if 0 is placed between 1 and 3 the diff will be the least and the numbers are respectively 135709, and 103579 and diff.=32130.

39. The first person gains R1. 11*a*. 6*p*. more.

39a. 455. **40.** 11 min. 45 sec. **41.** $1\frac{3}{8}$ ft. **42.** R5888.

43. 14. **44.** 4. **45.** 40 grains. **46.** 47.

47. 9600. **48.** R2790. 10*a*. ; $\frac{898}{1153}$. **49.** R14.

50. £22. 18*s*. ; £7. 12*s*. 8*d*. **51.** 42 boys ; 20 fruits.

52. $\frac{1}{4}$. **53.** 4 sq. ft. 18 in. **54.** $13\frac{1}{3}$ da. **55.** R3600.

56. R1847. 13*a*. **57.** £1. 7*s*. 1*d*. and 4*d*.

58. 55 min. **59.** R1. 10*a*. 6*p*. ; R1. 9*a*. $7\frac{3}{4}$*p*.

60. $9\frac{3}{4}$ weeks ; £341. 5*s*. **61.** 4 gall. **62.** $3\frac{3}{4}$ hr.

63. 11 P. M **64.** 1 P. M. ; 120 mi. from Cal.

65. 172800. **66.** 39. **67.** £13.

68. After $12\frac{1}{2}$ min. **69.** R2120. **70.** £2. 0*s*. 8*d*.

71. $2\frac{1}{2}$ mi. **72.** 128. **73.** 14 ; 28 ; 42.

74. 42 ft. **75.** $14\frac{2}{7}$ da.

76. Monday, 12 h. 8 m. P. M. ; 11 h. 56 m. A. M. **77.** 66 yd.

78. R2560. **79.** $59\frac{7}{12}$. **80.** 14 yd. ; 7 yd. ; 2 yd. 2 ft.

81. 1-15 o'clock. **82.** 2250. **83.** $1\frac{1}{2}$ mi. ; 2 hr.

84. 8 mi. per hr. **85.** 16 lb. **86.** $2\frac{2}{3}$ hr. **87.** 1008.

88. 72. **89.** 45. **90.** 6 : 5. **91.** $\frac{7}{1850}$. **92.** 5.

93. $55\frac{1}{2}$ sec. **94.** $29\frac{20}{27}$ yd. **95.** 10. **96.** 29 of wine to 41 of water.

97. *A*, R5. 4*a*. ; *B*, R17. 12*a*. ; *C*, R24.

98. $4\frac{10}{11}$ and $16\frac{10}{11}$ min. past 2. **99.** $30\frac{15}{23}$ sec. **100.** 18.

101. A cow, £1 ; a sheep, 5*s*. **102.** 7 : 17. **103.** $\frac{1}{8}$.

104. $7\frac{1}{2}$. **105.** 4 mi. per hr. **106.** *B* wins by $\frac{1}{16}$ yd.

107. 4 da. **108.** 2 oz. **109.** 2 gall.

110. $392\frac{17}{24}$. **111.** 55 min. **112.** 5 min. 15 sec.

113. 152 days. **114.** 4 gall. **115.** £491. 8*s*.

116. *A*, in 36 days ; *B*, 48 ; *C*, $28\frac{4}{5}$. **117.** 20 mi. per hr.

118. 360 sec. **119.** 15. **120.** 2 : 1.

Examples. 155.

1. R20 and R30. **2.** £80 and £140.

3. 12, 24, 36. **4.** 10*a*., R1. 14*a*., R3. 2*a*.

5. 200 md., 500 md., 900 md. 6. 6 sr., 8 sr., 10 sr.
7. R3, R7, R11, R15. 8. £6, £4, £3.

Examples. 156.

1. R1. 9*a*., R3. 2*a*., R4. 11*a*., R6. 4*a*.
2. £8. 2*s*., £6. 15*s*., £2. 14*s*., 18*s*.
3. 7, $4\frac{1}{2}$, $6\frac{2}{3}$, $7\frac{5}{6}$ tons. 4. 75, 100, $112\frac{1}{2}$, 120, 125.
5. £3, £1. 17*s*. 6*d*. 6. R106. 7. £66 ; £71. 10*s*. 8. $100\frac{4}{5}$ lb.
9. 250 lb. 10. 50,000. 11. R40, R30, R20.
12. R12, R16, R8. 13. R240, R80, R40.
14. R18, R6, R8. 15. £3, £6 16. 12, 10, 8.
17. R6, R10, R5. 18. 5*s*. $7\frac{1}{2}$*d*., 7*s*. $3\frac{3}{4}$*d*., 1*s*. $8\frac{1}{4}$*d*., 18*s*. 9*d*.
19. Each man 5*s*., each woman 3*s*., each boy 2*s*. 20. R2. 8*a*.
21. Men 27*s*., women 27*s*., children 11*s*. 3*d*. 22. £18, £12, £9.
23. $\frac{14}{71}$ cwt. 24. 20, 30, 40, 50. 25. 50.
26. 40 rupees, 48 eight-anna pieces, 64 four-anna pieces.
27. Each man R2. 8*a*., each woman R1, each child R$\frac{3}{4}$.
28. $\frac{3}{7}$, $\frac{2}{5}$, $\frac{1}{3}$. 29. R70, R42, R30.
30. The radii are $\frac{1}{\sqrt{3}}$ and $\frac{\sqrt{2}}{\sqrt{3}}$ ft. 31. 180 gr.
32. R25000. 33. 57.

Examples. 157.

1. R70, R100, R150. 2. R780, R520. 3. £1200.
4. R4500, R3000, R3000. 5. R3372. 8*a*. 6. £480, £360, £240.
7. £17. 10*s*., £15, £12. 8. R7, R6, R4. 8*a*.
9. £286, £163. 16*s*. 10. R$483\frac{21}{313}$, R$493\frac{116}{313}$, R$218\frac{146}{313}$.
11. £100. 12. £366. 13. R168. 12*a*. 14. 30.

Examples. 158.

1. In the ratio of 3 to 1. 2. 8 : 5. 3. In the ratio of 9 to 11.
4. 197 : 180. 5. In the ratio of 33 : 2. 6. 1 : 4.
7. $8\frac{1}{3}$ lb. of each. 8. 25 md. at R3, 35 md. at R2. 4*a*.
9. $4\frac{2}{3}$ gall. 10. 20 : 7 ; 5*s*. $1\frac{1}{2}$*d*. 11. In proportion of 3, 3, 2, 2.
12. In proportion of 1, 1, 5. 13. 10 gall.
14. In proportion of 4, 6, 9. 15. In proportion of 52, 78, 51, 68.
16. $\frac{3}{4}$, $\frac{1}{4}$. 17. $\frac{5}{8}$.

Examples. 159.

1. 3. 2. $13\frac{5}{8}$. 3. $7\frac{1}{3}$. 4. 4·34. 5. $11\frac{1}{2}$ years. 6. R4. 8*a*.
7. 125. 8. £2. 9*s*. $4\frac{4}{5}$*d*. 9. 10 stones. 10. R4. 8*a*. $9\frac{3}{5}$*p*.
11. $8\frac{3}{5}$ mi. 12. $10\frac{1}{2}$ st. 13. 14 yr. 14. $8\frac{1}{3}$ st.
15. 11 yr. 16. R5. 11*a*. 17. R7. 18. 63°, 75°.
19. $\frac{1}{3}$. 20. $44\frac{4}{5}$ mi.

Examples. 160.

1. $\frac{1}{4}$. 2. $\frac{2}{3}$. 3. $\frac{1}{2}$. 4. $\frac{3}{4}$. 5. $\frac{1}{25}$.
6. $\frac{1}{25}$. 7. $\frac{1}{20}$. 8. $\frac{1}{100}$. 9. 1. 10. $1\frac{1}{2}$.
11. 60 p. c. 12. 65 p. c. 13. $33\frac{1}{3}$ p. c. 14. 500 p. c.
15. 1 p. c. 16. 100. 17. 8. 18. 16. 19. 3.

Examples. 161.

1. $\frac{1}{8}$. 2. $\frac{1}{3}$. 3. $\frac{1}{400}$. 4. $\frac{3}{800}$. 5. $1\frac{1}{4}$. 6. R35.
7. £10. 10*s*. 8. 3*s*. 9. 1218. 10. $\frac{9}{20}$ sq. in.
11. 4 cwt. 1 qr. 12. R750. 13. 35929.
14. £600. 15. R51 . 15 . $7\frac{1}{3}$. 16. £450.

Examples. 162.

1. $16\frac{2}{3}$ p. c. 2. $3\frac{1}{3}$ p. c. 3. $42\frac{6}{7}$ p. c. 4. 35 p. c.
5. $88\frac{8}{9}$ p. c. 6. $19\frac{13}{33}$ p. c. 7. $468\frac{3}{4}$ p. c. 8. 138 p. c.
9. 50 p. c. 10. 20 p. c. 11. 20 p. c. 12. $57\frac{1}{7}$ p. c.
13. 210 p. c. 14. 50 p. c. 15. $87\frac{41}{57}$ p. c. 16. 24 p. c.
17. $12\frac{1}{2}$ p. c. 18. Nitre 75 p. c., sulphur 10, and charcoal 15.
19. $8\frac{1}{3}$ p. c.

Examples. 163.

1. 220. 2. 1200. 3. 25. 4. 10800. 5. 100.
6. $1296\frac{8}{37}$. 7. R4875. 8. R5000. 9. R78. 2*a*.

Miscellaneous Examples. 164.

1. 10*a*. 2. R8000. 3. $R4545\frac{5}{11}$. 4. 128. 5. $R1531\frac{11}{19}$.
6. 35 p. c. 7. $54\frac{3}{7}$ p. c. 8. $2\frac{244}{547}$ p. c. decrease. 9. 50 lb.
10. $9\frac{1}{11}$ p. c. 11. $18\frac{2}{11}$ p. c. 12. $9\frac{1}{11}$ p. c.
13. 15 lb. per sovereign. 14. 16 per rupee.
15. 5000. 16. 6000. 17. 70 p. c. boys ; 30 p. c. girls.
18. 562432. 19. 11355 nearly. 21. 392. 22. 20 p. c.

Examples. 165.

1. R175. 2. £245. 3. R75$\frac{2}{3}$. 4. R7003. 2*a*.
5. R28000. 6. £914$\frac{2}{7}$. 7. R3000. 8. £101 . 10 . 7$\frac{1}{2}$.
9. R10000. 10. £260. 11. £5154$\frac{6}{7}$; £154$\frac{6}{7}$.

Examples. 166.

1. R20 ; 20% 2. R50 ; 10% 3. R1000 ; 100% 4. 8*a*. ; 20%
5. 6*d*. ; 9$\frac{1}{11}$% 6. R20 ; 20% 7. R50 ; 10% 8. R100 ; 50%
9. 8*a*. ; 20% 10. 6*d*. ; 9$\frac{1}{11}$% 11. 25 p. c.
12. 25 p. c. 13. 25 p. c. 14. 33$\frac{1}{3}$ p. c. 15. 8$\frac{1}{3}$ p. c. loss.
16. 71$\frac{29}{31}$ p. c. gain. 17. 33$\frac{1}{3}$ p. c.

Examples. 167.

1. £210. 2. £410. 3. R13. 12*a*. 4. R462. 8*a*.
5. £6. 9*s*. 6. R80 ; 1*a*. 10$\frac{4}{5}$*p*. 7. 1*s*. 5$\frac{1}{4}$*d*.
8. 12. 9. 9*s*. 4$\frac{1}{2}$*d*. 10. 2*s*. 3$\frac{27}{35}$*d*.

Examples. 168.

1. R3. 3*a*. 2. R500. 3. 8 md.
4. 143 for R12. 5. 13$\frac{1}{3}$ p. c. loss.

Examples. 169.

1. R320. 2. $\frac{4}{7}$*s*. 3. R2 . 0 . 4$\frac{1}{2}$.
4. 8. 5. R2320$\frac{25}{33}$.

Examples. 170.

1. 6 p. c. gain. 2. $\frac{1}{2}$ p. c. gain. 3. 37$\frac{5}{31}$ p. c. gain.
4. 5$\frac{1}{53}$ p. c. gain. 5. 6$\frac{1}{4}$ p. c. gain. 6. 12 p. c. gain.

Examples. 171.

1. 50 p. c. 2. 2$\frac{38}{41}$*d*. 3. Loses 16 p. c. 4. 17 p. c.
5. 26$\frac{6}{19}$ p. c. 6. 16$\frac{2}{3}$ p. c. 7. R150. 8. R22$\frac{2}{9}$. 9. 25 yd.
10. Gains 30$\frac{1}{30}$ p. c. 11. 4 for 3*s*. ; 512. 12. 12$\frac{1}{2}$ p. c.
13. 2$\frac{2}{13}$*a*. 14. 1 lb. to 2 lb. 15. 45 and 36.
16. 16 sr. 17. 18$\frac{1}{3}$ lb. 18. 228 md.
19. £40. 20. 2*a*. 3*p*. 21. 17$\frac{1}{3}$ p. c. ; 2 : 1.
22. R23 . 5 . 4. 23. 19 : 12. 24. 1 : 2. 25. R1. 0*a*. 8*p*.

26. $5\frac{1}{135}$ p. c. 27. 280. 28. $9\frac{3}{8}$ p. c.
29. 50 p. c. for cash payment. 30. 21 p. c. 31. 460.
32. $33\frac{1}{3}$ p. c. 33. 32% gain. 34. $8\frac{2}{3}$% loss.

Examples. 172.

1. R7. 4*a*. 2. R21. 6*a*. 3. R45.
4. R263 . 10 . 9. 5. R11 . 12 . 6. 6. R270.

Examples. 173.

1. R24. 2. £60. 3. R315. 4. £57. 12*s*.
5. R222. 12*a*. 6. £112. 7. R40 . 13 . $8\frac{19}{25}$; R536. 1 . $8\frac{19}{25}$.
8. £32 . 10 . 6 ; £357. 15 . 6. 9. R108 . 5 . $7\frac{17}{25}$; R334 . 1 . $4\frac{17}{25}$.
10. R285. 11. £372. 8*s*. 12. R440 . 8 . $4\frac{4}{5}$.
13. £763 . 13 . $0\frac{3}{25}$. 14. £406 . 4 . $1\frac{257}{320}$. 15. £226 . 1 . 11.

Examples. 174.

1. R33 . 5 . 4. 2. £100. 3. £157. 10*s*.
4. £5 . 12 . 6. 5. R2 . 0 . 3. 6. R3. 14 . 7.

Examples. 175.

1. £2. 8*s*. 2. R20. 4*a*. 3. R4 . 13 . $1\frac{23}{25}$.
4. £5 . 4 . $6\frac{1}{7}$. 5. R6 . 14 . $1\frac{671}{1155}$. 6. R9 . 14 . $7\frac{13}{125}$.

Examples. 176.

1. $2\frac{1}{3}$. 2. $3\frac{1}{4}$. 3. $3\frac{11}{56}$. 4. $3\frac{9}{25}$.
5. 5. 6. $3\frac{1}{3}$. 7. $2\frac{1}{4}$. 8. 6*p*.

Examples. 177.

1. 3 yr. 2. $3\frac{1}{3}$ yr. 3. $3\frac{1}{2}$ yr. 4. 4 yr. 9 mo.
5. 2 yr. 3 mo. 24 da. 6. 97 days. 7. 64 yr.
8. 3 yr. 9. 5 yr. 10. 15th April. 11. 16 mo.

Examples. 178.

1. R750. 2. R4266 . 10 . 8. 3. £170 . 6 . 3. 4. £1050.
5. R400. 6. R730. 7. R800. 8. R150.
9. R265. 10. £33 . 13 . 4. 11. R672 . 4 . 4. 12. £1022 . 4 . 7.

Miscellaneous Examples. 179.

1. $6\frac{1}{4}$. 2. R500. 3. R570. 4. 3 yr. 5. 10 yr.
6. 6 p. c. 7. R9733 . 5 . 4. 8. R400 ; $7\frac{1}{2}$. 9. $8\frac{8}{45}$ yr.
10. R533 . 5 . 4. 11. £190. 12. £30000. 13. R19200. 14. 40 yr.

Examples. 180.

1. R41. 2. R42 . 6 . 11. 3. R38 . 6 . 6. 4. R141 . 2 . 8.
5. £731 . 3 . 3. 6. £343 . 4 . 5. 7. £641. 6. 3. 8. £260 . 9 . 1.
9. R14 . 2 . 2[illegible]. 10. £31 . 18 . 9 to the nearest penny.

Examples. 181.

1. R1102. 8*a*. 2. R327 . 13 . 1. 3. R772 . 4 . 2.
4. R855. 14*a*. 5. R2184 . 13 . 4. 6. R4328 . 7 . 7.
7. R1 . 0 . 10. 8. R11 . 1 . 7. 9. R3278 . 2 . 11.
10. R375 . 3 . 11.

Examples. 182.

1. £90 . 14 . 1 to the nearest penny. 2. £120. 3. £250.
4. £3125. 5. £815 . 3 . 3 to the nearest penny.
6. 15*s*. to the nearest penny. 7. $4\frac{1}{2}$. 8. 25.

Examples. 183.

1. 2 years. 2. 2 years. 3. $2\frac{1}{2}$ years. 4. $2\frac{1}{4}$ years.
5. 2·500 years. 6. 4 years. 7. 2 years. 8. $2\frac{3}{4}$ years.

Miscellaneous Examples. 184.

1. R2·432. 4. R625. 5. R3310. 2*a*. 7. £1499. 19*s*. 11·828*d*.
8. 8 years. 9. 85184. 10. R10000. 11. £1000.
12. R5000.

Examples. 185.

1. R170. 2. R1250. 3. R3562. 8*a*. 4. £1337. 10*s*.
5. £1416 . 13 . 4. 6. £1005 . 6 . 8. 7. £1600.
8. R182. 8*a*. 9. R20000. 10. £1000.

Examples. 186.

1. R5. 4*a*. 2. R80 . 3 . 4. 3. R151. 14*a*.
4. R105 . 6 . 8. 5. £20 . 4 . $8\frac{1}{2}$. 6. £17 . 8 . 2[illegible].

7. £4 . 2 . 4. 8. £1. 15s. 9. R708. 12a.
10. R482 . 14 . 8. 11. R1077 . 8 . 6. 12. £38 . 8 . 9.

Examples. 187.

1. 2 years hence. 2. $3\frac{47}{56}$ yr. 3. $3\frac{1}{2}$ yr. 4. 9 mo.
5. $2\frac{1}{3}$ yr. 6. $4\frac{1}{4}$ yr. 7. 3 mo.

Examples. 188.

1. 20 p. c. 2. $2\frac{1}{2}$ p. c. 3. $5\frac{1}{2}$ p. c. 4. $2\frac{3}{4}$ p. c.
5. 3 p. c. 6. 5 p. c. 7. $3\frac{1}{2}$ p. c.

Examples. 189.

1. R8134. 8a. 2. R53903 . 10 . 8. 3. £574 . 3 . 4.
4. 4 yr. 5. 19 mo. 6. $3\frac{1}{2}$ p. c. 7. R600. 8. R2800.
9. R450 ; $6\frac{1}{4}$ p. c. 10. £200 ; 5 yr. 11. R$134\frac{4}{81}$.
12. £858 . 6 . 8. 13. *B*'s offer. 14. R$60\frac{40}{41}$.
15. 50 : 51 ; R$49\frac{1}{51}$. 16. 20 p. c. 17. R$91\frac{2}{3}$.
18. £$17\frac{23}{41}$. 19. £188 . 13 . $5\frac{1}{4}$. 20. $12\frac{1}{2}$.
21. £375. 10s. 22. R7128 . 11 . 10, to the nearest pie.

Examples. 190.

1. R2 . 8 . 10. 2. £247. 10s. 3. £[illegible]. 8s. 4. R88. 13a.
5. $\frac{18}{1463}a$. 6. $\frac{64}{1825}s$. 7. R9504. 8. R337. 8a.
9. 20 p. c. 10. $16\frac{2}{3}$ p. c. 11. $12\frac{1}{2}$ p. c. 12. $33\frac{1}{3}$ p. c.

Examples. 191.

1. 7 mo. 2. $2\frac{9}{11}$ mo. 3. 8 mo. 4. 6 mo. 5. 9th June.

Examples. 192.

1. R1900. 2. £242 . 16 . 3. 3. R5034. 6a. 4. $93\frac{5}{8}$.
5. $106\frac{3}{8}$. 6. R1500. 7. R4500. 8. £7440. 9. R70.
10. £22. 10s. 11. £1248. 12. £$5177\frac{1}{2}$. 13. £5300.

Examples. 193.

1. R70. 2. R1641 . 5 . 3. 3. £40,000. 4. R270.
5. £921. 4s. 6. R779 . 2 . 8. 7. R6. 4a. 8. 17.
9. 105. 10. £20 increase. 11. R3750 stock ; R11. 4a. increase.

12. R34 decrease. 13. R20 gain. 14. No alteration.
15. £30,500. 16. R22,500. 17. R7200.
18. $93\frac{3}{4}$. 19. $129\frac{7}{8}$. 20. $78\frac{3}{8}$.

Examples. 194.

1. $4\frac{4}{5}$ p. c. 2. $4\frac{62}{187}$ p. c. 3. $3\frac{1}{4}$ p. c. 4. $3\frac{31}{56}$. 5. $72\frac{8}{11}$.
6. $74\frac{7}{8}$. 7. 99. 8. $86\frac{1}{3}$. 9. $4\frac{47}{128}$ p. c. 10. The latter.
11. The former. 12. $\frac{5}{11}$ p. c. 13. R7010. 14. ·£3100.

Miscellaneous Examples. 195.

1. $\frac{40}{171}$ p. c. 2. $2\frac{1}{2}$ p. c. 3. The former. 4. £32. 5*s*.
5. $77\frac{1}{2}$. 6. 190. 7. £1800 ; 2 years sooner.
8. R90,600. 9. R1824. 10. 91. 11. $82\frac{1}{4}$.
12. R840 13. 108 14. £9380. 15. R30,000.
16. £4 16*s*. ; 35 : 34. 17. 2261 : 2260. 18. R20,800.
19. 10. 20. R1000 and R2000. 21. £400, £1200.
22. R3200. 23. $3\frac{31}{48}$ p. c. 24. R100. 25. R2700.
26. £242914$\frac{242}{247}$. 27. £75,000. 28. $100\frac{20}{21}$.

Examples. 196.

1. £275 . 15 . 5. 2. R3705 . 7 . 6. 3. 360. 4. £4 . 17 . 4.
5. R2 . 13 . 4 per dollar. 6. 110. 7. R$1\frac{1}{3}$. 8. 14.
9. R25. 15*a*. 10. Advantageous through London.
11. £12 . 18 . $7\frac{13}{25}$. 12. 1 lose 10 p. c. 13. 8*s*. 2*d*.
14. £83 . 6 . 8. 15. £56. 5*s*. 16. R1 = 1*s*. 8*d* 17. £80.
18. £4687. 10*s*. 19. Gains £11. 5*s*. 20. 1*s*. 4*d*. per rupee.
21. 1 Gold Mohur = ·71... eagle. 22. 1 Napo. = 8·55 rupees.
23. R1. 8*a*. 24. 2*s*. 1*d*. 25. One of the former = 2 of the latter.

Examples. 197.

1. 8125 cm. 2. 5017 cm. 3. 9129 cm. 4. 6507 cm.
5. 9256 cm. 6. 2738 cm. 7. 326 dm. 8. 907·2 dm.
9. 653·7 dm. 10. 478 dm. 11. 659·3 dm. 12. 792·7 dm.
13. 5 m. 6 dm. 14. 6 dm. 5 cm. 2 mm. 15. 2 cm. 3 mm.
16. 35 m. 6 dm. 17. 6 cm. 6 mm. 18. 70 m. 5 dm. 0 cm. 7 mm.
19. (i) 3·015 m. (ii) 5·8 m. (iii) 5 m. (iv) 87·51 m.
(v) 7000 m. (vi) 11·5 m.

20. (i) 38200 cm. (ii) 500100 cm. (iii) 500000000 cm.
(iv) ·3 cm. (v) 80·5 cm. (vi) 239070 cm.

21. (i) 1·075 Km. (ii) ·005078 Km. (iii) ·00318 Km.
(iv) 80 Km. (v) 71 Km. (vi) 56·7808.

22. 6·8 m. 23. 9·06 m. 24. 5·43. 25. 84·2 m.

26. ·638 m. 27. 8·053. 28. 5060·080. 29. 703·009 m.

30. 10·101 m. 31. 6·038 Km. 32. 84·01 Km.

33. ·305 Km. 34. ·100935 Km. 35. 503·4 cm.

36. 10·8 cm. 37. 2345·6 cm. 38. 406080·2 cm.

39. (i) 3·425 m. (ii) 429·708 m. (iii) ·00785. (iv) ·0005 m.

40. (i) 38570 cm. (ii) 10870500 cm. (iii) ·000007 cm. (iv) ·078067 cm.

41. (i) 3000000 mm. (ii) 587000 mm. (iii) ·5 mm. (iv) ·00008 mm.

Examples. 198.

1. (i) 9 m. 9 dm. 7 cm. 6 mm. (ii) 1 Dm. 7 m. 5 dm. 5 mm.
(iii) 6 m. 2 dm. 6 cm. (iv) 2 m. 3 dm. 4 cm. 2 mm.
(v) 2 m. 1 dm. 7 cm. 6 mm.

2. (i) 1 m. 6 dm. (ii) 6 Hm. 1 Dm. 3 m. 1 dm. 8 cm.
(iii) 4 m. 1 dm. 3 cm. ·7 mm.

3. (i) (a) 2 m. 2 dm. 7 cm. 5 mm. ; (b) 2·275 m.
(ii) (a) (6 Dm.) 66 m. 8 dm. 7 cm. ; (b) 66·87 m.
(iii) (a) 2 m. 2 dm. 8 mm. ; (b) 2·208 m.
(iv) (a) 7 m. 2 dm. ; (b) 7·2 m.

4. (i) £14. 9f. 7c. 8m. 5. (i) £2. 9f. 2c. 8m. 6. (i) £17. 3f. 2c. 5m.
(ii) £15. 4f. 4c. 4m. (ii) £3. 0 f. 2c. 8m. (ii) £137. 7f. 8c. 8m.
(iii) £17. 5f. 6c. 4 mils. (iii) £2. 1f. 2c. 7m. (iii) £397. 9c. 6m.
(iv) £20. 3f. 9c. 9m. (iv) £2. 8f. 3c. 5m. (iv) £180. 8f.
(v) £30. 2f. 0 c. 9m. (v) £1. 6f. 2c. 1m. (v) £140. 7f. 9c.

7. 2·305 Km. 8. 3 Km. 4 Dm. 7 cm.

9. 120 Dm. 3 m. 2 dm. 7 cm. 10. 75073050 mm.

11. 30 Km. 7 Hm. 5 m. 8 cm. 6 mm. 12. 23000807 sq. m.

13. 500600·04 sq. Dm. 14. 4 ha. 7 a. 40 ca. 15. 80700 ca.

16. 36 ha. 30 a. 70 ca. 17. 3 cu. m. 12 cu. dm. 35 cu. cm.

18. 5027004000 cu. mm. 19. 40 Kl. 7 Hl. 3 dl. 2 ml.

20. 3 Mg. 4 Hg. 6 gr. 21. 13 fr. 7 dec. 5 cent. 22. 1·1 m.

23. 4125 times. 24. 5 days. 25. 8 Kg. 5 Hg. 26. 3 fr. 75 c.

Examples. 199.

1. 3 a. 5 ca. 2. 200 hectolitres. 3. ·914... metre.
4. ·621... mile. 5. 29·921276 inches. 6. 453·6... grams.
7. 1·2255... grams. 8. 4545·$\dot{4}\dot{5}$ cu. cm. 9. 8 tonneaux 825 kilo.
10. 1056·8... grams. 11. £7. 6*s*. 10½*d*. 12. 2·20 lb.
13. 10 lb. nearly. 14. 5·25 m. 15. 1050 cm.
16. 5 francs. 17. (1) 1000 ; (2) 1000000.
18. 37500 cu. cm. 19. $\dot{4}$ m. 20. $100\dot{0}$ grams.
21. 28·41. 22. 13·6 ; ·8. 23. 1·5 cm.
24. 13 times ; ·61 litres left. 25. 35·2.
26. 5 yd. 2 ft. 1·9051 in. 27. 121·8 ares. 28. 1·234 metres.
29. 453 grams. 30. 637·5 kilo. 31. 1·5 metres. 32. 1·8... francs.
33. £44. 34. 193·75 sq. yd. 35. 5·2 sq. metres.
36. (i) 2·54 cm. ; (ii) 1550 sq. in. ; (iii) 61 cu. in. ; (iv) 28 litres.
37. 16 grains. 38. 933·25 grams. 39. 1360000 grams.
42. 1 hr. 25 min. 20 sec. 43. 0·7716. 44. 3727 litres.

Examples. 199a.

3. 101° at 11 A. M., 99·5° at 11 P. M.
10. R71. 4*a*. nearly ; R93. 11*a*. nearly ; R130. 9*a*. nearly.

Examples. 199b.

1. R30. 3. 16 miles ; 5½ hours.
4. R166. 10*a*. 8*p*. (Hint : Take 1 small vertical div. = R50 income)
5. R62. 8*a*. ; R3000. (Hint : Take tax as R25 per every R800.)
6. About 59 inches ; 2 metres. 7. R124.
8. 2⅔ days ; $\frac{9}{20}$. (Hint : Take 9 small horizontal divs. = 1 day).
9. 12⅘ days. (Hint : Take 5 small horizontal divs. = 1 day).
10. 12·7 mins. 11. 40 yds. 12. 14 yds. 13. 30 mins.

Examples. 200.

1. 30. 2. R94. 3. R70. 4. 3. 5. 3½ mi.
6. R18. 7. 5*s*. 10*d*. 8. Tea 2*s*., coffee 1*s*. per lb.
9. Tea 2*s*., sugar 6*d*. per lb. 10. 2 and 5. 11. £900 and £300.
12. 25, 30 and 35 years. 13. 20, 10 and 15 years.
14. *A* R54, *B* R18, *C*, R8. 15. R150. 16. R342⁶⁄₇.
17. 95, 60. 18. 40, 60. 19. 50, 300. 20. R6. 4*a*.

21. 5*a*. **22.** 1 md. ; 5 md., 3 md. **23.** $40\frac{5}{33}$ mi. per hr.
24. $24\frac{507}{683}$ mi. **25.** 1122 ft. **26.** $15\frac{99}{500}$ min. **27.** $9\frac{27}{50}$ min.
28. 40. **29.** 20. **30.** 70 oz. **31.** 12 gr.
32. 11 oxen, 24 sheep. **33.** £8750. **34.** 20 years'.
35. 3 p. c. **36.** $3\frac{1}{3}$ weeks. **37.** 19. **38.** 15 lb. 10 oz.
39. 44 days ; 2 : 1. **40.** 200 cu. ft. **41.** 3 hours.
42. 3 hours. **43.** 65 gallons ; 13 hours.

Examples for Exercise. 201a.

1. Ten billion, thirty thousand two hundred million, seven hundred and twenty thousand, and twenty-one.
2. 48910. **3.** 47337*q*. **4.** $5^2 . 11^2 . 17$.
5. $\frac{13}{14}$. **6.** 23·0424 ; 22·9596. **7.** R4 . 7 . 9.
8. Three hundred and twenty crores, one lac, three thousand, one hundred and two.
9. 10091401. **10.** R2 . 7 . 3. **11.** 37. **12.** $1\frac{3}{4}$.
13. ·0001596 ; ·0051472. **14.** $1\frac{1}{2}d$. **15.** 18508934.
16. 49110419796. **17.** 17*s*. 9*d*. **18.** 48345. **19.** $5\frac{134}{459}$.
20. ·7045. **21.** $\frac{5}{12}$. **22.** CMXLIV ; 499.
23. 33211521848. **24.** 921. **25.** $1\frac{1}{2}$. **26.** 153·41134.
27. $\cdot 02\dot{6}$. **28.** 15. **29.** 765. **30.** £7. **31.** 32953856 dr.
32. $\frac{14}{54}$. **33.** $\frac{53}{55}$. **34.** ·212. **35.** £1. 3*s*. $5\frac{1}{4}d$. **36.** 13440.
37. R8. 3*a*. $2\frac{2}{3}p$. **38.** $\frac{1}{150}$. **39.** $\frac{1991}{2464}$. **40.** 3·0688259...
41. $\frac{69918}{94675}$. **42.** R3. 12*a*. **43.** 2. **44.** $142114\frac{1}{2}$.
45. $\frac{3}{7}, \frac{3}{8}, \frac{27}{70}$. **46.** $1\frac{7}{9}$. **47.** 4. **48.** $\cdot 0\dot{8}$.
49. 7. **50.** 324. **51.** 11. **52.** $3\frac{5}{6}$. **53.** 700310.
54. 1·2375. **55.** 125·56875*d*. **56.** 1 min. 30 sec.
57. 124727. **58.** R16. 13*a*. 3*p*. **59.** $\frac{3}{11}$.
60. 3 po. 4 yd. 2 ft. 3 in. **61.** 9 ; 7. **62.** 424·8936.
63. 14. **64.** 4536360. **65.** 52084. **66.** R110328. 1*a*. 6*p*
67. $22\frac{3}{8}$. **68.** $\frac{4}{9}$. **69.** $3\frac{3}{8}$. **70.** 4828·04...
71. 5456. **72.** 340 po. 5 yd. 1 in. **73.** R466. 9*a*.
74. $\frac{1}{16}$. **75.** 11*s*. $8\frac{3}{4}d$. **76.** 42·6. **77.** $\cdot\dot{7}0\dot{9}$. **78.** 137
79. R1. 7*a*. 4*p*. **80.** Saturday. **81.** $\frac{913}{5085}$. **82.** $\frac{1}{7}$.
83. $\frac{5}{7}$. **84.** $43\cdot\dot{3}$. **85.** 729. **86.** £125. 5*s*. **87.** $\frac{3}{4}$.
88. 9405. **89.** 120·712. **90.** $7702\frac{1}{3}$ in.

91. 934·12 sq. yd. **92.** R3. 8*a*. **93.** 5 and 7. **94.** $2\frac{3}{8}$.
95. 275 times ; rem. ·003. **96.** ·3125. **97.** 29400000.
98. 9, 6 and 4 times. **99.** 326764. **100.** 4*s*.
101. $1\frac{78}{133}$. **102.** $\frac{11}{18}$. **103.** $\cdot\dot{4}46153\dot{8}$. **104.** $112\cdot\dot{4}$.
105. 21 yd. 2 ft. $2\frac{5}{8}$ in. **106.** 1753. **107.** $\frac{1}{3}$.
108. 12*a*. **109.** $\cdot00000014285\dot{7}$. **110.** ·00759... **111.** ·8.
112. $\frac{6}{37}$. **113.** 1296. **114.** 1386 sq. yd. 3 ft. 96 in.
115. $\frac{2}{3}$. **116.** $3\frac{1}{2}$. **117.** ·8. **118.** R1. 8*a*. 8*p*. **119.** 220.
120. 48. **121.** 2*s*. $8\frac{1}{2}$*d*. **122.** $1\frac{11}{70}$. **123.** 13.
124. $\cdot330\dot{5}$. **125.** $3\cdot\dot{4}6153\dot{8}$. **126.** £182. 7*s*. 2*d*. **127.** 13.
128. Wednesday. **129.** 53. **130.** $4\frac{4}{15}$. **131.** 20.
132. ·0432. **133.** 3840. **134.** 2^2.3.5.7.673 ; 3.7.19.101 ;
G. C. M. 21 ; L. C. M. 2^2.3.5.7.19.101.673.
135. 26. **136.** 1. **137.** $\cdot\dot{0}5752\dot{8}$. **138.** $\frac{1}{175}$.
139. 4288·179204. **140.** 250 times.

Examples for Exercise. 201b.

1. 3210 ; 1023. **2.** 12. **3.** 3. **4.** $16\frac{3}{7}$ min.
5. $46\frac{2}{3}$. **6.** $\frac{1}{8}$. **7.** 5 p. c. **8.** 4, 7.
9. 4725. **10.** 1050 sq. yd. **11.** 6 h. $27\frac{9}{13}$ m. P. M.
12. R46. 4*a*. **13.** 3·2804. **14.** 4. **15.** 137.
16. 1250 ; ·0125 ; ·0000000125. **17.** R5. 10*a*.
18. Monday 8 P. M. ; $\frac{3}{13}$ min. to 6. **19.** 10*s*. ; 6*s*. 8*d*. ; 2*d*.
20. $\frac{67}{135}$. **21.** 17*s*. 6*d*. **22.** 1855. **23.** $8\frac{830}{4345}$.
24. 300 sq. yd. **25.** 8 hr. **26.** £22. 8*s*. **27.** 169 : 191.
28. $9\frac{1}{11}$ p. c. **29.** 999976 ; 100141. **30.** 172.
31. 19251, 18261, 17271, 16281, 15291, 15201, 14211, 13221, 12231, 11241, 10251. **32.** $3\frac{1}{2}$ hr. **33.** R9963. **34.** 11 : 9.
35. $33\frac{1}{3}$. **36.** 5. **37.** 14. **38.** R750. **39.** 7 h. 34 m. P. M.
40. £419 . 19 . 3. **41.** 401 : 544. **42.** 4 yr. **43.** 150.
44. $\frac{8}{11}$. **45.** 1015. **46.** $3\frac{1}{3}$ days. **47.** 9 days. **48.** 16 : 65.
49. £264. 6*s*. 8*d*. **50.** 14. **51.** 80. **52.** R156.
53. 1 hr. **54.** 70. **55.** 83 : 92 ; 92 : 153. **56.** £4800.
57. 429. **58.** $\cdot0\dot{2}$. **59.** $11\frac{1}{4}$ gallons. **60.** 11 P. M. **61.** 12 da.
62. In the first vessel ratio of wine to water is 1729 : 271 ; in the second 271 : 1729. **63.** £4840, £4400, £4000. **64.** 20.

65. 7·875. 66. 453750 tons. 67. 45 days. 68. 440 mi.
69. 7 : 1. 70. 53$\frac{1}{3}$. 71. 200. 72. 120. 73. 26.
74. 17$\frac{1}{2}$ mi. and 9$\frac{1}{2}$ mi. per hour. 75. 1*s*. 10$\frac{1}{2}$*d*.
76. Each man £3. 15*s*. ; each woman £2. 10*s*. ; each child £1. 5*s*.
77. 4 mo. hence. 78. 250. 79. 388 ; 11·32 gr.
80. ₨19. 8*a*. 81. Loses $\frac{1440}{1441}$ min. 82. 20 hr. 16 min.
83. 1200. 84. £276 . 6 . 1. 85. 8184 or 7434.
86. £10. 8*s*. 87. 126. 88. 12 hr.
89. 18$\frac{6}{13}$ days ; on the supposition that they work 13 hours a day.
90. *A* £540, *B* £360, *C* £240. 91. ₨621$\frac{9}{11}$. 92. ₨500.
93. 61000. 94. 24 yd. per min. 95. 9 hr.
96. 113$\frac{1}{533}$ gr. 97. ₨2. 13*a*., ₨4. 8*a*. 98. 10 for a rupee.
99. £1033. 100. 128·5016... 101. $\frac{1}{2}$ in.
102. The clock ought to have been set at 5 h. 30$\frac{2176}{5781}$ m. P. M.
103. 150 mi. 104. *A*, ₨48 ; *B*, ₨40 ; *C*, ₨35. 105. ₨26.
106. 63. 107. $\frac{875}{972}$. 108. 16 ft. 109. 12$\frac{1}{2}$ hr. ; *A*, 4$\frac{1}{2}$; *B*, 5$\frac{1}{2}$.
110. ₨1. 8*a*. 111. 4*a*., 8*a*., ₨1. 8*a*., ₨4. 8*a*., ₨13. 8*a*.
112. ₨24$\frac{7}{10}$. 113. ₨660. 114. ₨24000. 115. 73 times.
116. 5$\frac{1}{7}$ miles from *P*. 117. 10*s*. 118. *A*'s 1$\frac{2}{3}$ oz., *B*'s 2 oz.
119. ₨10. 120. £280. 121. ·0218... 122. 2 ft.
123. 7$\frac{9}{13}$ yd. 124. ₨9. 7*a*. 3*p*. 125. 40. 126. ₨3. 2*a*.
127. 46. 128. ·575. 129. £12. 10*s*. 130. 5$\frac{3}{11}$ days.
131. 4$\frac{1}{6}$ ft. 132. 8 ft. 133. Will lose 7 p. c.
134. 120. 135. 4$\frac{1}{2}$. 136. 15 yd. 137. 1$\frac{23}{28}$ hr.
138. £48. 15*s*. 139. 35, 15, 10, 25. 140. 47$\frac{1}{17}$ p. c.
141. ₨5. 142. 576·0297502224. 143. 50 times.
144. They will run a dead heat. 145. 25. 146. 9.
147. £10. 148. 3 gallons. 149. £30 . 14 . 8$\frac{2}{3}$. 150. 3 ft.
151. 23$\frac{1}{3}$ days. 152. 43 wk. 1 da. 2 hr. 153. 6 ft., 8 ft.
154. Loses 53$\frac{1}{3}$ p. c. 155. 78. 156. £8. 6*s*. 157. 121.
158. 21$\frac{3}{7}$ min. 159. ₨105000. 160. 6$\sqrt{2}$ in., 8$\sqrt{2}$ in. 161. 12$\frac{1}{2}$.
162. 42 gallons. 163. 279 ; $\frac{3}{8}$. 164. Breadth, 6 yd. ; height, 5 yd.
165. 25$\frac{15}{38}$ min. 166. ₨67. 8*a*. 167. 224, 336, 420. 168. 54$\frac{27}{31}$.
169. 72. 170. $\frac{14}{123}$. 171. 4 hr. 172. 21$\frac{3}{5}$ hr.
173. 66 min. 174. *A* must pay 1*s*. 3*d*. and *C* 1*s*. 6*d*. to *B*.

175. £40. **176.** 11. **177.** £2359. 15*s*. $2\frac{89}{187}$*d*. **178.** 1200.
179. 36 mi. and 24 mi. per hour. **180.** $2333283\frac{1}{3}$ francs.
181. £1327. 10*s*. **182.** 12. **183.** $2313\frac{15}{121}$. **184.** ·1115718.
185. $217\frac{1}{2}$ ft. ; 242 times. **186.** $11\frac{1}{3}$. **187.** 3. **188.** £75.
189. The former ; customer loses 2·05 oz. in 1 lb.
190. 58 miles. **191.** 79 wk. 1 da. 22·83 hr. **192.** $263\frac{3}{11}$.
193. $3\frac{21}{47}$ days. **194.** £10. **195.** R300. **196.** 6800 : 7221.
197. 20th Oct. 1855. **198.** 780 ac., 463 ac., 520 ac.
199. 3 times. **200.** 3426 yd. **201.** (i) 40 ; (ii) 60 ; (iii) 80.
202. *A*, R$2476\frac{4}{81}$; *B*, R$1523\frac{77}{81}$. **203.** $99\frac{5}{13}$; £$176\frac{48}{227}$.
204. $1\frac{3}{4}$*d*. **205.** ·125. **206.** 3175. **207.** *C* wins by $\frac{880}{3741}$ yd.
208. 19 ac. **209.** R345. **210.** R54. 14*a*. 4*p*. ; $3\frac{47}{105}$ p. c.
211. 14*s*. $7\frac{3}{4}$*d*. ; 9*d*. **212.** ·346574. **213.** 1 min. $51\frac{1}{3}$ sec.
214. 60 days. **215.** £606. **216.** After 6 months.
217. £15400. **218.** 2*s*. $2\frac{3}{4}$*d*. **219.** $\frac{1}{1285}$.
220. 5000 sq. ft. **221.** $322\frac{2}{3}$ yards. **222.** 29040 ft.
223. R76. **224.** Gains R$25\frac{205}{333}$. **225.** R550. 13*a*. 4*p*.
226. *A*, $1\frac{1}{15}$ of a chest ; *B*, $\frac{9}{10}$; *C*, $\frac{1}{30}$. **227.** 17 in.
228. 22 yd. **229.** $43\frac{17}{25}$. **230.** *A*, R76 ; *B*, R76 ; *C*, R40.
231. R770 ; 1. **232.** 10. **233.** £860. 3*s*. $11\frac{1}{2}$*d*.
234. 6 yd., 6 yd., 3 yd. **235.** After 9 min. **236.** 10.
237. 1 lb. to 2 lb. **238.** 12 ; R1460. **239.** R411. 12*a*.
240. 3*s*. $8\frac{935}{1688}$*d*. **241.** 7 in. each way ; 7776.
242. 2 min. $27\frac{3}{11}$ sec. ; 1080 yd. **243.** 10.
244. Better 20 lb., worse 40 lb. **245.** £500. **246.** 1152.
247. £2364. 12*s*. $4\frac{1}{2}$*d*. **248.** 2 ft. **249.** *B* wins by 88 yd.
250. R18. **251.** 12 bus., 12 bus., 36 bus.
252. R$5\frac{130}{399}$ decrease. **253.** R4. 3*a*. $1\frac{1}{3}$*p*. **254.** $10\frac{13}{27}$. **255.** 250 lb.
257. $13\frac{1}{3}$ days. **258.** 3 : 2, (by volume). **259.** R30780.
260. R276. 1*a*. 6*p*. **261.** 5*a*. $7\frac{1}{2}$*p*. ; R5493. 7*a*. **262.** 72 yd.
263. 1 min. **264.** R$43\frac{8}{11}$. **265.** 80 lb. **266.** R1726 . 10 . 8.
267. 4*a*. 3*p*. gain. **268.** £1123 . 15 . 2. **269.** 59 sq. ft. 21 in.
270. 39 yd. **271.** $10\frac{1}{3}$ da. ; $4\frac{76}{243}$ cu. ft. **272.** 65.
273. R95197. 2*a*. $1\frac{43}{53}$*p*. **274.** 2*s*. 3*d*. **275.** 6*p*. **276.** 12 yd.
277. 3 da. **278.** 27 da. **279.** 2 st. 7 lb. **280.** R16500.

281. $3\frac{3}{11}$ mi. **282.** 64. **283.** 9 cu. ft. $1397\frac{1}{5}$ in. **284.** $1\frac{1}{2}$ hr.
285. 27. **286.** 40 yr. **287.** 92. **288.** 60.
289. £1508. 15*s.* $7\frac{113}{160}$*d.* **290.** 2399 lb. $7\frac{53}{192}$ oz. **291.** 160 yd.
292. $4\frac{4}{135}$*a.* **293.** 1000 yd. **294.** 17000 ; 18067.
295. $3\frac{1}{3}$ pice. **296.** £1668. 7*s.* $1\frac{57}{175}$*d.* **297.** R2 . 9 . 8.
298. $5\frac{4}{7}$ da. **299.** 49. **300.** $26\frac{6}{19}$. **301.** £89 . 8 . 9.
302. 9. **303.** R370. **304.** 161 sq. ft. $21\frac{1}{3}$ in. **305.** 25 mi.
306. 2176. **307.** R1500. **308.** £1350. **309.** R2 . 15 . $7\frac{1}{3}$.
310. 14·5. **311.** 2 in. **312.** 5 min. ; $\frac{1}{4}$ mi. **313.** 68.
314. $10\frac{280}{5601}$ p. c. increase. **315.** 12 p. c. **316.** 4 yd.
317. $933\frac{1}{3}$ lb. **318.** $49\frac{1}{2}$ min. **319.** 18 da. **320.** $33\frac{1}{3}$.
321. R44000 decrease. **322.** R$1705\frac{15}{37}$; £$173\frac{67}{121}$. **323.** 1.
324. $\sqrt[3]{3}$, $\sqrt{2}$, $\frac{4}{3}$. **325.** Faster 99 yd. ; slower 77 yd.
326. £1 . 18 . 4. **327.** Just passes. **328.** R6 . 8 . $11\frac{4}{11}$.
329. $4\frac{16}{21}$. **330.** R2. 3*a.* **331.** £900. **332.** $5\frac{5}{143}$ mi.
333. $2\frac{9}{20}$. **334.** 72 gall. **335.** $4\frac{1}{2}$ p. c. **336.** 1*s.* 8*d.*
337. 9*a.* 3*p.* **338.** 144 ; 1*a.* **339.** 22 mi. **340.** $4\frac{1}{3}$.
341. R$9230\frac{10}{13}$. **342.** £7995. **343.** 1*s.* $9\frac{21}{100}$*d.* **344.** 5*a.* 4*p.*
345. £150. 15*s.* **346.** 80 min. **347.** 2601. **348.** R$1925\frac{4789}{5044}$.
349. £1073. 4*s.* 0·6560736*d.* **350.** R30.

Problems. 202.

1. 942. **2.** 10*d.* **3.** $11\frac{9}{41}$ in. **4.** 1083.
5. 80 guineas, 128 half-crowns. **6.** $\frac{1}{10}$. **7.** 132. **8.** £275.
9. $6\frac{1}{4}$; $156\frac{1}{4}$. **10.** 223·358... ; 20·057... oz. **11.** $34\frac{2}{3}$.
12. The latter. **13.** 3*s.* $11\frac{3}{25}$*d.* **14.** 15*s.* $11\frac{1}{4}$*d.*, 15*s.* 10*d*, 15*s.* 9*d.*
15. 3456, 2304. **16.** 126 qt. **18.** R5, R3, R2. **19.** 2632.
20. 3. **21.** 36. **22.** 424. **23.** 60. **24.** $1\frac{159}{2401}$ oz.
25. 120000. **26.** 11960 sq. yd. 4 ft. 20·41 in.
27. 10 ft. **28.** 10*a.* 8*p.* **29.** 1319·472 ft. **30.** $33\frac{1}{3}$ lb.
31. 8*s.* **32.** R1·025... **33.** 395. **34.** $46\frac{1}{2}$ hr.
35. R1026. **36.** 6 hr. 59 min. 15 sec. **37.** 54 times.
38. 11 days. **39.** *B* ; $\frac{1}{35}$. **40.** 13. **41.** 50. **42.** $\frac{8}{11}$ mi.
43. 1 mile 980 yards ; $13\frac{3}{8}$ miles. **44.** $2\frac{178}{318}$ hr. **45.** £20.
46. $36\frac{2}{3}$ mi. per hr. ; 8 h. 37 m. A. M. **47.** $29\frac{13}{33}$ mi., $15\frac{8}{35}$ mi.
48. $9\frac{23}{330}$ mi. per hr. **49.** $10\frac{2}{3}$ mi. **51.** 115 min.

52. 167 min. 53. 25 mi. 54. 11·30 A. M.
55. In 10 min. more. 56. *A* £162, *B* £118, *C* £104.
57. *A* £1296, *B* £1872, *C* £1044. 58. 30. 59. 3.
60. R720, R1280. 61. $\frac{25}{71}$. 62. 11, 22 and 33 days.
63. Tea 1*s*. $5\frac{1}{2}$*d*., coffee 5*s*. 10*d*. 64. 30 and 18.
65. 8 and 12. 66. 2·20 lb. 67. 10 gall.
68. Man R250, each woman R62. 8*a*., each child R15. 10*a*.
69. R24, R15, R1. 70. 30 yr. and 25 yr. 71. 10 p. c.
72. 1021*d*. 73. R5 . 7 . $1\frac{5}{11}$. 74. 30 times. 75. 12*s*.
76. £5000. 77. $4\frac{1}{2}$ mi. per hr. 78. $42\frac{4}{33}$.
79. 23 carats fine. 80. $4\frac{1}{2}$ mi. per hr. 81. R$1\frac{31}{385}$.
82. 9 gall. 83. 2 : 1. 84. 12 gall. 85. $5\frac{2}{5}$ gall.
86. 1 : 1. 87. 3145 : 6424 : 1431. 88. 2*s*. 4*d*. per stone.
89. R16060. 90. R2. 8*a*. ; 2*a*. 8*p*. 91. R7678. 2*a*. ; 10*a*. 2·85*p*.
92. £7. 15*s*. $7\frac{661}{1152}$*d*. 93. 10, 25, 50, 75. 94. 18*s*.
95. *A* R2400, *B* R900, *C* R240, *D* R60. 96. 28800 ft.
97. 15 rich, 85 poor. 98. $27\frac{269}{315}$ cu. in. 99. R$3923\frac{1}{13}$.
100. R820. 101. 133. 102. $7\frac{1}{32}$; $4\frac{5}{18}$. 103. £818. 8*s*.
104. R12960, R11220. 105. £48000. 106. $6\frac{5}{6}$ p. c.
107. 48 mi. 108. £10. 109. $5\frac{1}{4}$. 110. R10538 . 12 6.
111. R14508, R12090, R12896, R9672. 112. £$19\frac{5}{7}$.
113. R$4942\frac{8}{41}$. 114. 45 mi. per hr. 115. The steamer ; 16 hr.
116. 25. 117. 76. 118. 35 measures. 119. 30 seers.
120. £690. 121. 52. 122. R9180. 123. 1050.
124. 15 ; $\frac{73}{4000}$ cu. in 125. £5. 14*s*. 126. 8400. 127. 144.
128. R5000. 129. 25. 130. $3\frac{5}{9}$ md. 131. $2\frac{2}{3}$ p. c. 132. 2*d*.
133. R1. 9*a*. 134. R450. 135. The second is R20 less.
136. 7. 137. 20 da. 138. R7. 8*a*., R10. 139. R7. 8*a*., R9.
140. 30. 141. R2. 142. 7 and 1. 143. R3. 12*a*.
144. By 3*d*. 145. $56306\frac{1}{4}$; $12577571\frac{353}{1109}$.
146. $1166\frac{2}{3}$, 1169, 1000, 1002. 147. 48 centres, 31 outers.
148. £4. 4*s*., £3, £1. 16*s*. 149. R8. 150. R4500.
151. R49. 152. 89. 153. 11. 154. $\frac{16}{25}$ in.
155. Each man, R2 ; woman, R2 ; boy, 12*a* ; girl, 8*a*.
156. 7 : 40. 157. 10, 15, 20. 158. 75 p. c. and 25 p. c.
159. $6\frac{3}{8}$ cwt. alloy, $2\frac{1}{2}$ cwt. lead, $\frac{1}{8}$ cwt. tin. 160. 8*a*., 6*a*., 4*a*.

161. 1 md. 162. R2. 163. 6*a*. 164. 15 hr.
165. $5\frac{5}{19}$ hr. 166. 4 hr. 20 min., 7 hr. 35 min.
167. R46 . 10 . 8. 168. $3\frac{9}{10}$ mi. 169. 4-25 P. M.
170. 18 mi. per hr. 171. $2\frac{1}{2}$ mi. 172. R46. 8*a*. 173. R37350.
174. 120. 175. $7\frac{41}{121}$ gr. 176. R$5065\frac{15}{16}$ decrease.
177. 140, 168, 160 ; 840. 178. R15. 179. 20. 180. R400.
181. $15\frac{5}{8}$. 182. £412. 10*s*. 183. English navvies ; £4000.
184. £1050. 185. £34 . 8 . $11\frac{17}{125}$. 186. 1199·365234375 sq. yd.
187. $18\frac{8}{135}$. 188. $123\frac{1}{3}$. 189. 2*s*. 8*d*. 190. $33\frac{1}{3}$. 191. 12.
192. 48 of each kind. 193. 90 mi. 194. 60 p. c. 195. 31.
196. 21420. 197. R10022. 4*a*. $6\frac{2}{3}$*p*. 198. £1239. 13*s*. $4\frac{80}{121}$*d*.
199. £353. 11*s*. $7\frac{7}{17}$*d*. 200. 3*s*. $7\frac{1}{3}$*d*. 201. £2000.
202. 11*s*. $7\frac{3}{13}$*d*. 203. 78 p. c. 204. £$4654\frac{6}{11}$, £$135\frac{25}{33}$, £$9\frac{25}{33}$.
205. 320. 206. £3 . 17 . $10\frac{1}{2}$; 5*s*. $1\frac{1}{2}$*d*. 207. 1100 ft. per sec.
208. $1\frac{1}{2}$ mi. and $\frac{3}{8}$ mi. per hr. 209. $2\frac{2}{3}$ days after 2nd starts.
210. £13116 . 6 . 8. 211. 250.
212. 8 min. 4 sec. ; 8 min. 15 sec. ; 8 min. 26 sec. 213. 14 min.
214. R$22\frac{2}{3}$. 215. $9\frac{19}{73}$ min. 216. R200. 217. 15 : 9 : 5.
218. 75 sec. 219. $29\frac{252}{1324}$ mi. per hr. 220. £7 . 11 . 3.

ANSWERS TO CALCUTTA MATRICULATION PAPERS.

1920.

Compulsory Paper.

2. (1) 1 ; (2) $\frac{1}{24}=\cdot04\dot{1}\dot{6}$. 3. (1) £84. 9*s*. $4\frac{1}{2}$*d*. ; (2) R5000.

Additional Paper.

2. (*a*) 9 hours ; (*b*) R80000.

1921.

Compulsory Paper.

1. (*a*) (2) 133 ; (*b*) (2) 360 sec.
2. (1) (i) $\frac{1}{24}$, (ii) $0\cdot04\dot{1}\dot{6}$; (2) 60 feet. 3. (1) R317. 4*a*. $6\frac{3}{4}$*p*. ; (2) $5\frac{1}{3}$ p.c.

Additional Paper.

1. (*a*) 5·37...cm. ; (*c*) 1*s*. $11\frac{1}{4}$*d*.
2. (*a*) 0·0258 ; (*b*) *A* £1139, *B* £439, *C* £275.

1922.

Compulsory Paper.

1. (*a*) (2) 305 ; (*b*) (1) 600, (2) 5040. 2. (1) 75 ; (2) 50.

3. (1) (i) £23. 14*s*. $6\frac{1}{4}$*d*., (ii) R965. 5*a*. 4*p*. ; (2) R750.

Additional Paper.

1. (*a*) 1234 ; (*b*) 0·474. 2. (*a*) 2·718 ; (*b*) $42\frac{6}{7}$ p. c.

1923.

Compulsory Paper.

1. (1) 2695248661200. (2) 305.
   (*b*) (1) 11587 times ; Rem. 4. (2) 94.

2. (1) 1. (2) $\cdot\dot{3}$. 3. (1) R442. 7*a*. $7\frac{1}{2}$*p*. (2) 20 years.

Additional Paper.

1. 1679. (*b*) ·226. 2. ·368. (*b*) $17\frac{1}{11}$ p. c.

1924.

Compulsory Paper.

1. (*a*) (1) 568, (2) 44. (*b*) (1) 11, (2) 120.

2. (1) $\frac{1}{2}$, (2) ·3345. 3. (*a*) (1) R44. 12*a*. (2) 56 yd. ; (*b*) 4 p. c.

Additional Paper.

1. (*a*) 13579 ; (*b*) 5 p. c. 2. (*a*) ·10536 ; (*b*) 49.

1925.

Compulsory Paper.

1. (*a*) (1) 104478661118590 ; (2) 6. (*b*) 12. 2. $\frac{2}{3}$; 560.

3. (*a*) (1) R80. 11*a*. $2\frac{7}{10}$*p*. ; (2) R7300. (*b*) 10 hrs. 42 min.

Additional Paper.

1. (*a*) 1234 ; (*b*) 2·41. 2. (*a*) 2·7183 ; (*b*) $6\frac{1}{2}$ p. c.

1926.

Compulsory Paper.

1. (*a*) (1) R959488 : (2) 472787. (*b*) (1) 22154 ; (2) 37.

2. (*a*) (1) 1 ; (2) $\frac{89}{99}$. (*b*) 11 days.

3. (1) R1632. 0*a*. 8*p*. (2) He loses $2\frac{2}{25}$ p. c ; after 8 min.

Additional Paper.

1. 18 ; ·99999. 2. ·20273.

1927.

Compulsory Paper.

1. 84 ; 5. 2. (1) $\frac{1}{4}$, (2) ·01236 ; 14 weeks.

3. (1) R545. 5*a*. ; (2) R77$\frac{11}{17}$, 6$\frac{1}{4}$ p. c.

Additional Paper.

1. 1·41069. 2. 124 ; 2·1875 × $\sqrt{7}$.

1928.

Compulsory Paper.

1. (1) 17548, 14911 ; (2) 32. 2. (1) 1 ; (2) $\cdot\dot{1}4285\dot{7}$; 15 days.

3. (1) R81. 0*a*. 9*p*. . (2) 4 yrs. ; 30 hrs.

Additional Paper.

1. 1·413 ; 1·0000. 2. 1·411.

1929.

Compulsory Paper.

1. (i) 523 ; (ii) 12 ; *A* 18, *B* 57, *C* 33.

2. (i) 1 ; (ii) $\cdot\dot{1}4\dot{8}$; 401 : 544.

3. (i) R217. 10*a*. 8*p*. ; (ii) 44·$\dot{4}$ p. c. ; 38 ac.

Additional Paper.

1. ·53452 ; ·00428. 2. ·2554 ; ·001.

1930.

Compulsory Paper.

1. (i) 11 ; (ii) At 10 P. M. on the day after next ; R225.

2. (i) R9. 12*a*. 6*p*. ; 1. (ii) 3 days.

3. (i) £192. 15*s*. 10*d*. (ii) 19800 ; 4*s*. 6*d*.

Additional Paper.

1. 2·6180 ; ·53203. 2. 1·0750 ; 11$\frac{1}{4}$ miles.

1931.

Compulsory Paper.

1. (i) 48 ; 2472. (ii) 39 ; 1121.

2. (i) 5 ; R51. 14*a*. 3*p*. (ii) R48. 14*a*. 10*p*.

3. (i) 4 p. c. (ii) 6 days.

Additional Paper.

1. ·1057 ; 1·0560. 2. R300. 12*a*. 6*p*. ; £105.

1932.

Compulsory Paper.

1. (i) $\frac{1}{3}$. (ii) R36 ; 18720.
2. (i) R12. 12*a*. $\frac{1}{3}p$. (ii) 355 years after 1921 ; R500.
3. (i) ·005208$\dot{3}$. (ii) R75 ; $3\frac{1}{2}$ ch. per seer.

Additional Paper.

1. (i) ·02543. (ii) 1·1090 ; 190. 2. (i) R990. (ii) 40 miles away from *A* ; *A* = R160, *B* = R240, *C* = R600.

1933.

1. 0 ; 1. 2. 504 ; 98280.
3. (i) ·3647. (ii) 16 : 15 ; (i) R405. 6*a*. $7\frac{1}{3}p$. (ii) 24000.

Additional Paper.

1. (i) ·173... ; 12*a*. (ii) R4. 9*a*.
2. (i) 1·1540... ; 11 miles 925 yds. (ii) 12 hours.
3. 8*a*. $1\frac{6}{25}p$.

1934.

Compulsory Paper.

1. R2558 ; 10017. 2. 1 ; 14.
3. R5215. 11*a*. $2\frac{13}{16}p$. ; (i) 1818 ; (ii) 6759.
4. R800, $7\frac{1}{2}$ p. c. ; 8 days.

Additional Paper.

1. (i) 9·798 ; 3733 yd. 1 ft. ; (ii) 1·2796.
2. (i) 2500 Kg. : R546. 14*a*., R492. 3*a*., R710. 15*a*. (ii) R80.
3. (i) After 2 hr. $51\frac{9}{11}$ min. ; 480 yd. from the starting place along the 1st man's course and 504 yd. along that of the 2nd man. (ii) After 5 hr. $43\frac{7}{11}$ min. ; 960 yd. off the starting place along the 1st man's course and 24 yd. beyond the starting place along the 2nd man's course.

ANSWERS TO PUNJAB ENTRANCE PAPERS.

1900.

1. 2000·301. 2. R7352. 15*a*. nearly. 3. 10 yd. ; 22 yd.

1901.

1. £25. 0*s*. 4*d*. 2. 1·5789 : 1 or 1 : ·6$\dot{3}$. 3. 12 per cent. 4. 79 ; 49.

1902.

1. 3.5.7.11.13.37. 2. 5760. 3. R4800.
4. R2. 7*a*. 9*p*. 5. 15 per cent.

1903.

1. Prime ; 28721 × 373. 2. 2$\frac{1}{3}$ hr. 3. R355. 12*a*. 11$\frac{13}{33}$*p*.
4. *A*, R105$\frac{33}{535}$; *B*, R131$\frac{797}{2528}$; *C*, R109$\frac{2171}{5056}$. 5. R2$\frac{131}{200}$.

1904.

1. $2^{10}.3^2.7^2$; $2^5.3.7$; 5.7.9. 2. ·014 ; 14789·35.
3. R223 . 14 . 3. 4. R256 . 3 . 2$\frac{5}{8}$. 5. $\frac{41}{50}$.

1905.

1. (1) 2·43$\dot{2}$4$\dot{3}$; (2) 90017. 2. Man, R167. 1*a*. ; woman, R97. 7*a*. 3*p*. ; boy, R69. 9*a*. 9*p*. ; girl, R41. 12*a*. 3*p*.
3. R114. 13*a*. 6*p*. 4. R812. 8*a*. 5. (i) Sunday ; (ii) 24 times.

1907.

1. 53. 2. £5115 . 2 . 4$\frac{1}{8}$. 3. R14285$\frac{5}{7}$.
4. R126720. 5. ·4$\dot{1}$9024$\dot{2}$.

1908.

1. 192$\frac{1}{2}$ ft. 2. ·00000292... 3. 9$\frac{1}{11}$ p. c. 4. 6·125 p. c.

1909.

1. $\frac{14}{407}$; ·04375. 2. R1. 8*a*. 1$\frac{61}{155}$*p*. 3. 16$\frac{56}{155}$ days.
4. R1161. 6*a*. ; R492. 10*a*.

1910.

1. 2525. 2. $\frac{10}{316}$; ·0003125. 3. R1689. 11*a*. 5$\frac{1}{3}$*p*.
4. P. W. = £1002. 15*s*. 5$\frac{25}{31}$*d*. ; D = £33. 8*s*. 6$\frac{6}{31}$*d*. 5. R736.

1911.

2. 32·907... ; $\frac{25}{588}$. 3. £11870. 3*s*. 4·4925*d*.
4. 2 months hence. 5. 5$\frac{2}{3}$ days.

1912.

1. (*b*) 89. 2. 1 ; $\sqrt{17}$. 3. 8 days. 4. 2$\frac{8}{11}$ p. c. ; R12500.
5. 13$\frac{1}{11}$ min. past 2.

1913.

1. 109, 113 ; 544. 2. (*b*) 1·4162. 3. R3 ; R2. 10*a*. ; R1. 6*a*.
4. R2269. 15*a*. 11*p*. 5. 150·8.

1921.

1. 188121 ; $8\frac{3}{194}$. 2. R45. 5*a*. 4*p*. ; $\frac{1}{15}$.
3. R6. 2*a*. ; R10. 2*a*. 8*p*. 4. $4\frac{22}{51}$ years. 5. 3 ; 6*s*. 4*d*.

1922.

1. 192 ; $6\frac{3}{55}$. 2. £3. 9*s*. $5\frac{3}{7}$*d*. ; $\frac{5833}{4180}$*d*. 3. £11. 18*s*. 5*d*.
4. $7\frac{1}{2}$ p. c. ; $12\frac{4}{5}$ oranges.
5. *A*, R3762 ; *B*, R2280 ; *C*, R6498.

1923.

1. ·6606. 2. £39. 13*s*. 4*d*. ; $\frac{17}{21}$. 3. R623. 3*a*. 8*p*.
4. 9 p. c. ; $\frac{9}{10}$ sr. 5. 1456849 sq. yd. ; R227632. 10*a*. 6*p*.

ANSWERS TO ALLAHABAD ENTRANCE PAPERS.

1900.

1. $\frac{1125}{1875552}$; ·067. 2. ·2995670̇ ; $\frac{1}{40}$ 3. R197. 12*a*. $0\frac{3}{4}$*p*. ; 3·165
4. R3200 ; R3889. 9*a*. 11·04*p*. 5. R289. 5*a*. nearly.

1901.

1. $\frac{143}{140}$ ft. ; £8. 1*s*. 2. $\frac{1}{32}$; ·03391. 3. 160 yd.
4. R625. 5. R16800.

1902.

1. ·000279 ; 25234·713. 2. 44. 3. R3285.
4. *A*, 48 ; *B*, 84. 5. $7\frac{7}{30}$ days. 6. $8\frac{1}{2}$ years.

1903.

1. (*a*) 34 ; $2\frac{1}{3}$ in. (*b*) ·5. 2. (*a*) ·0051472 ; ·9375. (*b*) 2·5.
3. 30 days. 4. 10-15 P. M. 5. $4\frac{1}{2}$ p. c.

1904.

1. (*a*) $11\frac{35}{41}$; (*b*) 000125. 2. (*a*) £1714 . 15 . 3 ; (*b*) 3·1624.
3. £75. 4. R1157. 10*a*. 5. £19425.

1905.

1. (*a*) $8\frac{44}{117}$; (*b*) ·3536. 2. (*a*) 4 cwt. 1 qr. 9·89 lb. ; ·0216...
(*a*) £6 . 5 . $4\frac{1}{2}$. 3. $15\frac{1}{2}$ p. c. 4. $R9\frac{91}{101}$. 5. £$2\frac{1}{2}$ loss.

1906.

1. 42. 2. $16\frac{1}{12}s$. 3. 25 yr. 4. 3981.

1907.

1. Yes. 3. 94·91. 4. 2·236...

1908.

1. 324513. 2. $\frac{1}{2}$. 3. See Art. 108.

1909.

1. ·355431. 2 44 p. c. 3. 1*s*. 4*d*. 4. R5. 8*a*. increase.

ANSWERS TO ALLAHABAD MATRICULATION PAPERS.

1910.

1. 99,000,099,099,099 ; 845·9. 2. 315 ; 8735. 3. £700.

1911.

1. 9,000,089,009,010 ; 1. 2. 1·609344 Km. 3. 34021.

1912.

1. Three billion, two hundred and three thousand and six hundred million, and four hundred thousand.

2. (1) $1\frac{26}{175}$; (2) £1. 0*s*. $8\frac{1}{4}d$. 3. $85\frac{270}{873}$.

1913.

1. 1[illegible]. 2. 11. 3. R2880. 4. £81.

1914.

1. (1) $\frac{80}{281}$. (2) 3*s*. $3\frac{5}{7}d$. ; ·16. 2. 6·015. 3. 55*s*. $6\frac{3}{5}d$. 4. 54.

1915.

1. (1) 55·1. (2) ·316 2. 14364. 3. 36 ft. 4. 8 p. c.

1916.

1. 2 tons 2 cwt. 2 qr. 2 lb. 2. 15 ft. 3. 8*s*. 4*d*.

1917.

1. (1) $\frac{44}{47}$; (2) 0·027. 2. (1) 0·25 ; (2) 1·4142....

3. $\frac{1}{2}$ quart. 4. R781. 4*a*.

1918.

1. (*a*) 99 960 ; (*b*) R369$\frac{163}{172}$. 3. 288 ; 720.
3. August 17, 2 h. 21$\frac{8}{17}$ m. A. M. right time.

1919.

1. (*a*) 11 and 13. (*b*) $1\frac{1}{16}$; 0·2. 4. R635. 4*a*. 3. The former.

1921.

1. (*a*) 389 ; (*b*) 350. 2. 5[illegible] ft. 3. R708. 12*a*.

1922.

1. (*a*) ·0045 ; (*b*) 1·001. 2. R198. 2*a*. $9\frac{7}{12}$*p*. 3. $37\frac{7}{84}$ p. c.
4. 20 p. c.

1923.

1. (*a*) $39\frac{459}{1760}$ mi. ; (*b*) ·00005. 2. 16 ft., 12 ft., 12 ft. 3. 16$\frac{1}{2}$.

ANSWERS TO PATNA MATRICULATION PAPERS.

1918.

Compulsory Paper.

1. (*a*) 496783793655 ; 5000. (*b*) 9163 ; 125.
2. (*a*) 1 ; £906. 7*s*. 11*d*. (*b*) ·043225, ·011875 ; 15*s*. 9*d*, ·1575.
3. (*a*) R500 ; 25 years. (*b*) 10 days.

Additional Paper.

1. ·001333. 2. 453 litres nearly. 3. 4055. 4. 176400.

1919.

Compulsory Paper.

1. (*a*) 604356745368450 ; 939663, 100203. (*b*) 875 ; 9*d*.
2. (*a*) 1 ; ·2083. (*b*) £2. 15*s*. 11$\frac{1}{4}$*d*, ; £183. 12*s*. 4*d*.
3. (*a*) £315. 10*s*. 8*d*. ; 9*s*. 0 8*d*. (*b*) Half-an-hour ; 5 hours.

Additional Paper.

1. ·0499976 ; £220. 2. 8 Km. 47 m. ; ·6321.

1920.

Compulsory Paper.

3. [illegible]. 4. 1$\frac{1}{12}$. 5. ·0203125. 6. £1571. 9*s*. 0$\frac{3}{4}$*d*. 7. £3200.

Additional paper.

1. (*a*) ·0006 ; (*b*) 142 yards.

1921.

2. $\frac{43}{1520}$. 3. 31·371539 ; 2312. 4. £2. 15*s*. $11\frac{1}{4}$*d*. ; £10. 7*s*. 6*d*.
5. $4\frac{5}{8}$ p. c. ; $3703\frac{19}{27}$ guineas.

1922.

2. 47. 3. R420. 4. R3289. 6*a*. $9\frac{1}{13}$*p*. ; R472.
5. 51·98 ; 21 years 4 months.

1923.

1. 709. 2. 257040. 4. £37·97925 ; R4527. 3*a*. $2\frac{1}{3}$*p*.
5. 2 times ; R45 each.

1924.

2. 997920. 3. R2650. 5*a*. $3\frac{1}{2}$*p*. ; 9·45. 4. 55 men.
5. 967·5 francs ; 23 p. c.

1925.

Compulsory Paper.

1. 7256 ; 48900. 2. 42 ; 8177. 3. 152 yds. ; 1.
4. R550, 5 p. c. ; $10\frac{13}{27}$. 5. R1725, R769. 8*a*.

Additional Paper.

1. 321602520470 ; 2500. 2. (*a*) 56 yds. ; (*b*) ·542708$\dot{3}$.
3. 210 mi. ; $1\frac{215}{495}$. 4. 3 yrs. 5. 5·403 ; 10 ft.

1926.

Compulsory Paper.

1. 100359 ; 10 ft. 2. Prime number ; ·03396875.
3. R103. 8*a*. $6\frac{1}{4}$*p*. ; R50. 4. R384 ; 52 004.
5. R1120 ; 6 min.

Supplementary Paper.

1. 1243 ; R7 and R4. 8*a*. 2. 115·23, ·00568 ; $\frac{25}{891}$.
3. R2056. 2*a*. 8*p*. ; $1\frac{1}{21}$. 4. R10000 ; 7564.
5. 24 ft. ; 400 mi.

1927.

1. (*a*) 670734472184 ; 24604661. (*b*) 117 ; 720.
2. (*a*) $\frac{11}{16}$; ·08$\dot{3}$. (*b*) £164. 5*s*. $6\frac{5}{14}$*d*. ; ·09091.
3. R13888. 2*a*. ; 8 hr.

1928.

1. (*a*) 5643 ; *A*, R457 ; *B*, R685. 8*a*. ; *C*, R514. 2*a*. ;
   (*b*) $\frac{9}{16}$, R1. 2*a*. 9*p*. per lb.
2. (*a*) £11346. 9*s*. 11$\frac{3}{4}$*d*. ; 7$\frac{1}{2}$ p. c. (*b*) 1·0003 ; $\cdot 0215\dot{9}\dot{0}$.
3. (*a*) *B* beats *A* by 1$\frac{9}{11}$ yds. and *C* by 3$\frac{8}{11}$ yds. ; R373. 3*a*. 3$\frac{15}{107}$*p*.
   (*b*) *A*, 27$\frac{1}{2}$ p. c. ; *B*, 35 p. c. ; *C*, 37$\frac{1}{2}$ p. c.

1929.

1. (*a*) 453550073305 ; 1266000. (*b*) 5438698 ; 6*s*. 4*d*.
2. (*a*) $\frac{1}{3}$; ·0074007. (*b*) £6. 6*s*. 3*d*. ; 6$\frac{1}{4}$ yr.
3. (*a*) 10$\frac{5}{8}$ p. c. loss ; at 5-20 hr.

1931.

Supplementary Paper.

1. (*a*) 4460 ; 999948, 999936, 100050, 100032. (*b*) 488432 ; ·0072.
2. (*a*) 20 p. c. ; R1680. 9*a*. 5$\frac{5}{8}$*p*. (*b*) £5380. 3. 13 weeks.

1932.

1. (*a*) 12203568. (*b*) 4$\frac{8}{25}$. (*a*) 9395200. (*b*) 11·280781...
2. (*a*) R669. 4*a*. 7$\frac{3}{7}$*p*. (*b*) R293. 5*a*. ;
   (*a*) 2·4606 ft. (*b*) R1121. 5*a*. 4*p*.
3. 58·82 p. c. ; 16$\frac{2}{3}$ min.

ANSWERS TO SCHOOL LEAVING CERTIFICATE EXAMINATION, UNITED PROVINCES.

1910.

1. (*a*) $\cdot 005208\dot{3}$ of a rupee ; (*b*) (1) ·8125 of a rupee ;
   (2) ·5625 of a rupee ;
   (3) ·13 of a rupee correct to two decimal places.
2. 1152. 3. ·8921. 4. Profit of R107. 8*a*. 5. 10 yrs.

1911.

1. (*a*) (i) 40011·43 ; (ii) ·000312 ; (*b*) (i) The right-hand side expression should be multiplied by ·1 ; (ii) ·00017 in the right-hand side expression should be multiplied by 10^6.
2. 4·42 lb. too much ; 1·06 lb. too much ; ·012 lb. too little.
3. $\frac{1}{3}$ of the mixture. 4. R1210. 1*a*. 8*p*. ; 13 p. c.

1912.

1. 326·4. 3. 2800 sq. yd.

1913.

1. 104·1. 2. $9\frac{3}{5}$ cu. ft. 3. R180.

1914.

1. R813. 6*a*. 2. *A* arrives earlier than *B* by ·18 min.
3. R5668. 6*a*. 5*p*. 4. 35 p. c.

1915.

1. R93. 12*a*. 2. 20·85 min. after his start. 3. $3\frac{1}{4}$ p. c.

1916.

1. R360. 6*a*. 5*p*. ; R320. 12*a*. 10*p*. 2. Each son's share : R1791. 0*a*. 9*p*. ; each daughter's share : R895. 8*a*. 4*p*. ; wife's share : R1343. 4*a*. 7*p*. ; each brother's share : R447. 12*a*. 2*p*. ; each uncle's share : R298. 8*a*. 1*p*. 3. $4\frac{1}{3}$ p. c.

1917.

1. (2) 3·16. 2. £6377. 7*s*. $4\frac{1}{2}$*d*. 3. 6*a*. per dozen.

1918.

1. ·00759. 2. (*a*) $12\frac{1}{2}$ p. c. ; (*b*) £195. 6*s*. 3*d*.
3. £4730. 15*s*. 9*d*.

1919.

1. (*a*) [illegible] ; (*b*) 544. 2. (*a*) [illegible] greatest ; [illegible] least ; (*b*) 10.
3. 30[illegible] cu. ft. ; 32[illegible] md. 4. (*a*) R700 ; (*b*) 25 mi.
6. 5 must be subtracted.

1920.

1. (i) 2197. (ii) 19*s*. 2[illegible]*d*. 2. (i) 391 ; (ii) 18, 12, 10.
3. 3[illegible]. 4. 12 p. c. 5. 5 P. M. 6. 4 ; 2 mi.

1921.

1. (*a*) 145·250. (*b*) R342. 14*a*. 3*p*. [illegible]
2. [illegible] 3. £14908. [illegible] 5. [illegible]

1922.

1. (1) R1. 4*a*. (2) ·308. 2. R249. 15*a*. 7*p*.
3. $\frac{2}{5}$, $\frac{1}{3}$, $\frac{4}{15}$; all will do in 4 days. 4. (1) $1\frac{1}{4}$% (2) 84 yd.

1923.

1. (i) 4·338 ; (ii) 365, 735. 2. 6 p. c. 3. £24.
4. 88 yd. ; 22 mi. 5. 18 sheep.

1924.

1. (i) $\frac{1}{2}$, (ii) 2. 2. 7 days. 3. 130. 4. R5100 6 p. c. 5. 6 ; 8.

ANSWERS TO HIGH SCHOOL EXAMINATION PAPERS OF THE BOARD OF HIGH SCHOOL AND INTER. EDUCATION, UNITED PROVINCES.

1925.

1. (*a*) 21·69. (*b*) R13. 13*a*. 10*p*. nearly. 2. R2833. 12*a*. 9*p*.
3. R384 4. R46. 15*a*. 6*p*.

1926.

1. (*a*) $\frac{2}{165}$, (*b*) 2. 2. boy, R1 ; woman, R1 8*a*. ; man, R2. 8*a*.
3. *A* gave R4000. 4. 304 ac. 9 sq. yd.

1927.

1. (*a*) R7. 14*a*. 1*p*. ; (*b*) ±·002. 2. (*a*) 1·01940 ; (*b*) 21.
3. He secured first class marks in Mathematics, Vernacular and Optional. 4. 5 p. c. ; R250.

1928.

1. (*a*) 3 ; (*b*) 88. 2. (*a*) 1 414 ; (*b*) 29[illegible] p. c. gain.
3. R81. 8*a*. 5*p*. 4. R6. 6*a*. 4*p*.

1929.

1. (*a*) ·0123... ; (*b*) R5. 7*a*. 6*p*.
2. (*a*) 1·6 ; (*b*) *A*, 162 ; *B*, 108 ; *C*, 72.
3. R251. 8*a*. 11*p*. ; 12[illegible] ac.

1930

1. (*a*) ·050 ; (*b*) [illegible] 2. (*a*) 87 ; [illegible]

3. (*a*) Man, R[illegible]; woman, R1, 8*a*. ; boy, R1 ; (*b*) [illegible]

4. (*a*) R[illegible]000 ; (*b*) R[illegible]000.

1931

1. (*a*) [illegible] ; (*b*) [illegible] hundred and eleven millions, and fourteen [illegible]

2. (*a*) 10[illegible]444 ; (*b*) R160. 10*a*. 9*p*. nearly.

3. (*a*) 6 hr. ; (*b*) R[illegible]000. 4. (*a*) R5100 ; (*b*) 288 ft.

1932

1. (*a*) $\frac{3}{8}$; (*b*) 1600. 2. (*a*) 2 mi. p. hr. ; (*b*) R750.

3. (*a*) £500 ; (*b*) R20. 13*a*. 4*p*.